From a New 3rd Order Chaotic Circuit (Gene Circuit) to a Complete Hyperchaotic Circuit Family

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Abstract — A new 3rd order piecewise linear autonomous chaotic circuit (ideal and practical model) is presented. One can unfold this new circuit into a standard 4th order hyperchaotic circuit and then generate some complete hyperchaotic circuit families. Three kinds of hyperchaotic circuit families are shown by computer simulation respectively. In all these hyperchaotic circuits (up to 9th order), there are at least two positive Lyapunov exponents.

I Introduction

The first example of hyperchaos was provided by Rössler [1]. The first computer simulation and experiment confirmation of a 4th order hyperchaotic circuit was demonstrated by Matsumoto [2]. A summary of hyperchaotic circuits is shown in [5]. Researchers are interested in finding hyperchaotic circuits and applications. However, where does hyperchaos come from and how to find a complete hyperchaotic family? From simplicity to complexity rule, the author first found a new 3rd order chaotic circuit with which the standard 4th order hyperchaotic circuit can be developed immediately by general unfolding. Because of different unfolding structures, only some hyperchaotic families are presented. Chaotic and hyperchaotic pictures are shown by computer simulation. For all kinds of hyperchaotic circuits there are at least two positive Lyapunov exponents.

2 A New 3rd Order Chaotic Circuit

2.1 Ideal Circuit Equation and Attractor

The ideal circuit is shown in Fig.1 (a) where the constitutive relation of the nonlinear resistor is given by Fig.1 (b). The circuit dynamics is described by

$$C_1 dv_{C1}/dt = -Gv_{C1} - g(v_{C1} - v_{C2}) \quad (1a)$$
$$C_2 dv_{C2}/dt = -i_{L2} + g(v_{C1} - v_{C2}) \quad (1b)$$
$$L_2 di_{L2}/dt = v_{C2} \quad (1c)$$

where v_{C1}, v_{C2}, and i_{L2} denote the voltage across C_1, C_2, and the current through L_2, respectively, and $i = g(v)$ is the piecewise linear function in Fig. 1(b).

$$g(v_{C1}-v_{C2}) = G_b(v_{C1}-v_{C2}) + (G_a-G_b)(|v_{C1}-v_{C2}+B_p| - |v_{C1}-v_{C2}-B_p|)/2 \quad (2)$$

Fig. 2 shows the chaotic attractor (v_{C1}-v_{C2}) by solving (1) and (2) with $C_1 = 0.5$, $C_2 = 0.05$, $L_2 = 2/3$, $G = -1/3$, $Ga = -0.2$, $G_b = 3$, and $B_p = 1$.

The Lyapunov exponents are $\mu_1 \approx 0.32$, $\mu_2 \approx 0$, and $\mu_3 \approx$ -54.66. The Lyapunov dimension is $d_L = 2.00576$.

2.2 The Practical Circuit State Equation

By adding a resistor r in L_2 branch we obtain a practical model. With r the circuit may be canonical and a well-known example in double scroll circuit by adding r is in [3]. Via the rescaling, equation (1) is transformed into the following simpler dimensionless form:

$$dx/dt = -\alpha[x+f(x-y)] \quad (3a)$$
$$dy/dt = f(x-y)-z \quad (3b)$$
$$dz/dt = \beta y - \gamma z \quad (3c)$$

where

$$f(x) = b(x-y)+a-b \quad \text{if} \quad x-y \geq 1 \quad (4a)$$
$$f(x) = a(x-y) \quad \text{if} \quad |x-y| \leq 1 \quad (4b)$$
$$f(x) = b(x-y)-(a-b) \quad \text{if} \quad x-y \leq -1 \quad (4c)$$

After calculating the eigenvlalues, you will find that they are quite different from other famous chaotic attractors, such as double scrolls [4].

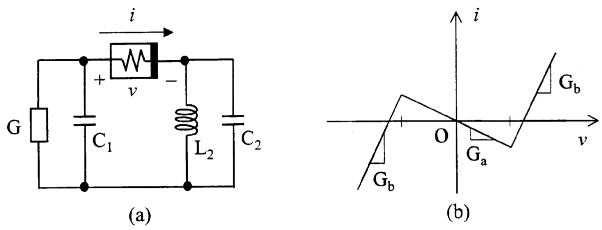

Fig. 1 (a) A new 3rd order chaotic circuit (b) The characteristic curve of the nonlinear resistor

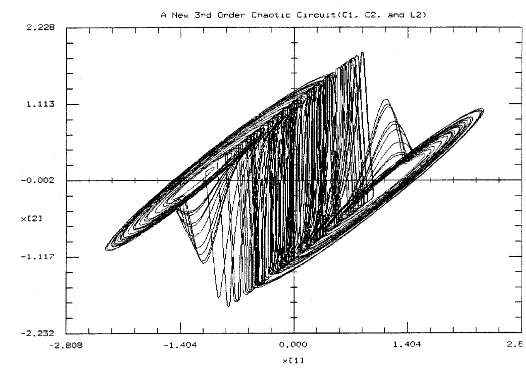

Fig. 2 The chaotic attractor of the new 3rd circuit (v_{C1} – v_{C2})

3 4^{th} order Hyperchaotic Circuit

There are some methods to create 4^{th} order hyperchaotic circuits. Here, the circuitry is formed by directly paralleling an inductor $L_{11} = 5$ across the left part (capacitor C_1 and Conductor G) in Fig 1(a), as shown in Fig.4. So, we name it the standard 4^{th} hyperchaotic circuit. The state equation is:

$$C_1 dv_{C1}/dt = -(i_{L11}+Gv_{C1})-g(v_{C1}-v_{C2}) \quad (6a)$$
$$C_2 dv_{C2}/dt = -i_{L2}+g(v_{C1}-\underline{v_{C2}}) \quad (6b)$$
$$L_{11} di_{L11}/dt = v_{C1} \quad (6c)$$
$$L_2 di_{L2}/dt = v_{C2} \quad (6d)$$

Fig. 5 shows the hyperchaotic attractor with $C_1 = 0.5$, $C_2 = 0.05$, $L_{11} = 5$, $L_2 = 2/3$, $G = -1/3$, $Ga = -0.2$, $G_b = 3$, and $Bp = 1$. The Lyapunov exponents are $\mu_1 \approx 0.23$, $\mu_2 \approx 0.047$, $\mu_3 \approx 0$, and $\mu_4 \approx -49.92$. The Lyapunov dimension is given by $d_L \approx 3.00558$. There are two positive Lypunov exponents, so it is hyperchaotic.

Note that this 4^{th} order hyperchaotic circuit is obtained by unfolding the left parallel branch C_1 and G in Fig. 1(a) – the new 3^{rd} chaotic circuit. Except $L_{11} = 5$, all the other parameters are the same as in the original 3^{rd} new chaotic circuit. Here inductor L_{11} plays the rule of exchanging chaos and hyperchaos. This is really an interesting example.

Fig. 5 is the hyperchaotic attractor. Comparing Fig. 2 (3^{rd} order circuit) with Fig. 5 (4^{th} order circuit) , we can see that these attractors are very similar. It seems that this kind of figure is the symbol of hyperchaos.

Now from this 4^{th} order basic hyperchaotic circuit and the new 3^{rd} chaotic circuit, we will demonstrate the following three different hyperchaotic circuit families.

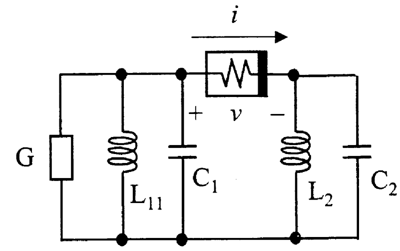

Fig. 4 The 4^{th} order hyperchaotic circuit

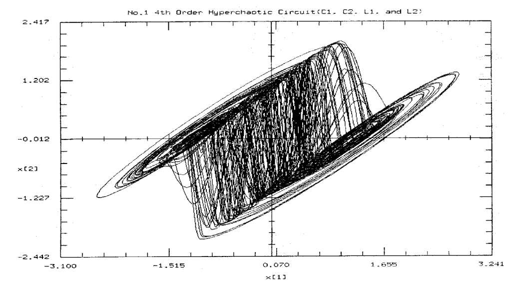

Fig. 5 The 4^{th} order hyperchaotic attractor ($v_{C1} - v_{C2}$)

4 No.1 Hyperchaotic Circuit Family

4.1 No.1 5^{th} Order Hyperchaotic Circuit

Here, we obtain the 5^{th} order hyperchaotic circuit by inserting a capacitor C_{11} in the inductor L_{11} branch in Fig. 4. Then we have the circuit shown in Fig. 6. The state equation can be described as follows:

$$C_1 dv_{C1}/dt = -i_{L11}-Gv_{C1}-g(v_{C1}-v_{C2}) \quad (7a)$$
$$C_2 dv_{C2}/dt = -i_{L2}+g(v_{C1}-v_{C2}) \quad (7b)$$
$$C_{11} dv_{C11}/dt = i_{L11} \quad (7c)$$
$$L_{11} di_{L11}/dt = v_{C1}-v_{C11} \quad (7d)$$
$$L_2 di_{L2}/dt = v_{C2} \quad (7d)$$

Set $C_{11} = 0.1$, and keep all the other parameters the same as in the 4^{th} order circuit (Fig. 4). The computer simulation result, a chaotic attractor is shown in Fig. 7. The two largest positive Lyapunov exponents are $\mu_1 \approx 0.154$ and $\mu_2 \approx 0.0562$. The smallest negative Lyapunov exponent is $\mu_5 \approx -56.15$.

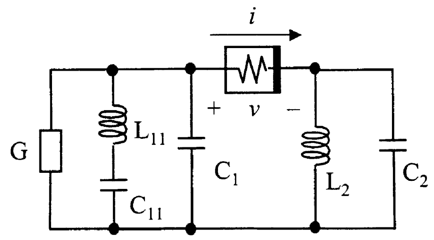

Fig. 6 No.1 5^{th} hyperchaotic circuit

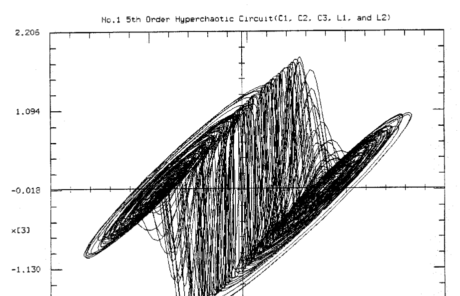

Fig. 7 No. 1 5^{th} hyperchaotic attractor ($v_{C1} - v_{C2}$)

4.2 No.1 (2N)-th Order Hyperchaotic Circuit

To save space, we only draw (2N)-order circuit, as shown in Fig. 8. When N = 3, we have the No.1 6^{th} order hyperchaotic circuit. Its state equation is:

$C_1 dv_{C1}/dt = -i_{L11} - Gv_{C1} - g(v_{C1} - v_{C2})$ (8a)

$C_2 dv_{C2}/dt = -(i_{L2} + i_{L3} + \ldots + i_{LN}) + g(v_{C1} - v_{C2})$ (8b)

$C_3 dv_{C3}/dt = i_{L3} + i_{L4} + \ldots + i_{LN-1} + i_{LN}$ (8c)

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$L_{N-1} di_{LN-1}/dt = v_{C2} - (v_{C3} + \ldots + v_{CN-1})$ (8_{N-1})

$L_N di_{LN}/dt = v_{C2} - (v_{C3} + \ldots + v_{CN-1} + v_{CN})$ (8_N)

With $Bp = 1$, $C_1 = 0.5$, $C_2 = 0.05$, $C_3 = 0.025$, $L_{11} = 5.5$, $L_2 = 2/3$, $L_3 = 1$, and $R = -3.5$, $\mu_1 \approx 0.129$ and $\mu_2 \approx 0.072$.

When N = 4, we have the No.1 8^{th} order hyperchaotic circuit. With $Bp = 1$, $C_1 = 0.5$, $C_2 = 0.05$, $C_3 = C_4 = 0.025$, $L_{11} = 5.5$, $L_2 = 2/3$, $L_3 = L_4 = 1$, and $R = -3.5$, the first three largest Lyapunov exponents are $\mu_1 \approx 0.087$, $\mu_2 \approx 0.05$, and $\mu_3 \approx 0.034$. When N becomes larger and larger, more and more positive Lyapunov exponents will appear. Fig. 8 will be named as the No.1 even order hyperchaotic circuit family branch.

4.3 No.1 (2N+1)-th Order Hyperchaotic Circuit (N≥3)

We only present (2N+1)-th order circuit, as shown in Fig.9. The state equation can be represented as:

$C_1 dv_{C1}/dt = -i_{L11} - Gv_{C1} - g(v_{C1} - v_{C2})$ (9a)

$C_{11} dv_{C11}/dt = i_{L11}$ (9b)

$C_2 dv_{C2}/dt = -(i_{L2} + i_{L3} + \ldots + i_{LN}) + g(v_{C1} - v_{C2})$ (9c)

$C_3 dv_{C3}/dt = i_{L2} + i_{L3} + \ldots + i_{LN-1} + i_{LN}$ (9d)

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$L_{N-1} di_{LN-1}/dt = v_{C2} - (v_{C3} + \ldots + v_{CN-1})$ (9_{N-1})

$L_N di_{LN}/dt = v_{C2} - (v_{C3} + \ldots + v_{CN-1} + v_{CN})$ (9_N)

With N = 3, $Bp = 1$, $C_1 = 0.1$, $C_2 = 0.05$, $C_3 = C_4 = 0.05$, $Ga = -0.2$, $Gb = 3$, $L_1 = 5$, $L_2 = L_3 = 2/3$, and $R = -3$, we have the 7^{th} order circuit with the two largest Lyapunov exponents $\mu_1 \approx 0.088$ and $\mu_2 \approx 0.0443$.

With $Bp = 1$, $C_1 = 0.1$, $C_2 = C_3 = C_4 = C_5 = 0.05$, $Ga = -0.2$, $Gb = 3$, $L_1 = 5$, $L_2 = L_3 = L_4 = 2/3$, and $R = -3$, we have the 9^{th} order circuit with $\mu_1 \approx 0.048$ and $\mu_2 \approx 0.0275$.

In other words, there are also at least two positive Lyapunov exponents in the (2N+1)-th circuits, so it is hyperchaotic. Similarly, we name Fig. 9 as No.1 odd order hyperchaotic circuit family.

Therefore, combining the new third chaotic circuit (Fig.1 – basic circuit), the 4^{th} hyperchaotic circuit (Fig.4), No.1 5^{th} hyperchaotic circuit (Fig.6), Fig.8 and Fig.9, we have No. 1 hyperchaotic circuit family.

5. No.2 Hyperchaotic Circuit Family

5.1 No.2 5^{th} Order Hyperchaotic Circuit

Here we obtain another 5^{th} order hyperchaotic circuit by inserting a capacitor C_0 in the nonlinear element branch in Fig. 4. We only write the state equation as

$C_0 dv_{C0}/dt = g(-v_{C0} + v_{C1} - v_{C2})$ (10a)

$C_1 dv_{C1}/dt = -i_{L11} - Gv_{C1} - g(-v_{C0} + v_{C1} - v_{C2})$ (10b)

$C_2 dv_{C2}/dt = -i_{L2} + g(-v_{C0} + v_{C1} - v_{C2})$ (10c)

$L_{11} di_{L11}/dt = v_{C1}$ (10d)

$L_2 di_{L2}/dt = v_{C2}$ (10e)

Let $C_0 = 5$, and keep all the other parameters the same as in the 4^{th} order chaotic circuit (Fig. 4), the computation results are $\mu_1 \approx 0.154$, $\mu_2 \approx 0.0627$, $\mu_3 \approx 0$, $\mu_4 \approx -0.0176$, and $\mu_5 \approx -56.57$.

5.2 No.2 (2N)–th Order Hyperchaotic Circuit (N≥3)

Here we only describe $2N^{-th}$ order circuit equation as:

$C_0 dv_{C0}/dt = g(-v_{C0} + v_{C1} - v_{C2})$ (11a)

$C_1 dv_{C1}/dt = -i_{L11} - Gv_{C1} - g(-v_{C0} + v_{C1} - v_{C2})$ (11b)

$C_2 dv_{C2}/dt = -(i_{L2} + \ldots + i_{LN-1}) + g(-v_{C0} + v_{C1} - v_{C2})$ (11c)

$C_3 dv_{C3}/dt = i_{L2} + i_{L3} + \ldots + i_{LN-2} + i_{LN-1}$ (11d)

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$L_{N-1} di_{N-1}/dt = v_{C2} - v_{C3} - \ldots - v_{CN-1} + v_{CN}$ (11N)

With $Bp = 1$, $C_0 = 5$, $C_1 = 0.5$, $C_2 = 0.05$, $C_3 = 3$, $Ga = -0.2$, $G_b = 3$, $L_1 = 5$, $L_2 = 2/3$, and $R = -3$, we have the No.2 6^{th} order hyperchaotic circuit with the three largest Lyapunov exponents $\mu_1 \approx 0.1325$, $\mu_2 \approx 0.056$, and $\mu_3 \approx 0.014$. Obviously, the No.2 6^{th} order circuit is also hyperchaotic.

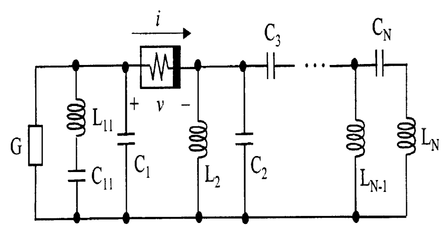

Fig. 8. No. 1 (2N)-th order hyperchaotic circuit

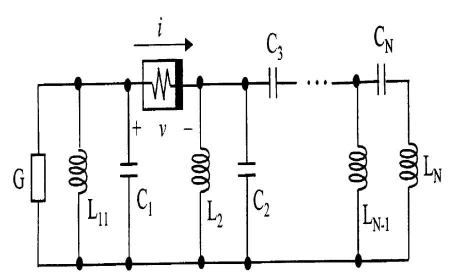

Fig.9 No.1 (2N+1)-th hyperchaotic circuit

5.3 No.2 (2N+1)-th Order Hyperchaotic Circuit (N≥3)

The (2N+1)-th order circuit state equation is:

$$C_0 dv_{C0}/dt = g(-v_{C0}+v_{C1}-v_{C2}) \quad (12a)$$
$$C_1 dv_{C1}/dt = -i_{L11}-Gv_{C1}-g(-v_{C0}+v_{C1}-v_{C2}) \quad (12b)$$
$$C_2 dv_{C2}/dt = -(i_{L2}+\ldots+i_{LN})-g(-v_{C0}+v_{C1}-v_{C2}) \quad (12c)$$
$$C_3 dv_{C3}/dt = i_{L3}+\ldots+i_{LN-1}+i_{LN} \quad (12d)$$
$$\cdots\cdots$$
$$L_N di_N/dt = v_{C2}-v_{C3}-\ldots-v_{CN-1}-v_{CN} \quad (12_N)$$

With $Bp = 1$, $C_0 = 10$, $C_1 = 0.5$, $C_2 = C_3 = 0.05$, $Ga = -0.2$, $G_b = 3$, $L_{11} = 5$, $L_2 = 2/3$, $L_3 = 1$, and $R = -3$, we have the No.2 7th order hyperchaotic circuit with the two largest Lyapunov exponents $\mu_1 \approx 0.1577$ and $\mu_2 \approx 0.059$.

With $Bp = 1$, $C_0 = 10$, $C_1 = 0.5$, $C_2 = C_3 = C_4 = 0.05$, $Ga = -0.2$, $G_b = 3$, $L_{11} = 5$, $L_2 = 2/3$, $L_3 = L_4 = 1$, and $R = -3$, we have the No.2 9th order hyperchaotic circuit with the four largest Lyapunov exponents $\mu_1 \approx 0.1296$, $\mu_2 \approx 0.071$, $\mu_3 \approx 0.0334$, and $\mu_4 \approx 0.0133$.

Combining Fig.1, Fig. 4, and the corresponding circuits, we have No. 2 hyperchaotic complete hyperchaotic circuit family.

6 No.3 Hyperchaotic Circuit Family Branch

6.1 No.3 5th order Hyperchaotic Circuit

We obtain the No.3 5th order hyperchaotic circuit by inserting C_{11} in upper left junction of C_1 and L_1 in Fig. 4. Then the circuit state equation is as follows:

$$C_1 dv_{C1}/dt = -(i_{L11}+G(v_{C1}+v_{C2}))+g(v_{C1}-v_{C2}) \quad (13a)$$
$$C_{11} dv_{C11}/dt = -(i_{L11}+G(v_{C1}+v_{C11})) \quad (13b)$$
$$C_2 dv_{C2}/dt = -i_{L2}+g(v_{C1}-v_{C2}) \quad (13c)$$
$$L_{11} di_{L11}/dt = v_{C1}+v_{C11} \quad (13d)$$
$$L_2 di_{L2}/\mathrm{dt} = v_{C2} \quad (13e)$$

Let $C_{11} = 5$, and keep all the other parameters the same as in the 4th order chaotic circuit, we obtain two largest Lyapunov exponents $\mu_1 \approx 0.178$ and $\mu_2 \approx 0.028$.

6.2 No.3 (2N)-th Order Hyperchaotic Circuit (N≥3)

Then the 2N-th order circuit state equation is:

$$C_1 dv_{C1}/dt = -(i_{L1}+i_{L11}+G(v_{C1}+v_{C11}))-g(v_{C1}-v_{C2}) \quad (14a)$$
$$C_{11} dv_{C11}/dt = -(i_{L1}+(v_{C1}+v_{C11})/R) \quad (14b)$$
$$C_2 dv_{C2}/dt = -(i_{L2}+i_{L3}+\cdots+i_{LN})+g(v_{C1}-v_{C2}) \quad (14c)$$
$$\cdots\cdots$$
$$L_N di_{LN}/dt = v_{C2}-v_{C3}-\ldots-v_{CN-1}-v_{CN} \quad (14N)$$

With $Bp = 1$, $C_1 = 0.5$, $C_{11} = 5$, $C_2 = 0.05$, $Ga = -0.2$, $G_b = 3$, $L_1 = 6$, $L_2 = 1/2$, and $R = -3$, we have the No.3 6th order hyperchaotic circuit with $\mu_1 \approx 0.132$, $\mu_2 \approx 0.015$, and $\mu_3 \approx 0.0057$.

6.3 No. 3 (2N+1)-th Order Hyperchaotic Circuit (N≥3)

The (2N+1)-th order circuit state equation is:

$$C_1 dv_{C1}/dt = -(i_{L1}+G(v_{C1}+v_{C11}))-g(v_{C1}-v_{C2}) \quad (15a)$$
$$C_{11} dv_{C11}/dt = -(i_{L1}+G(v_{C1}+v_{C11})) \quad (15b)$$
$$C_2 dv_{C2}/dt = -(i_{L2}+i_{L3}+\ldots+i_{LN})+g(v_{C1}-v_{C2}) \quad (15c)$$
$$\cdots\cdots$$
$$L_N di_{LN}/dt = v_{C2}-v_{C3}-\ldots-v_{CN-1}-v_{CN} \quad (15N)$$

With $Bp = 1$, $C_1 = 0.5$, $C_{11} = 5$, $C_2 = C_3 = 0.5$, $Ga = -0.2$, $G_b = 3$, $L_1 = 5$, $L_2 = 2/3$, $L_3 = 1$, and $R = -3$, we have No.3 7th order circuit with $\mu_1 \approx 0.132$ and $\mu_2 \approx 0.049$. Similarly, by combing the new 3rd order circuit (Fig.1), the 4th order circuit (Fig. 4), No.3 5th hyperchaotic circuit, and the corresponding circuits, we have No.3 complete hyperchaotic circuit family.

7 Discussion

The attractors of the new 3rd order chaotic circuit become the basic block in creating hyperchaotic family because its attractor is similar to hyperchaos. Perhaps there exists a hyperchaotic gene in it. One can immediately obtain a standard 4th order hyperchaotic circuit by unfolding. The 4th order circuit is much simpler than the first hyperchaotic circuit. And the complete hyperchaotic families are formed by inserting a *CL* pairs into the new 3rd and 4th order circuit respectively. Finally, we obtained at least two positive Lyapunov exponents in all these hyperchaotic circuits (up to 9th order circuit). What's the mechanism of this new 3rd order chaotic circuit and its theoretical and practical meaning ?

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