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Do Heterogeneous Duopolists Prefer Chaos?

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1. Introduction

This paper deal with a non-tâtonnement adjustment process of heterogeneous duopolists with discrete time. In the Cournot tâtonnement process, chaotic patterns of output adjustments have been observed; see Rand[11] and Dana and Montrucchio[3] Witteloostuijn and Lier[12] (hereafter, W-L), to name a few. In addition, the microfoundations of nonlinear reaction patterns of Cournot oligopolists have been provided by Puu[9], [10] and Kopel[6].

The salient feature of the Cournot tâtonnement process is that the agents adjust the output in response to a difference between their expectations and outputs actually offered, and actual production takes place on when the expectation is identical with output observed. A chaotic tâtonnement, in spite of working of output adjustment, does not converge to an equilibrium output but fluctuates around it while remaining bounded. Therefore no actual production is carried out because no convergence is brought about.

One of ways to deal with this problem is to admit intermediate production on the way of the output adjustment process. In particular we construct a Cournot duopoly model and consider an economic implication of output chaos in the non-tâtonnement process. Since production occurs in non-clearing market, an agent may be better off or worse off than she could be at the steady state, depending on the prevailing market output. So our primary concern is to investigate whether, from the long-run point of view, the gain from the chaotic output instability can dominate the loss. For this purpose, we compare the Long-run Average Profit (LAP, hereafter) and Nash-Equilibrium Profit (NEP, hereafter) in chaotic non-tâtonnement process.

Recently, it is investigated that heterogeneity of agents more likely induce chaotic patterns in their interactions. We also concern that how heterogeneity of duopolists affects the result of comparing LAP and NEP in chaotic non-tâtonnement process.

This paper proceeds as follows: In Section2, we construct a dynamic model of Cournot duopoly. Section 3 examines steady states. In Section introduce some numerical example which calculate the LAP. Section 5 gives concluding remarks.

2. Model

Consider two firms, X and Y namely, producing a homogeneous good and facing linear demand function:

$$p = a - b(x + y) \tag{1}$$

where p is the market price and x respectively y denotes the output of firm X respectively Y. We assume that for both of X and Y, the marginal costs of production are independent to their own but dependent to their rival's output. That is, for example, firm X's marginal cost c_x is a function of y. Therefore, firm X's profit function $\Pi^x$is defined by

$$\Pi^x(x; y) = p \cdot x - c_x(y) \cdot x. \tag{2}$$

We assume that the maximization problem of firm X is uniquely solved for given nonnegative y. Thus the reaction function of firm X denoted by $r_x(y)$ is given by

$$r_x(y) = \arg\max_x \Pi^x(x; y). \tag{3}$$

It is apparent that the shape of $r_x(y)$ crucially depends on the specification of $c_x(y)$. It can be linear or even be constant. We however more concerns the case that $r_x(y)$ is nonlinear in particular U-shaped. Thus we introduce two types of U-shaped reaction curves.

Proposition 1 *If firm X has a marginal cost of*

$$c_x(y) = a - by - 2b\alpha\left(y + \frac{1}{\sqrt{\alpha}} - 1\right)^2, \tag{4}$$

then X is called imitator-dualist (I-dualist hereafter) and the reaction function of firm X is given by

$$r_x(y) = a\left(y + \frac{1}{\sqrt{\alpha}} - 1\right)^2. \tag{5}$$

□

If firm X is an I-dualist, $c_x(y)$ is decreasing function of y when $\alpha = 1$ and then $r_x(y)$ is an increasing function of y[1]. For $\alpha > 1$, $c_x(y)$ becomes inverted U-shaped and the reaction curve will change to U-shape[2].

Proposition 2 *If firm X has a marginal cost of*

$$c_x(y) = a - 2b - by\left(2\alpha\left(y - \frac{2}{\sqrt{\alpha}}\right) + 1\right), \tag{6}$$

then X is called accommodator-dualist (A-dualist hereafter) and the reaction function of firm X is given by

$$r_x(y) = \left(\sqrt{\alpha}y - 1\right)^2. \tag{7}$$

□

If firm X is an A-dualist, $c_x(y)$ is an increasing function of y when $\alpha = 1$ and then $r_x(y)$ is a decreasing function of y[3]. For $\alpha > 1$, $c_x(y)$ becomes inverted U-shaped and then $r_x(y)$ becomes U-shaped.

We consider the dynamical system of this Cournot duopoly market with discrete time. Firm X and Y updated their output iteratively. In the end of each period, both of firms observe their rival's actual output. That is, for example, in period t firm X put their output level x_t to maximize Π^x for given observation of y_{t-1}. Thus we define the dynamical system:

$$x_{t+1} = r_x(y_t), \tag{8}$$
$$y_{t+1} = r_y(x_t). \tag{9}$$

3. Analysis

In this section, we examine the steady state points (SSP hereafter) of the adjustment process for three different cases. We assume that the parameters of both firm X and Y's marginal costs are symmetric, that is $\alpha = \beta = \gamma$.

[1]In this case, firm X receives positive externalities from Y's output. W-L[12] call a firm of this type as an *imitator*.

[2]A firm with U-shaped reaction curves receives negative-positive externalities from rival's behavior and W-L call the firm as a *dualist*

[3]Therefore X receives negative externalities from Y's output and W-L call a firm of this type as *accommodator*.

3.1. Homogeneous case 1

Suppose that both of duopolists are I-dualists, then the dynamical system of output adjustment becomes

$$x_{t+1} = a\left(y_t + \frac{1}{\sqrt{\gamma}} - 1\right)^2, \tag{10}$$
$$y_{t+1} = a\left(x_t + \frac{1}{\sqrt{\gamma}} - 1\right)^2. \tag{11}$$

This system has two SSP for minimal and four for maximal. All of them are non-negative real roots of the system of quadratic equations:

$$x^* = a\left(y^* + \frac{1}{\sqrt{\gamma}} - 1\right)^2, \tag{12}$$
$$y^* = a\left(x^* + \frac{1}{\sqrt{\gamma}} - 1\right)^2. \tag{13}$$

When $1 \leq \gamma < \gamma_1 \simeq 2.25$, two SSP $S_1 = (1, 1)$ and $S_2 = (s^*, s^*)(0 \leq s^* < 1)$ exists and S_1 is stable and S_2 is unstable. For $\gamma_1 \leq \gamma < \gamma_2 \simeq 2.97$, two other SSP both of which are stable and appear and alternatively S_2 becomes unstable. For $\gamma_2 \leq \gamma \leq 4$ all of four SSP are unstable and chaotic bifurcation appears.

3.2. Homogeneous case 2

Suppose that both of duopolists are A-dualists, then the dynamical system of output adjustment becomes

$$x_{t+1} = \left(\sqrt{\gamma}y_t - 1\right)^2, \tag{14}$$
$$y_{t+1} = \left(\sqrt{\gamma}x_t - 1\right)^2. \tag{15}$$

This system has three SSP for minimal and four for maximal. All of them are non-negative real roots of the system of quadratic equations:

$$x^* = \left(\sqrt{\gamma}y^* - 1\right)^2, \tag{16}$$
$$y^* = \left(\sqrt{\gamma}x^* - 1\right)^2. \tag{17}$$

When $1 \leq \gamma < \gamma_1 \simeq 1.56$, three SSP exist One is symmetric and the others are asymmetrical. Symmetric SSP is unstable and two asymmetrical SSP are unstable. For $\gamma_1 \leq \gamma < 4$, two asymmetrical SSP become unstable and chaotic bifurcation appears. Fourth SSP appears only if $\gamma = 4$.

3.3. Heterogeneous case

Consider a heterogeneous case. Suppose that firm X is I-dualists and firm Y is A-dualist. The dynamical system of output adjustment now becomes

$$x_{t+1} = a\left(y_t + \frac{1}{\sqrt{\alpha}} - 1\right)^2, \tag{18}$$
$$y_{t+1} = \left(\sqrt{\alpha}x_t - 1\right)^2. \tag{19}$$

This system has one SSP for minimal and four for maximal. All of them are non-negative real roots of the system of quadratic equations:

$$x^* = a\left(y^* + \frac{1}{\sqrt{\gamma}} - 1\right)^2, \tag{20}$$

$$y^* = (\sqrt{\gamma}x^* - 1)^2. \tag{21}$$

When $1 \leq \gamma < \gamma_1 \simeq 3.01$, One SSP exists and it is unstable. If $\gamma = \gamma_1$, the second SSP appears and it is stable. For $\gamma_1 < \gamma < \gamma_2 \simeq 3.23$, the second SSP splits into two and one of them remains stable while the other becomes unstable. For $\gamma_2 \leq \gamma \leq 4$ all of three SSP are unstable and the forth SSP appears only if $\gamma = 4$.

Stable and unstable regions of this system in all domain of parameters α and β are illustrated in Fig.1[4].

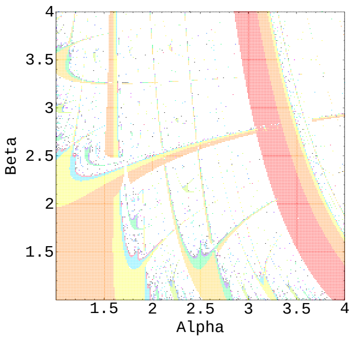


Figure 1: Bifurcation diagram of heterogeneous case

4. Numerical Example

In numerical example, we evaluate the LAP of duopolists for each two homogeneous and heterogeneous cases. Here we also assume that the parameters of marginal cost functions are symmetric($\alpha = \beta = \gamma$).

4.1. Homogeneous case 1

Variations of LAP and NEP for a case of homogeneous with I-dualists with respect to γ are illustrated in Fig. 2[5]. For $1 \leq \gamma < \gamma_1$ S_2 is unique and stable SSP. For $\gamma_1 \leq \gamma < \gamma_2$, S_2 becomes unstable and S_3 and S_4 are stable. Then, the problem of initial points occurs. For $\gamma \geq \gamma_2$, chaotic bifurcation occurs and then LAP deviates from NEP. For both firm, the LAP evaluated along chaotic trajectories are worse off for desirable NEP and better off for undesirable NEP.

[4]In Fig.1, red colored area depicts stable region and orange, yellow and blue colored areas depict respectively 2-, 4-, and 8-periodic cycle regions.

[5]In Fig. 2 and 3, S_2 depicts the NEP of symmetric SSP and S_3 and S_4 depict the NEP of asymmetric SSP.

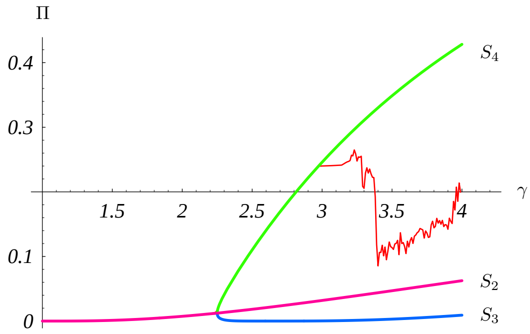


Figure 2: LAP of homogeneous case 1

4.2. Homogeneous case 2

Variations of LAP and NEP for homogeneous case with A-dualists are illustrated in Fig.3. Essential features of case 2 is same as case 1. For $1 \leq \gamma < \gamma_1$, the problem of initial points occurs. Here S_2 is unstable and S_3 and S_4 are stable. For both firm, the LAP evaluated along chaotic trajectories are worse off for desirable NEP and better off for undesirable NEP.

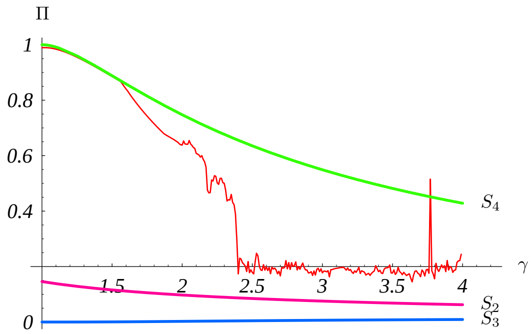


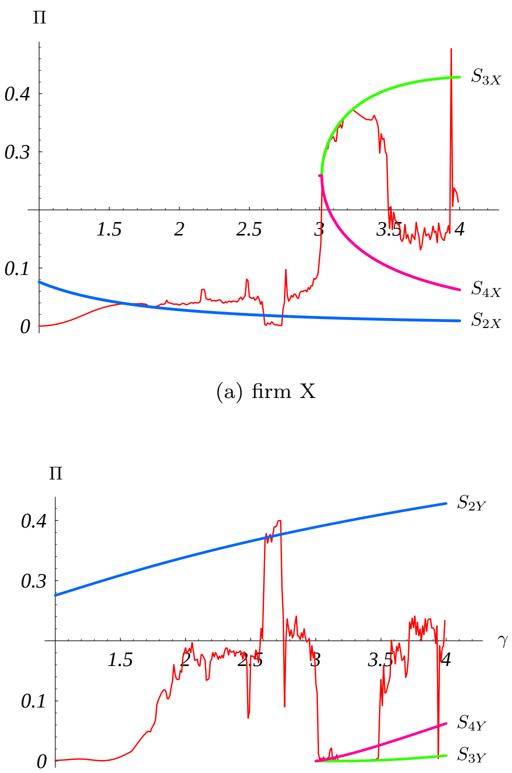
Figure 3: LAP of homogeneous case 2

4.3. Heterogeneous case

Figure 4 illustrates LAP and NEP of each heterogeneous duopolists. Firm X is an I-dualist and firm Y is an A-dualist. In our example, stable SSP appears when $\gamma_1 < \gamma < \gamma_2$ and it is unique. Therefore the problem of initial points does not occur in heterogeneous case. In addition, deviation of LAP from the NEP is worse off for firm X but better off for firm Y. In this sense, we could say that firm Y prefer chaos than a steady state.

5. Concluding Remarks

In homogeneous cases two asymmetric SSP provide contrastive NEP to duopolists and for both of firms



(a) firm X

(b) firm Y

Figure 4: LAP of heterogeneous case

chaotic LAP lies between desierable NEP and undesierable NEP. This situation will cause the problem of initial condition by necessity while those two SSP are stable. When they lose stability controlling chaos to a steady state will be taken into a consideration. Therefore game theoretical argument will be worthy of note in homogeneous.

In a heterogenous case, on the other hand, only one SSP become stable, the problems of initial point or controlling chaos will not occur. Furthermore, one of firms better off in chaotic LAP than NEP. This inclination will be clear when we compare chaotic LAP and NEP for wide range of parameters.

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