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General Properties of Resistive Nonlinear One-Ports: the Role of Symmetry of Embedded Elements

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Abstract

In order to calculate the characteristic of a resistive nonlinear one-port, it is of great importance to be able to determine previously if the one-port is connected or no, especially if it is of great dimension. Purpose of this paper is to show how the symmetry of embedded nonlinear characteristics and the symmetry of the one-port topology can reflect on the properties of connection, unicursality and structural stability of the characteristic of the one-port under investigation. In particular, by means of two significant circuit structures, already considered in literature, we will see how the symmetry of nonlinear elements can guarantee the connection of the characteristic of the one-port, even if the one-port is structurally unstable.

1. Introduction

An important problem widely investigated in literature is the calculation of the characteristic of a resistive nonlinear one-port. The solution of this problem is strictly related to the general properties (for example, see [1]) satisfied by the one-port under investigation. If the one-port is unicursal (roughly speaking the characteristic can be traced by a pencil in one continuous stroke) and structurally stable, a path-following method can be adopted to draw it [2]-[3]. Instead, if the characteristic is formed by more branches crossing in some singular points, but it is still connected, a path following method is applicable if it can get over the singular points [4],[5]. In this case, the one-port may be structurally unstable. Also general methods of calculation of a characteristic have been successfully developed [6],[7], but their use is limited by the dimension of the one-port, that is by the number of embedded nonlinear elements. On the other hand, some applications require the solution of one-ports of greater dimensions [5]. So, it is of great importance to be able to determine previously if the one-port is connected or not. If the answer is affirmative, then a homotopy method is applicable.

Purpose of this paper is to show how the symmetry of embedded nonlinear characteristics and the symmetry of one-port topology can reflect on the properties of connection, unicursality and structural stability of the v-i characteristic of the one-port under investigation.

2. Preliminaries

In this paper, we will examine autonomous one-ports embedding N resistors, of which M are nonlinear voltage-controlled. Constitutive relations of nonlinear elements are cubic functions of the form: $i = \psi(v) = av^3 + bv^2 + cv + d$, that can be approximated by cubic-like PWL characteristics. The N voltages and currents of embedded elements are denoted, respectively, by N-vectors $\mathbf{v}_R$ and $\mathbf{i}_R$. Considering also variables v and i at the port, the one-port is so completely described by $N+1$ voltages and $N+1$ currents, grouped, respectively, in vectors $\mathbf{v}$ and $\mathbf{i}$.

The space $\mathcal{K} \subset \Re^{N+1} \times \Re^{N+1}$ of Kirchhoff laws is defined by the set of equations: $\mathbf{Ai} = \mathbf{0}$ and $\mathbf{Bv} = \mathbf{0}$, where $\mathbf{A}$ is the reduced incidence matrix and $\mathbf{B}$ is the fundamental loop matrix. $\mathcal{K}$ is a linear vector subspace symmetric with respect to origin. This property will play a crucial role in our paper, as we will discuss in next sections; other important symmetries may be present.

The space $\mathcal{B}_R \subset \Re^N \times \Re^N$ is an N-dimensional (ND) manifold defined by the constitutive relations of embedded components. We can so define the set $\mathcal{B} = \{(\mathbf{v}, \mathbf{i}) \in \Re^{N+1} \times \Re^{N+1} | (\mathbf{v}_R, \mathbf{i}_R) \in \mathcal{B}_R\}$, a $(N+2)$D manifold.

Configuration space $\Sigma \subset \Re^{N+1} \times \Re^{N+1}$, defined as: $\Sigma = \mathcal{K} \cap \mathcal{B}$, is formed by the set of points $\mathbf{v}$ and $\mathbf{i}$ satisfying both the one-port constitutive relations and Kirchhoff equations. The projection of Σ on plane (v, i) defines characteristic $\mathcal{C}$ of the one-port, while projection on space $\Re^M$, the space of nonlinear resistors voltages $\mathbf{v}_R$, defines curve $\mathcal{C}_M$. If the one-port is transparent [8], the properties of connection and unicursality of Σ can be transfered directly to $\mathcal{C}$. A one-port is said transparent if any point (v, i) of $\mathcal{C}$ is the projection of only one admissible point $(\mathbf{v}, \mathbf{i})$ of Σ. Otherwise, the situation must be examined carefully.

The properties of a one-port [1] can be deduced from spaces $\mathcal{K}$, $\mathcal{B}$, and Σ. The structural stability property depends on the intersection of spaces $\mathcal{K}$ and $\mathcal{B}$: if they are transversal, the one-port is structurally stable. It means that Σ does not change configuration under small perturbations of the internal resistors constitutive relations. Another very important property is unicursality. Roughly speaking, the term unicursal means that a curve can be traced by a pencil in one continuous stroke, namely the curve constitutes a 1D manifold. In our paper, all examined one-ports are regular, so they have a configuration space Σ formed by one or more 1D manifolds, also called branches. If Σ is formed by one manifold, than it is unicursal and connected; otherwise, if it is formed by two or more manifolds, it is no more unicursal, but it can be still connected, if the manifolds have some points in common. These ones are singular points, since the jacobian matrix (associated to Σ) has not maximum rank. Note that all these properties of Σ can be transposed to characteristics $\mathcal{C}$ and $\mathcal{C}_M$.

These properties are important when characteristic $\mathcal{C}$ must be calculated and drawn, since the choice of the algorithm depends on them. If $\mathcal{C}$ is regular and unicursal, a path-following algorithm can be adopted, while if $\mathcal{C}$ is regular, not unicursal but conected, the algorithm must be able to deal with singular points. At last, if $\mathcal{C}$ is regular but not connected, a general algorithm must be adopted.

The nonlinear characteristics have been approximated with PWL functions, such that one-ports characteristics have been calculated with a computer program written in C++ language, based on [6].

3. Series connection of two tunnel diodes

Let us consider a resistive one-port $\mathcal{S}_1$ composed by the series connection of two tunnel diodes $D_1 : i_1 = \psi_1(v_1) = 0.43v_1^3 - 2.69v_1^2 + 4.56v_1$ and $D_2 : i_2 = \psi_2(v_2) = 2.5v_2^3 - 10.5v_2^2 + 11.8v_2$. Σ is a regular curve in $\Re^3 \times \Re^3$ space ($N = M = 2$), described by the following 5 equations: $v = v_1 + v_2$; $i = i_1 = i_2$; $i_1 = \psi_1(v_1)$, $i_2 = \psi_2(v_2)$. Each diode has been approximated by a PWL characteristic of 10 equi-spaced segments. The v-i characteristic $\mathcal{C}$, projection of Σ, is shown in Fig. 1, while curve $\mathcal{C}_2$ in Fig. 2. Examining the two curves, it is clear that $\mathcal{C}_2$ is not connected, but it is composed by two separate branches. Of course, also $\mathcal{C}$ is not connected, because it is a direct function of $\mathcal{C}_2$. Furthermore, It is easy to check that the internal resistors constitutive relations and the space of Kirchhoff laws are transversal, that is $\mathcal{C}_2$ (and $\mathcal{C}$) is structurally stable. This configuration is due to the relative position of spaces $\mathcal{B}$ and $\mathcal{K}$. The close branch, clearly shown in Fig. 2, is located around the local minima and maxima of ψ_1 and ψ_2: indeed, quite these convex and concave pieces of $\mathcal{B}$ give rise, by the intersection with the linear space $\mathcal{K}$, to it. So, the main features of this example are: cubic functions non symmetric respect to the origin (local maxima and minima set at the same side with respect the origin) and different for the two internal nonlinear resistors, absence of any other topological symmetry. The result is that $\mathcal{C}$ is regular, but not connected (nor unicursal). If we set $\psi_2(v_2) = \psi_1(v_2)$, we obtain a characteristic $\mathcal{C}$ not unicursal, formed by two branches, but connected, with two singular points. The close branch has been shifted towards the open branch.

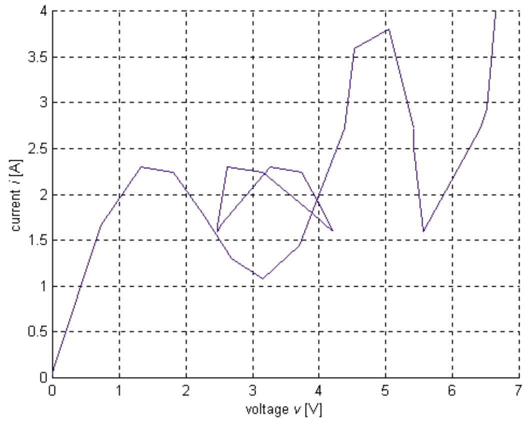

Figure 1: Characteristic $\mathcal{C}$ of one-port $\mathcal{S}_1$.

If we shift the constitutive relations of both resistors D_1 and D_2 setting, respectively, the origin of axes in their flex points, we obtain one-port $\mathcal{S}_2$. Both $\mathcal{C}$ and $\mathcal{C}_2$, shown respectively in Fig. 3 and 4, become connected, being formed by one single branch. $\mathcal{S}_2$ is also structurally stable, for the same reasons exposed for $\mathcal{S}_1$. Note that a consequence of structural stability is that both $\mathcal{C}_2$ and $\mathcal{C}$ form a 1D manifold in $\Re^2$. Furthermore, the jacobian matrix of $\mathcal{C}_2$ (of Σ) has always full rank, that is there are no singular points in the curve. It is so easy to draw $\mathcal{C}$ by means of a homotopy method, while is not suitable to draw $\mathcal{C}$ of $\mathcal{S}_1$, because the two branches are completely separated. In this case, the main features are: cubic functions symmetric respect to the origin and different for the two internal nonlinear resistors, absence of any other topological symmetry. Symmetry of internal constitutive relations is sufficient to guarantee connection.

4. Autonomous nonlinear rings

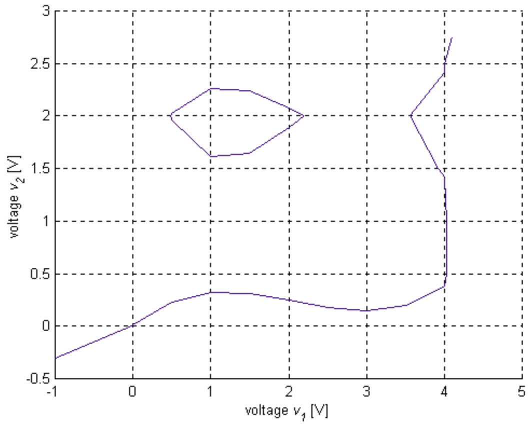

Figure 2: Curve $\mathcal{C}_2$ of one-port $\mathcal{S}_1$.

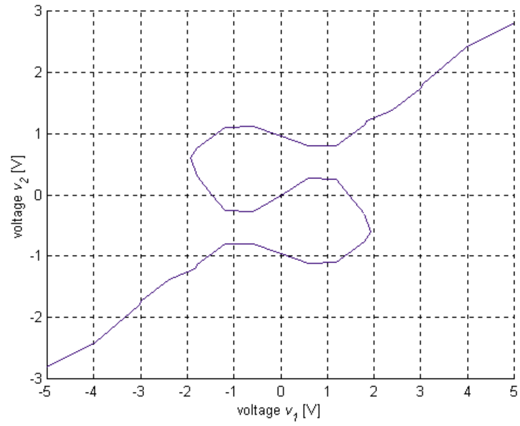

Figure 4: Curve $\mathcal{C}_2$ of one-port $\mathcal{S}_2$.

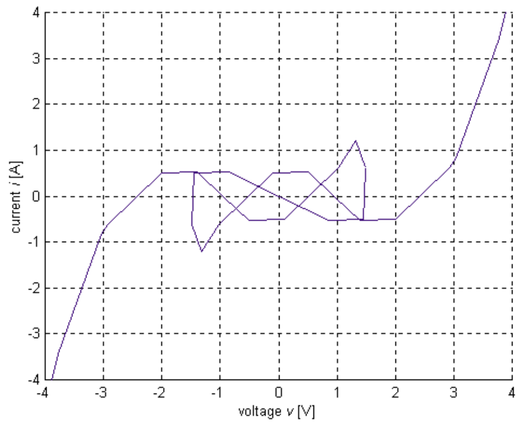

Figure 3: Characteristic $\mathcal{C}$ of one-port $\mathcal{S}_2$.

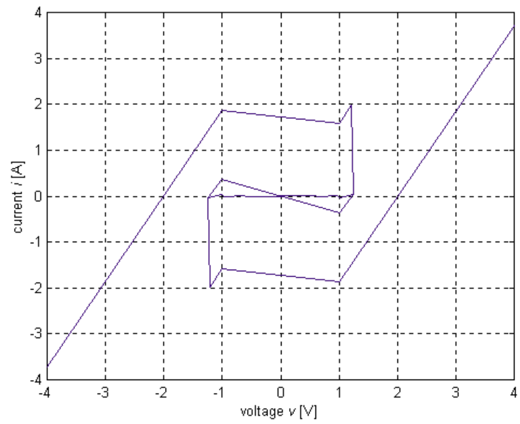

Figure 5: Characteristic $\mathcal{C}$ of one-port $\mathcal{P}_1$.

Let us consider a resistive one-port [5], composed by 5 shunt nonlinear resistors $\mathcal{R}_m$ coupled by 5 positive linear resistors. All the 5 resistors $\mathcal{R}_m$ have the same cubic characteristic $i = \psi(v) = av^3 - bv$, with $a, b > 0$. Resistor R_1 acts as a parameter, while the other 4 linear resistors have the same fixed value R.

In our example, Let us consider one-port $\mathcal{P}_1$, with $R = R_1 = 1.6\ \Omega$, while the cubic-like PWL characteristic is formed by 3 segments with two finite breakpoints at (-1,1) and (1,-1). The first and last segments are infinite with slope 1. One-port $\mathcal{P}_1$ presents a high level of symmetry, both in internal constitutive relations and in topology. Indeed, it is easy to prove that $\mathcal{C}_5$ is symmetric with respect to the line defined by $v_{M-1} = v_1$, $v_{M-2} = v_2$, ... Therefore, we can guess that Σ is connected, even if not unicursal. Characteristic $\mathcal{C}$ in Fig. 5 and the projection on plane (v_1, v_4) of $\mathcal{C}_5$ in Fig. 6 confirm this assumption, showing that there are two branches, one close and the other open. The characteristic is now structurally unstable, with the presence of two singular points.

If we break the symmetry of linear resistors values, by setting $R_1 = 4.8\ \Omega$, we obtain one-port $\mathcal{P}_2$. Its characteristic $\mathcal{C}$ is shown in Fig. 7. It is unicursal, connected, without singolar points, because they have been broken by changing linear resistor R_1 value. We can break singular points of $\mathcal{P}_1$ also modifying the above PWL characteristic (R_1 unchanged), shifting one finite breakpoint from (-1,1) to (-1.5,1.0) and setting the slope of the first infinite segment to 0.5. We obtain one-port $\mathcal{P}_3$, whose characteristic $\mathcal{C}$ is shown in Fig. 8. Note that $\mathcal{C}$ is unicursal and structurally stable, because nonlinear constitutive relations are not symmetric with respect the origin, but they have still the local maxima and minima set around the origin. This is sufficient to guarantee connection and unicursality.

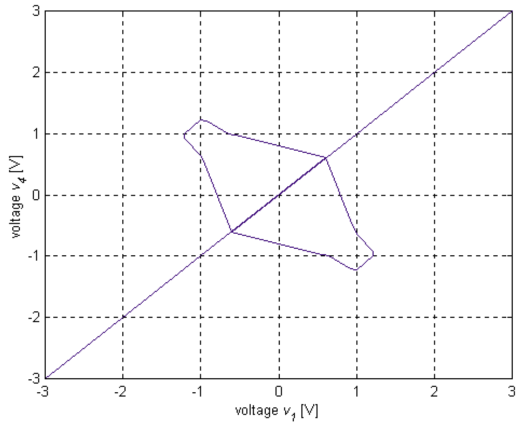

Figure 6: Projection of $\mathcal{C}_5$ of one-port $\mathcal{P}_1$.

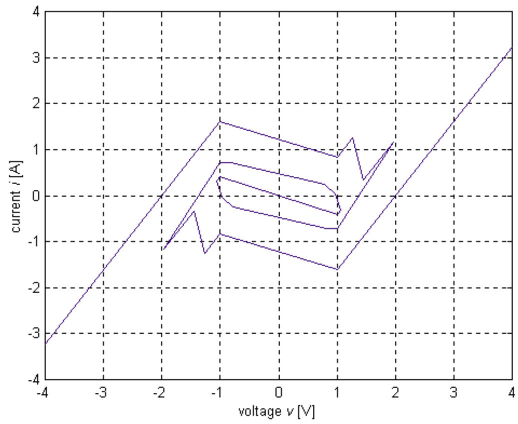

Figure 7: Characteristic $\mathcal{C}$ of one-port $\mathcal{P}_2$.

5. Conclusions

We have seen how the symmetry of nonlinear elements can influence the connection of the characteristic of a one-port, both if it is structurally stable and unstable. Furthermore, the breaking of symmetry, for instance changing the value of a linear resistor, can make characteristic unicursal and stable, because of the breaking of the singular points. A further devolopment can involve a time-simulation of these one-ports, comparing the results obtained when the one-port under investigation is unicursal or not.

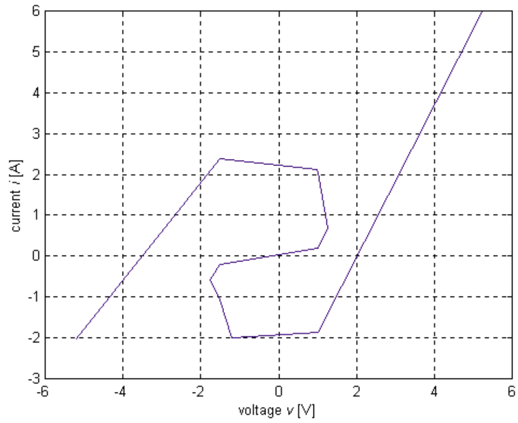

Figure 8: Characteristic $\mathcal{C}$ of one-port $\mathcal{P}_3$.

References

[1] L. O. Chua, T. Matsumoto, and S. Ichiraku, "Geometric Properties of Resistive Nonlinear n-Ports: Transversality, Structural Stability, Reciprocity, and Anti-Reciprocity," IEEE Trans. Circuits Syst., vol. CAS-27, no. 7, pp. 577-603, July 1980.

[2] E. Ikeno and A. Ushida,"The arc-length method for the computation of characteristic curves", IEEE Trans. Circuits Syst., vol. 23, March 1976, pp. 181-183.

[3] A. Ushida and L. O. Chua.,"Tracing solution curves of nonlinear equations with sharp turning points", Inter. Journal of Circuit Theory and Applications, Vol, 12, pp. 1-21, 1984.

[4] A. Ushida,"Analysis of Bifurcation Points of Nonlinear Resistive Circuits by Curve Tracing Method", Proc. of International Symposium on Circuits and Systems, London, 1994, vol. 6, pp. 121-124.

[5] S. Pastore, "Detection of All the Equilibrium Points of Dynamic 1D Autonomous Rings", Proceedings of the 14th European Conference on Circuit Theory and Design, ECCTD '99, Stresa, Italy, August 29 - September 2, 1999, Part II, pp. 699-702.

[6] S. Pastore and A. Premoli, "Capturing All Branches of Any One-Port Characteristic in Piecewise-Linear Resistive Circuits", IEEE Trans. Circuits Syst., Part I, Vol. CAS-43, No. 1, pp. 26-33, January 1996.

[7] L. Vandenberghe, B. L. De Moor and J. Vandewalle, "The Generalized Linear Complementarity Problem applied to the Complete Analysis of Resistive Piecewise-Linear Circuits", IEEE Trans. Circuits Syst., vol. 36, no. 11, pp. 1382-1391, November 1989.

[8] M. Hasler and J. Neirynck, "Nonlinear Circuits", Norwood, MA, Artech House, 1986.