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Multi-User Detection Techniques for Multiple Access Chaos-Based Digital Communication Systems

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Abstract

In a multiple access spread-spectrum communication system, the performance is usually limited by the mutual interference between users. The use of multiuser detection is an effective technique to reduce such interference. In this paper, the bit error performance of a multiple access chaos-shift-keying communication system employing multiuser detectors is studied. Two types of detectors, namely decorrelating detector and minimum mean-square-error detector, are evaluated. An approach to calculating the bit error rates (BERs) is also presented. Finally, brute-force simulations are performed to verify the predicted BERs.

Keywords — Chaos communication, multiple access, multi-user detection.

1. Introduction

Several chaos-based techniques to provide spread-spectrum digital communications have been studied recently [1]. Upon application of chaotic modulation, the original digital signal spreads to occupy a much wider bandwidth. As a consequence, there is spare capacity to accommodate more users within this bandwidth [2, 3]. Due to the non-orthogonal nature of the chaotic signals employed by different users, the system performance is normally limited by the interference between users. To reduce such interference, multi-user detection schemes can be applied. The analysis and performances of the multiuser detectors when applied to chaos-based spread-spectrum communication systems, however, are not available in the literature.

In this paper, two multi-user detection schemes for conventional direct-sequence code-division MA systems, namely decorrelating detector (DD) and minimum mean-square-error (MMSE) detector [4, 5], are employed for the detection of a MA antipodal chaos-shift-keying (MA-CSK) communication system. In particular, we use a simple cubic map to generate the chaotic sequences for all users. Due to the aperiodic nature of the chaotic sequences, the auto-correlation and cross-correlation of the chaotic sequences vary temporally and the conventional analytical methods do not apply. An approach to calculating the bit error rates is developed for a 2-user system. For systems with more users, a mixed analysis-simulation technique applies. For comparison and verification, we also present the bit error rates calculated from brute-force simulations which give the true system performance.

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2. System Description

2.1. Transmitter Structure

Consider the N-user antipodal MA-CSK communication system shown in Fig. 1. Denote the chaotic sequence generated by the ith generator by $\{x_k^{(i)}\}$. If the transmitted symbol is "+1", $\{x_k^{(i)}\}$ is transmitted; otherwise, $\{-x_k^{(i)}\}$ is sent. Define $d_m^{(i)} \in \{-1, +1\}$ as the mth transmitted symbol for user i which is assumed to be equiprobable for "+1" and "−1". Assuming a spreading factor of β, the transmitted signal for user i at time k ($k = (m-1)\beta+1, (m-1)\beta+2, \ldots, m\beta$) can be expressed as $v_k^{(i)} = d_m^{(i)} x_k^{(i)}$. The total transmitted signal of the system at time k is given by

$$v_k = \sum_{i=1}^{N} v_k^{(i)}. \tag{1}$$

2.2. Multiuser Receiver

The received signal is given by $r_k = v_k + \xi_k$ where ξ_k is the additive white Gaussian noise with zero mean and variance $\frac{N_0}{2}$. The synchronized chaotic sequences are assumed to be reproduced at the receiver, as shown in Fig. 1. The output of each of the correlators ($j = 1, 2, \ldots, N$) is given by

$$z_m^{(j)} = \sum_{k=(m-1)\beta+1}^{m\beta} r_k x_k^{(j)}. \tag{2}$$

Define $\boldsymbol{R}_m$ as the correlation matrix, the (i,j)th element of which, denoted by $R_{i,j,m}$, equals $\sum_{k=(m-1)\beta+1}^{m\beta} x_k^{(i)} x_k^{(j)}$ and corresponds to the correlation between users i and j. Note that in a chaos-based digital communication system,

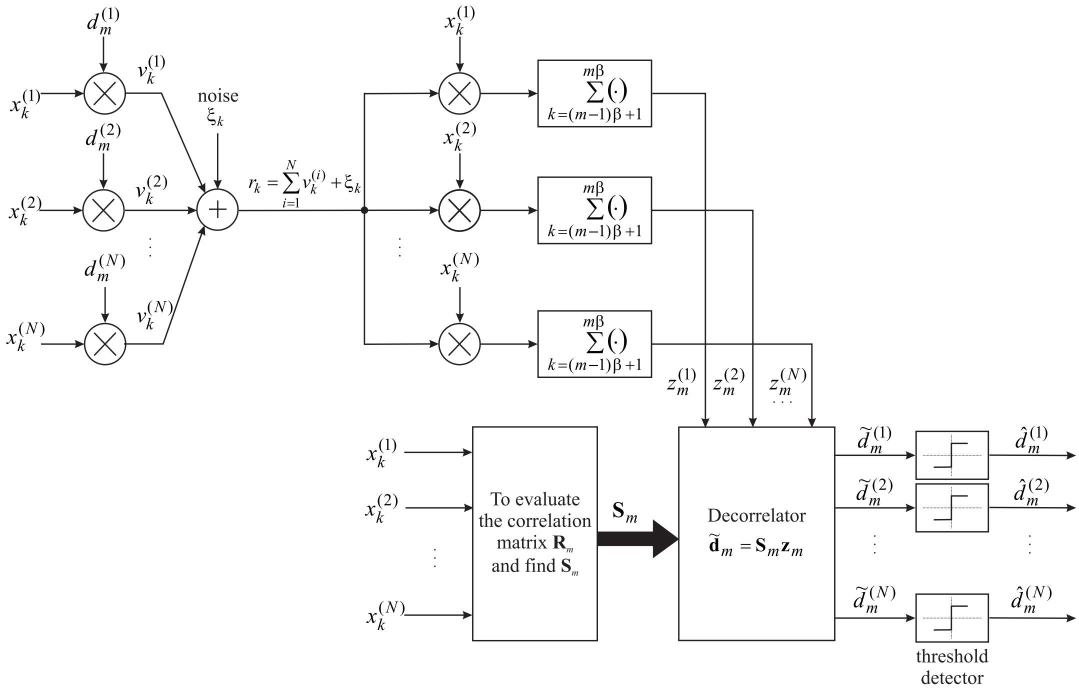

Fig. 1: Multiuser detection of a multiple access chaos-shift-keying digital communication system.

the correlation matrix $\boldsymbol{R}_m$ changes from bit to bit. Therefore, $\boldsymbol{R}_m$ needs to be re-calculated for every bit duration. The outputs of the correlator can now be expressed as

$$\boldsymbol{z}_m = \boldsymbol{R}_m \boldsymbol{d}_m + \boldsymbol{n}_m \tag{3}$$

where

$$\boldsymbol{z}_m = \begin{bmatrix} z_m^{(1)} & z_m^{(2)} & \cdots & z_m^{(N)} \end{bmatrix}^T \tag{4}$$

$$\boldsymbol{d}_m = \begin{bmatrix} d_m^{(1)} & d_m^{(2)} & \cdots & d_m^{(N)} \end{bmatrix}^T \tag{5}$$

$$\boldsymbol{n}_m = \begin{bmatrix} n_m^{(1)} & n_m^{(2)} & \cdots & n_m^{(N)} \end{bmatrix}^T \tag{6}$$

$$n_m^{(j)} = \sum_{k=(m-1)\beta+1}^{m\beta} \xi_k x_k^{(j)} \tag{7}$$

and T denotes matrix transposition.

2.2.1. Conventional Single-user Detector

For the conventional single-user detector, the symbols are decoded using

$$\hat{\boldsymbol{d}}_m = \text{sgn}[\boldsymbol{z}_m] \tag{8}$$

and sgn[.] represents the sign function. In other words, if $z_m^{(j)} > 0$, "+1" is detected for user j; otherwise, "-1" is decoded.

2.2.2. Decorrelating Detector (DD)

For a given $\boldsymbol{d}_m$, we assume that $\boldsymbol{z}_m$ is jointly Gaussian with mean $\boldsymbol{R}_m \boldsymbol{d}_m$ and covariance matrix $\frac{N_0}{2}\boldsymbol{R}_m$. The N-dimensional probability density function is then given by

$$p(\boldsymbol{z}_m|\boldsymbol{d}_m) = \frac{1}{\sqrt{(\pi N_0)^N \det[\boldsymbol{R}_m]}} \exp\Big[-\frac{1}{N_0}(\boldsymbol{z}_m - \boldsymbol{R}_m \boldsymbol{d}_m)^T \boldsymbol{R}_m^{-1}(\boldsymbol{z}_m - \boldsymbol{R}_m \boldsymbol{d}_m)\Big]. \tag{9}$$

where det[.] denotes the determinant operator. The optimum solution that maximizes the probability of a correct decision is equivalent to that minimizes the likelihood function

$$\Omega(\boldsymbol{d}_m) = (\boldsymbol{z}_m - \boldsymbol{R}_m \boldsymbol{d}_m)^T \boldsymbol{R}_m^{-1}(\boldsymbol{z}_m - \boldsymbol{R}_m \boldsymbol{d}_m), \tag{10}$$

from which we obtain

$$\tilde{\boldsymbol{d}}_m = \boldsymbol{R}_m^{-1} \boldsymbol{z}_m \tag{11}$$

where $\tilde{\boldsymbol{d}}_m = [\ \tilde{d}_m^{(1)} \quad \tilde{d}_m^{(2)} \quad \cdots \quad \tilde{d}_m^{(N)}\ ]^T$. The symbols are then decoded from

$$\hat{\boldsymbol{d}}_m = \text{sgn}[\tilde{\boldsymbol{d}}_m]. \tag{12}$$

2.2.3. Minimum Mean-Square-Error (MMSE) Detector

An MMSE detector balances the effect of noise and multi-user interference. The main principle is to find a mapping $\boldsymbol{H}_m$ minimizing the mean-square-error $J(\boldsymbol{d}_m)$ between the actual and the estimated transmitted symbols. The equation linking $\boldsymbol{H}_m$ and $J(\boldsymbol{d}_m)$ is given by

$$\begin{aligned} J(\boldsymbol{d}_m) &= \mathrm{E}[(\boldsymbol{d}_m - \hat{\boldsymbol{d}}_m)^T(\boldsymbol{d}_m - \hat{\boldsymbol{d}}_m)] \\ &= \mathrm{E}[(\boldsymbol{d}_m - \boldsymbol{H}_m \boldsymbol{z}_m)^T(\boldsymbol{d}_m - \boldsymbol{H}_m \boldsymbol{z}_m)] \end{aligned} \tag{13}$$

where $\mathrm{E}[\psi]$ denotes the mean value of ψ. The mapping that satisfies the requirement can be shown equal to

$$\boldsymbol{H}_m = (\boldsymbol{R}_m + \frac{N_0}{2}\boldsymbol{I})^{-1} \tag{14}$$

where $\boldsymbol{I}$ is the identity matrix. The best choice for $\boldsymbol{d}_m$ to make a decision is thus

$$\tilde{\boldsymbol{d}}_m = \boldsymbol{H}_m \boldsymbol{z}_m \tag{15}$$

and the symbols are decoded according to

$$\hat{\boldsymbol{d}}_m = \mathrm{sgn}[\tilde{\boldsymbol{d}}_m]. \quad (16)$$

In general, the decoded symbols in the linear multiuser detector shown in Fig. 1 can be expressed as

$$\hat{\boldsymbol{d}}_m = \mathrm{sgn}[\boldsymbol{S}_m \boldsymbol{z}_m] \quad (17)$$

where $\boldsymbol{S}_m$ is the linear mapping. For the conventional single-user, decorrelating and MMSE detectors, $\boldsymbol{S}_m$ is represented by $\boldsymbol{I}$, $\boldsymbol{R}_m^{-1}$ and $(\boldsymbol{R}_m + \frac{N_0}{2}\boldsymbol{I})^{-1}$, respectively. In a noiseless condition, the MMSE detector is equivalent to the decorrelating detector. When N_0 is very large, the MMSE detector is similar to the conventional single-user detector.

3. Performance Analysis

We assume that the cubic map given by

$$x_{k+1} = 4x_k^3 - 3x_k \quad (18)$$

is used by all users, and each uses a different initial condition. For the conventional single-user detector, the analytical bit error rates (BERs) have been derived in [2]. For the DD and MMSE detectors, we have developed techniques to find the BERs for the 2-user systems, and in general, the N-user systems. For the N-user systems, in particular, we make use of both analysis and simulation to find the BERs (refer to as "mixed analysis-simulation" (MAS) technique). Due to the shortage of space, we only state the BER equations for the N-user systems here.

3.1. Decorrelating Detector (DD)

Define P_s as the average power given by

$$P_s = \mathrm{E}[(x_k^{(1)})^2] = \mathrm{E}[(x_k^{(2)})^2] = \cdots = \mathrm{E}[(x_k^{(N)})^2], \quad (19)$$

and denote the average bit energy βP_s by E_b. For an N-user system with $N > 2$, the BER of user j is given by

$$\mathrm{BER}^{(j)} = \frac{1}{2}\mathrm{erfc}\left(\left[E_b\left(\frac{E_b}{N_0}\right)^{-1}\mathrm{E}[P_{j,j,m}]\right]^{-\frac{1}{2}}\right) \quad (20)$$

where $P_{j,j,m}$ denotes the (j,j)th element of $\boldsymbol{P}_m$, which is the inverse of the correlation matrix $\boldsymbol{R}_m$, and erfc(.) represents the complimentary error function. In (20), to find the BER, we need to first attain the mean value of term $P_{j,j,m}$ over all m by simulation.

3.2. MMSE Detector

For an N-user system, given a particular $\boldsymbol{d}_m$, the mean and variance of $\tilde{d}_m^{(j)}$ are given, respectively, by

$$\mathrm{E}[\tilde{d}_m^{(j)}|\boldsymbol{d}_m] = d_m^{(j)} - \frac{N_0}{2}\sum_{i=1}^{N} d_m^{(i)}\mathrm{E}[H_{j,i,m}] \quad (21)$$

and

$$\begin{aligned}\mathrm{var}[\tilde{d}_m^{(j)}|\boldsymbol{d}_m] &= \frac{N_0}{2}\mathrm{E}[H_{j,j,m}] - \frac{N_0^2}{4}\sum_{i=1}^{N}\mathrm{E}^2[H_{j,i,m}] \\ &+ \frac{N_0^2}{4}\sum_{i=1}^{N}\sum_{u=1,u\neq i}^{N} d_m^{(i)}d_m^{(u)}\mathrm{cov}\left[H_{j,i,m}, H_{j,u,m}\right]\end{aligned} \quad (22)$$

where $H_{j,i,m}$ denotes the (j,i)th element of $\boldsymbol{H}_m$ and $\mathrm{cov}[A,B]$ represents the covariance of A and B. By making use of simulations to find the values of the mean and covariance terms, and assuming $\tilde{d}_m^{(j)}|\boldsymbol{d}_m$ is normal, the BER of user j is then evaluated from

$$\begin{aligned}&\mathrm{BER}^{(j)} \\ &= \frac{1}{2^{N+1}}\sum_{\check{\boldsymbol{d}}_m^{(j)}}\left[\mathrm{erfc}\left(\frac{\mathrm{E}\left[\tilde{d}_m^{(j)}|d_m^{(j)}=+1,\check{\boldsymbol{d}}_m^{(j)}\right]}{\sqrt{2\mathrm{var}\left[\tilde{d}_m^{(j)}|d_m^{(j)}=+1,\check{\boldsymbol{d}}_m^{(j)}\right]}}\right)\right. \\ &\left.+\ \mathrm{erfc}\left(\frac{-\mathrm{E}\left[\tilde{d}_m^{(j)}|d_m^{(j)}=-1,\check{\boldsymbol{d}}_m^{(j)}\right]}{\sqrt{2\mathrm{var}\left[\tilde{d}_m^{(j)}|d_m^{(j)}=-1,\check{\boldsymbol{d}}_m^{(j)}\right]}}\right)\right]\end{aligned} \quad (23)$$

where $\check{\boldsymbol{d}}_m^{(j)}$ is given by

$$\check{\boldsymbol{d}}_m^{(j)} = [\ d^{(1)}\ \ \cdots\ \ d^{(j-1)}\ \ d^{(j+1)}\ \ \cdots\ \ d^{(N)}\ ]^T. \quad (24)$$

4. Results and Discussions

In Figs. 2 and 3, the BERs for the DD and MMSE detectors are plotted, respectively. It is observed that the MAS and the brute-force simulation results are very close to each other. Also, the BER is slightly higher when the number of users increases.

Fig. 4 plots the brute-force simulated BER versus E_b/N_0 for different number of users when the conventional single-user, decorrelating and MMSE detectors are used, respectively. The curve with $N = 1$ corresponds to the interference-free case and is the lower bound of the BER under all scenarios. In Fig. 4(a), it can be observed that the BERs for the decorrelating and MMSE detectors are very close. Also, their performances are substantially better than that of the conventional single-user detector where no interference reduction scheme is applied. In particular, when E_b/N_0 is larger than 8 dB, the BERs for the decorrelating and MMSE detectors with 10 users is lower than that for the conventional single-user detector with only two users. For $N = 30$, the results are plotted again in Fig. 4(b). The decorrelating detector can minimize the interference. However, it leads to an increase in the noise component. Therefore, for a low E_b/N_0, the BER for the DD is similar to that for the conventional single-user detector. With a high E_b/N_0, the improvement of the performance is large. Also, the BER curves indicate that the MMSE detector slightly outperforms the decorrelating detector. When the E_b/N_0 increases, the difference becomes smaller.

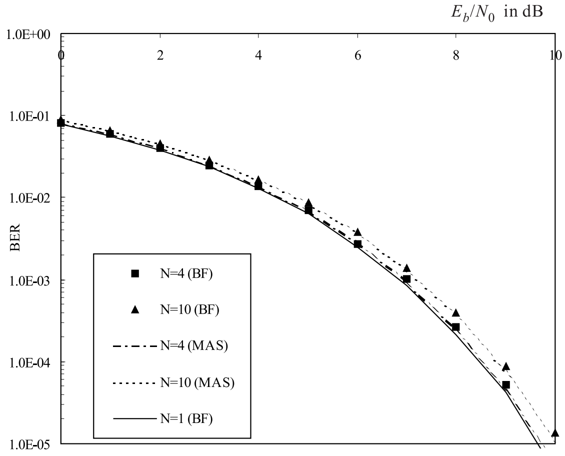

Fig. 2: BERs obtained by mixed analysis-simulation (MAS) and brute-force simulation (BF) versus E_b/N_0 for the decorrelating detector when $\beta = 100$.

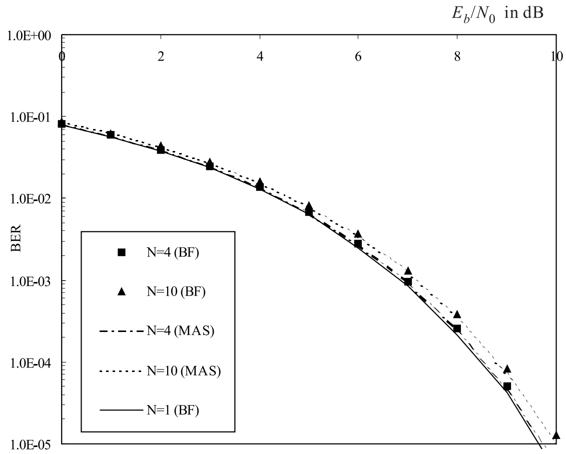

Fig. 3: BERs obtained by mixed analysis-simulation (MAS) and brute-force simulation (BF) versus E_b/N_0 for the MMSE detector when $\beta = 100$.

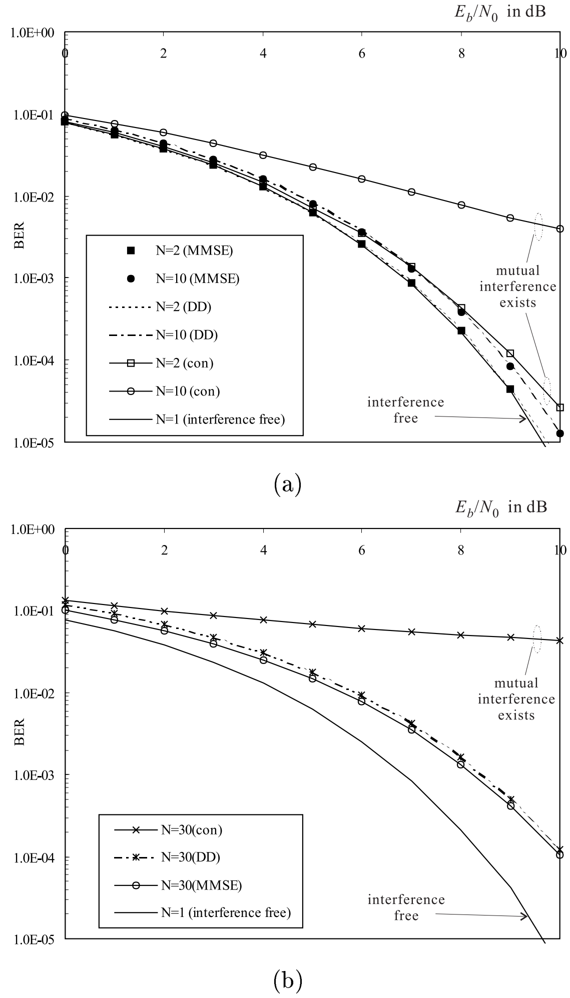

Fig. 4: Brute-force simulated BER versus E_b/N_0 for the conventional single-user (con), decorrelating (DD) and MMSE detectors when $\beta = 100$. (a) $N = 1, 2, 4, 10$; (b) $N = 30$.

5. Conclusions

In this paper, we have evaluated the performance of two multi-user detection techniques (decorrelating detector (DD) and minimum mean-square-error (MMSE) detector) when applied to a multiple access chaos-based digital communication system. A mixed analysis-simulation technique is developed for systems with more than two users. Results show that applying the multiuser detection techniques can substantially improve the bit error rate performance of the system. Also, the MMSE detector slightly outperforms the decorrelating detector.

References

[1] M.P. Kennedy, R. Rovatti and G. Setti, Eds., *Chaotic Electronics in Telecommunications,* Boca Raton, FL: CRC Press, 2000.

[2] W.M. Tam, F.C.M. Lau, C.K. Tse and M.M. Yip, "An approach to calculating the bit-error rate of a coherent chaos-shift-keying digital communication system under a noisy multiuser environment," *IEEE Trans. on Circuits and Systems I,* Vol. 49, No. 2, Feb. 2002, pp. 210–223.

[3] L.S. Tsimring, A.R. Volkovskii, S.C. Young and N.F. Rulkov, "Multi-user communication using chaotic frequency modulation," *Proc., International Symposium on Nonlinear Theory and Its Applications,* (NOLTA'01), Miyagi, Japan, Oct. 2001, pp. 561–564.

[4] R. Lupas and S. Verdú, "Linear multiuser detectors for synchronous code-division multiple-access channels," *IEEE Trans. on Information Theory,* Vol. 35, No. 1, Jan. 1989, pp. 123–136.

[5] Z. Xie, R.T. Short and C.K. Rushforth, "A family of suboptimum detectors for coherent multiuser communications," *IEEE Journal on Selected Areas in Communications,* Vol. 8, No. 4, May 1990, pp. 683–690.