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Simulation of transmission lines terminated with nonlinear resistors by wave digital filters

Masato OGATA and Tetsuo NISHI

† Department of Computer Science and Communication Engineering
Kyushu University
6-10-1 Hakozaki, Higashi-ku, Fukuoka, Japan
Phone:+81-92-642-3852, Fax:+81-92-642-4081
Email: {mogata,nishi}@csce.kyushu-u.ac.jp

1. Introduction

The increasing developments in high-speed networks highlight the previously negligible effects of interconnects, such as ringing, signal delay, distortion, reflections, and crosstalk. It is predicted that the signal degradation in high-speed networks is mainly caused by these effects of interconnects[1]. The term "high-speed" can be defined in terms of the signals frequency. If an impressed trapezoidal pulse has a rise/fall time of 0.1ns, the maximum frequency of the signal is approximately 10 GHz, that is, the wave length is about 3 cm. This implies that an ordinary wire presumed to effectively short two connected circuits at low frequency has too much capacitive/inductive effects at present.

These interconnect effects can cause logic glitches and analog signal distortion such that it fails to meet specification, accurate prediction of these effects in the entire high-speed networks is a necessity for designs. The interconnects should be modeled as distributed-constant networks, while logic circuits are regarded as nonlinear lumped-constant networks. Thus we consider that it is important to simulate the mixed lumped- and distributed-constant networks with transmission lines efficiently and accurately.

There are many methods to simulate these mixed lumped- and distributed-constant networks, e.g. asymptotic waveform evaluation (AWE), asymptotic equivalent circuit (AEC), and numerical inverse Laplace transformation, etc. The finite-difference time-domain (FDTD) methods are also applied. While most FDTD methods are based on Yee's algorithm[2], Fettweiss proposed the FDTD methods based on the principle of wave digital filters(WDFs)[3],[4]. WDFs are known to have very good properties concerning stability under finite-arithmetic conditions due to their inherent passivity[5]. Some papers report that more accurate numerical solutions of Maxwell's equation are obtained by the WDFs method in comparison with other FDTD methods[7]. Nonlinear lumped-constant circuits are also simulated by the WDFs method [8]. In this paper we apply the FDTD method based on the MD WDFs to simulate transmission lines terminated by linear/nonlinear lumped-constant elements.

2. Preliminaries

In this paper, wave quantities are adopted as signal parameters instead of voltages and currents. When v and i are the voltage and the current at a port, respectively, the incident and the reflected waves are defined by

$$a = v + Ri \qquad \text{and} \qquad b = v - Ri, \qquad (1)$$

respectively, where R is usually a positive constant, called port resistance. In this section, the realization of several elements by wave quantities is mentioned.

2.1. Linear building blocks

Linear elements and sources used in classical electrical circuits can be simulated by the principles of WDFs. As examples, we consider only a resistance, an inductance, and parallel/series interconnects.

The voltage-current characteristic of a resistance R_0 is

$$v(t) = R_0 i(t). \qquad (2)$$

If the port resistance is equal to the element value R_0, then we have

$$b(t) = 0. \qquad (3)$$

The $v - i$ characteristic of an inductance L_0 is

$$v(t) = L_0 \frac{\mathrm{d}i(t)}{\mathrm{d}t} \qquad (4)$$

Application of the trapezoidal rule amounts to replacing Eq.(4) by

$$v(t) + v(t-T) = R\{i(t) - i(t-T)\} \qquad (5)$$

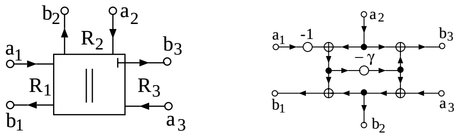

Figure 1: 3-port parallel adaptor and signal-flow diagram

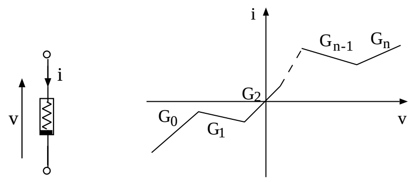

Figure 2:

where $R = \frac{2L_0}{T}$ and T is a positive shift of t. From the definitions of wave quantities, Eq.(5) can be written as

$$b(t) = -a(t-T). \tag{6}$$

The simulation of the interconnections present in the reference circuit is achieved by the so-called adaptors. An n-port parallel (resp. series) adaptor serves to simulate a parallel (resp. series) connection of n-ports. In Fig.1, there is an example of a 3-port parallel adaptor where $R_3 = \frac{R_1 R_2}{R_1+R_2}$ and $\gamma = \frac{R_3}{R_1}$.

2.2. Realization of two-dimensional (2D) elements

When we simulate the transmission lines by WDF, there are 2D elements in the reference circuit from which the WDF is derived. The $v-i$ characteristic of the 2D element is described by

$$v(x,t) = \left(\pm\alpha\frac{\partial}{\partial x} + \beta\frac{\partial}{\partial t}\right) i(x,t) \tag{7}$$

where α and β are positive constant. Integrating Eq.(7) along the line l connecting points $(x \mp X, t-T)$ and (x,t) and applying the generalized trapezoidal rule, we have

$$\begin{aligned} &\frac{\sqrt{X^2+T^2}}{2}\{v(x,t) + v(x \mp X, t-T)\} \\ &\quad = \sqrt{X^2+T^2}\frac{\alpha}{X}\{i(x,t) - i(x \mp X, t-T)\} \end{aligned} \tag{8}$$

where positive constants α and β are assumed to satisfy $\frac{\alpha}{X} = \frac{\beta}{T}$. If the port resistance R is chosen equal to $\frac{2\alpha}{X}$, then we obtain

$$b(x,t) = -a(x \mp X, t-T). \tag{9}$$

2.3. Realization of nonlinear resistors[8]

Here we mention the realization of nonlinear resistors according to Meerkötter-Scholz[8]. Assume the current i is expressed as a single-valued piece-wise linear function of the voltage v with $n+1$ segments as shown in Fig.2. If the port resistance R has to meet one of the following two inequalities:

$$\frac{1}{R} > -\min_{\nu} G_\nu \qquad\qquad \frac{1}{R} < -\max_{\nu} G_\nu, \tag{10}$$

then, the realization of the nonlinear resistance in terms of wave quantities can be written in the form[8]

$$b(a) = b(0) + c_0 a + \sum_{\nu=1}^{n} c_\nu(|a - a_\nu| - |a_\nu|) \tag{11}$$

where c_0, c_ν, a_ν, and $b(0)$ derived from the slopes G_ν are given as

$$c_0 = (\varrho_0 + \varrho_n)/2 \qquad c_\nu = (\varrho_\nu - \varrho_{n-1})/2,\ \nu = 1 \text{ to } n$$
$$\varrho_\nu = (1 - RG_\nu)/(1 + RG_\nu),\ \ \nu = 0 \text{ to } n$$
$$a_\nu = v_\nu + Ri(v_\nu),\ \ \nu = 1 \text{ to } n$$
$$b(0) = -(1 + c_0)Ri(0) - \sum_{\nu=1}^{n} c_\nu(|Ri(0) - a_\nu| - |a_\nu|).$$

3. FDTD method for transmission lines base on MD WDFs

In this section we derive the FDTD method based on the principle of MD WDFs for simulating transmission lines.

A single transmission line with the length d can be written by the following telegraph equation:

$$-\frac{\partial v}{\partial x} = L\frac{\partial i}{\partial t} + Ri, \qquad -\frac{\partial i}{\partial x} = C\frac{\partial v}{\partial t} + Gv \tag{12}$$

where L, C, R, and G are assumed be time- and space-invariant for the sake of simplicity. We hereafter define two partial differential operators D_x and D_t:

$$D_x \equiv \frac{\partial}{\partial x} \qquad\qquad D_t \equiv \frac{\partial}{\partial t}. \tag{13}$$

Replacing $i(x,t)$ and $v(x,t)$ by $i_a(x,t)$ and $ri_b(x,t)$, respectively, where r is an arbitrary positive value, Eq.(12) can equivalently be expressed as

$$\begin{pmatrix} LD_t + R & rD_x \\ rD_x & r^2(CD_t + G) \end{pmatrix} \begin{pmatrix} i_a \\ i_b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{14}$$

Regarding i_a and i_b as loop currents, we obtain a multidimensional reference circuit as shown in Fig.3, satisfying Eq.(14) as the mesh equation. In Fig.3, we introduce

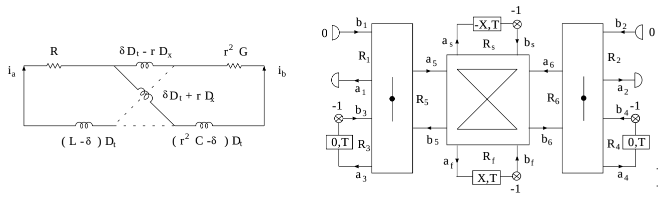

Figure 3: Reference circuit and a MD WDF for single transmission lines

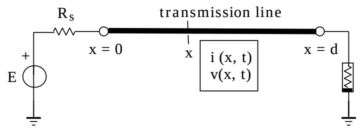

Figure 4: Mixed lumped- and distributed-constant circuit

an arbitrary positive constant δ and this constant δ satisfies:

$$0 \leq \delta \leq L \qquad \text{and} \qquad 0 \leq \delta \leq r^2 C. \tag{15}$$

From the conditions in (15), all inductances in Fig.3 are positive. This implies that all port resistances of the corresponding WDF are positive and full robustness of numerical analysis is achieved[3].

From the reference circuit in Fig.3, the WDF arrangement in Fig.3 is immediately obtained by applying procedures mentioned in Sec.2. Now we can simulate the transmission line in Eq.(12) by the FDTD method based on the principle of MD WDF in Fig.3, i.e. wave quantities are updated according to MD WDF.

4. Realization of boundary conditions

In this section we consider the realization for the lumped-constant networks at the boundaries $x = 0$ and $x = d$ by the MD WDF. Here we treat only the output port ($x = d$) because the realization of the input port ($x = 0$) can be treated in a similar way as in the output port.

As an example, let us consider a mixed lumped- and distributed-constant circuit in Fig.4. This transmission line is terminated by the voltage-controlled nonlinear resistance at the output port. Assume the the current i of the nonlinear resistor can be expressed as a single-valued function of the voltage v, i.e.,

$$i(d, t) = f(v(d, t)) \tag{16}$$

Eq.(16) can be rewritten as

$$i_a(d, t) = f(ri_b(d, t)). \tag{17}$$

Eq.(17) can be represented as the circuit consisting of ideal gyrators, ideal transformers, a current-controlled voltage source, and the nonlinear resistor as shown in Fig.5 where $R_a = R_1 + R_3 + R_5$, $R_b = R_2 + R_4 + R_6$, $m = 1/R_a$, and $n = r/R_b$. Applying the wave quantities to the circuit in Fig.5, we get the MD WDF at $x = d$ in Fig.5. Using these proposed MD WDFs at the boundaries, we can derive the FDTD method for the mixed lumped- and distributed-constant circuits in Fig.fig:fig6.

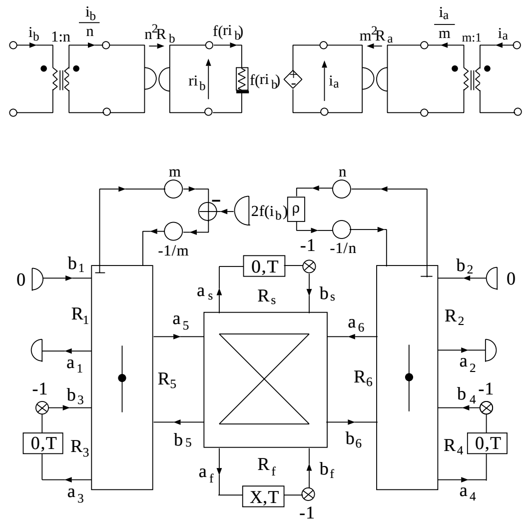

Figure 5: Reference circuit and MD WDF for the boundary at $x = d$

5. Numerical examples

5.1. Example 1

Let us consider the circuit in Fig.4 with $R_S = 0(\Omega)$. Per-unit-length parameters are given as:

$$\begin{array}{ll} R = 0(\Omega/m) & L = 1.0 \times 10^{-6}(H/m) \\ G = 0(\mho/m) & C = 1.0 \times 10^{-10}(F/m). \end{array}$$

The length of the transmission line denoted by d is $0.25(cm)$. Here we assume that the transmission line is terminated by the linear resistor R_L. The output voltage $v(d, t)$ is shown in Fig.6.

5.2. Example 2

Let us consider the circuit in Fig.4. Per-unit-length parameters, the length d and the resistor R_s are the same values as in Example 1. The transmission line is assumed to be terminated by the passive nonlinear resistor whose $v - i$ characteristic is given as:

$$i = f(v) = 1.5 \times 10^{-2} v + 0.5 \times 10^{-2} \{|v - 0.5| - 0.5\}.$$

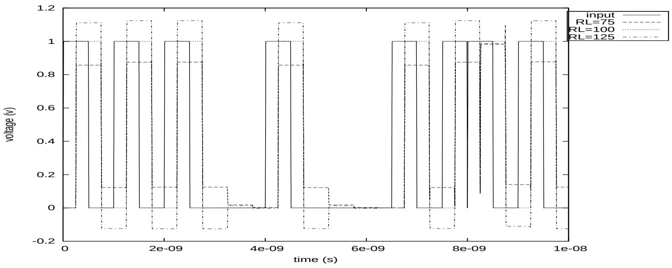

Figure 6: v(d,t) in Example 1

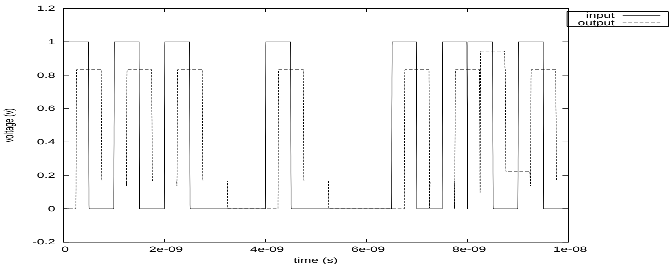

Figure 7: v(d,t) in Example 2

The output voltage is illustrated in Fig.7.

5.3. Example 3

Let us simulate the circuit in Fig.4 terminated by the locally active nonlinear resistor. Per-unit-length parameters, the length d and the resistor R_s are the same values as in Example 1. The $v-i$ characteristic of the terminated nonlinear resistor is shown in Fig.8. The output voltage is illustrated in Fig.9.

6. Conclusion

This paper gives the FDTD method based on the principle of MD WDFs for simulating mixed lumped- and distributed-circuits. We propose the MD WDF for the boundary condition with nonlinear lumped-constant elements(Fig.5).

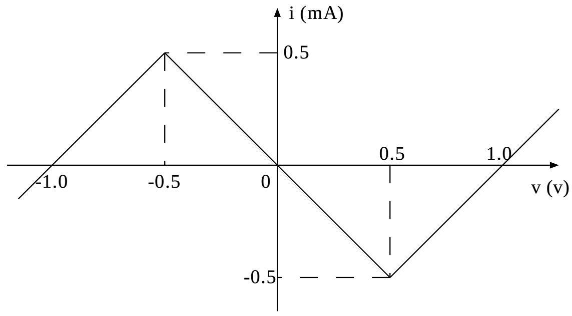

Figure 8:

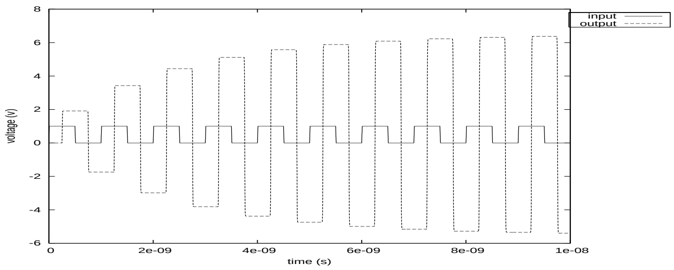

Figure 9: v(d,t) in Example 3

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