Autonomous Distributed Control System utilizing Reaction Diffusion Field

Akio Suzuki

National Institute of Advanced Industrial Science and Technology
1-2 Namiki, Tsukuba, Ibaraki, 305-8564 Japan
Email: akio.suzuki@aist.go.jp

***Abstract*—** The design method of a mechanical displacement reaction diffusion field by spatially distributed nonlinear control cells is proposed. Excitable wave propagations in a two-dimensional hexagonal lattice system are investigated, and a control method is proposed. As a first step of the application, a sensing function emergence and an autonomous conveying system are demonstrated.

1. Introduction

In a centralized control system, with the increase of a system scale, the complexity of a control law and the load of a master controller increase at an exponential rate. So, flexible and dynamic adaptations to troubles and environmental changes become difficult. Therefore, the concept of distributed control was appeared, and various systems were proposed. Especially, the concept of emergence system, which makes up itself only by local interactions, is attractive.

Spatiotemporal pattern formation by the local interactions of elements and its control are the key technology of the emergence system. A reaction diffusion system, which is observed in various chemical and biological systems, is the typical representative of a dynamical system that generates the spatiotemporal patterns. Various interesting spatiotemporal patterns and their interactions have been observed in reaction diffusion systems [1]. These nonlinear waves can provide the basis for information processing. For example, it is shown that elemental logic devices, a memory device and a time-difference device can be created with reaction diffusion fields by the suitable arrangement of excitable and diffusion fields [2].

We have the interest for a distributed mechanical system that does various works by the cooperation of a great number of mechatronic units. Applying reaction diffusion processing to the control principal of a distributed mechanical system is the objective of this study.

In this paper, we propose the design method of discrete reaction diffusion field by spatially distributed piecewise linear control cells. Excitable wave propagations in a two-dimensional hexagonal lattice are investigated, and a control method is proposed. As a first step of the application, a sensing function emergence and an autonomous conveying system are demonstrated.

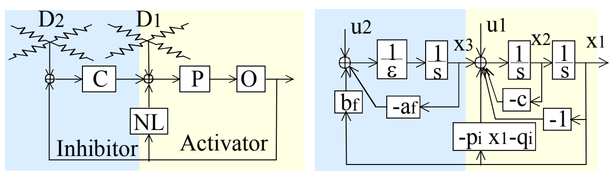
2. Nonlinear Control Cell

2.1. Cell configuration

Excitable and oscillatory kinetics of the reaction diffusion field is caused by a cubic nonlinear element and two subsystems, an activator and an inhibitor, in which the time scale greatly differs (Fig.1a). A nonlinear feedback element and a linear compensator can give these properties to an any object. Therefore, it is possible to generate the spatially discrete reaction diffusion field of various physical quantities utilizing actuators, electromagnets and heating devices.

In this paper, a linear mechanical oscillator is chosen as a controlled object, and a mechanical displacement reaction diffusion field is realized. Applying a nonlinear feedback, the mechanical oscillator becomes an activator. An inhibitor is a low pass filter with time constant which is very longer than natural frequency of the mechanical oscillator. The nonlinear element used in this paper is not cubic function but three region piecewise linear function. The block diagram of the proposed control cell is shown in Fig.1b. The state equations of a dimensionless form are as follows.

$$
\begin{aligned}
&\dot{x}_1 = x_2, \\
&\dot{x}_2 = -x_1 - c \cdot x_2 + x_3 - f(x_1) + u_1, \qquad (1)\\
&\varepsilon\,\dot{x}_3 = b_f \cdot x_1 - a_f \cdot x_3 + u_2, \\
&f(x_1) = \begin{cases} p_1 \cdot x_1 + q_1 & \qquad R_1 : B_{p1} \le x_1 \le B_{p2}, \\ p_2 \cdot x_1 + q_2 & \text{Region} \quad R_2 : x_1 < B_{p1}, \\ p_3 \cdot x_1 + q_3 & \qquad R_3 : x_1 > B_{p2}, \end{cases}
\end{aligned}
$$



a) Cell block diagram b) Dimensionless form

Fig. 1 Nonlinear control cell

O: Controlled Object, P: Pre-compensator, S: Slow Controller, NL: Nonlinear Feedback, D2,D3: Coupling Constant

$$q_1 = q_0\,,$$
$$q_2 = (p_1 - p_2)B_{p1} + q_0\,, \qquad (2)$$
$$q_3 = (p_1 - p_3)B_{p2} + q_0\,,$$

where x_1 and x_2 are the displacement and velocity of the oscillator, x_3 is the output of a low pass filter, c is a damping coefficient, f(x) is a piecewise linear position feedback function, a_f and b_f are the pole and gain of the filter, ε is a parameter for controlling the time constant of the inhibitor block, u_1 and u_2 are external inputs. In order to simplify description, p_i+1 is redefined p_i in following sections.

2.2. Bifurcation characteristics

The equilibrium points of Eqs.(1) are obtained by setting the derivatives equal to zero and looking for intersections of nullclines,

$$x_2 = 0,$$
$$x_3 = p_i\,x_1 + q_i - u_1\,, \qquad (3)$$
$$x_3 = \frac{b_f\,x_1 + u_2}{a_f}\,.$$

The equilibrium points are obtained as follows.

$$x_1^o = \frac{a_f(-q_i + u_1) + u_2}{a_f\,p_i - b_f}, \qquad x_2^o = 0,$$
$$x_3^o = \frac{b_f(-q_i + u_1) + p_i\,u_2}{a_f\,p_i - b_f}. \qquad (4)$$

Depending on the values of the piecewise linear feedback parameters and external inputs, the number of the equilibrium points are determined. If the equilibrium point exists, using Routh-Hurwitz criterion, its stability conditions are given as follows.

$$c + a'_f > 0, \quad c\,a'_f + p_i > 0, \quad p\,a'_f - b'_f > 0,$$
$$c\,a'^2_f + c^2\,a'_f + c\,p_i + b'_f > 0, \qquad (5)$$

where $a_f' = a_f/\varepsilon,\ b_f' = b_f/\varepsilon$.

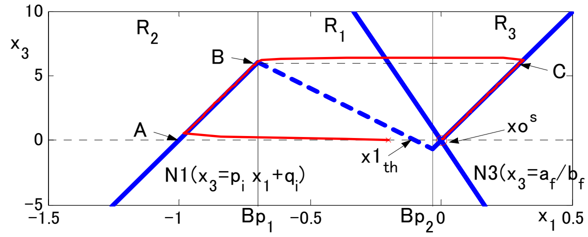
2.3. Excitable and oscillatory cell design

2.3.1. Excitable cell

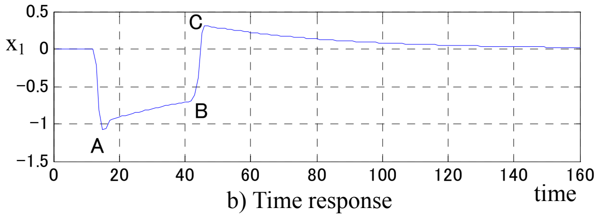
In order to make the piecewise linear function f(x) be a cubic, gain parameters must be assigned as p_1<0, p_2>0, p_3>0. The equilibrium point of an excitable cell is one only, and it must be stable. We assigned it at the origin. The typical phase portrait which plot nullclines $\dot{x}_1 = 0$, $\dot{x}_3 = 0$ (N1 and N3) in the plane x_2=0 is shown in Fig.2. The continuous line means a stable nullcline, and the broken line means an unstable one. A small perturbation is damped out, but a perturbation over the threshold x_{th} triggers a long excursion. The excitation variable x_1 decreases rapidly until the trajectory approaches the leftmost branch of N1. At this point the trajectory moves slowly up the nullcline as the recovery variable x_3 increases. When the trajectory reaches the top of the left branch of N1, the trajectory moves to the rightmost branch as variable x_1 rapidly increases, which is followed by a slow decrease of x_3 back to the rest state. The shape of the response can be designed by assigning the point A, B, C. With the increase of parameter p_2, the bottom of the excitation wave becomes flat. With the increase of parameter p_3, the overshoot value C of the response decrease. We assigned to x_{th}=-0.1, A=-1.0, C=0.25. As the parameter which satisfied these conditions, following values were obtained. p_1=−10, p_2=20, p_3=p_2, Bp_1=−0.7, Bp_2=−1/30, q_0=−1. Other parameters were assigned to a_f=1, b_f=−30, c=4, ε=100. The phase portrait and the response are shown in Fig.2.

2.3.2. Oscillatory cell

An oscillatory cell is obtained by the disappearance of the stable equilibrium point in region R_3, and the appearance of an unstable equilibrium point in region R_1. By the



a) Phase portrait



b) Time response

Fig.2 Excitable Cell

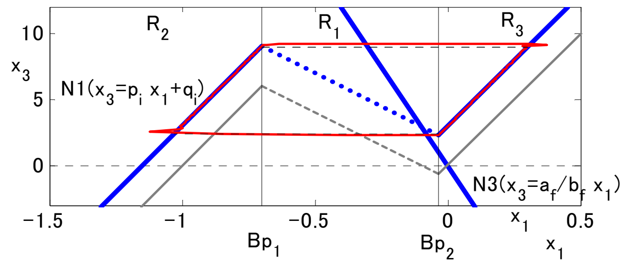


Fig.3 Oscillatory cell

variation of external input u_2, u_2 and offset value q_0 of the piecewise linear feedback function, it is possible to cause this bifurcation as shown in phase portrait Fig.3. The condition is as follows.

$$Bp_1 < x_1^{\,o} = \frac{q_0 - u_1}{b_f / a_f - p_1} < Bp_2 \tag{6}$$

The state variables of the oscillator alternately move between the two stable branches of nullcline N1, because the stable equilibrium point does not exist.

3. Excitable Wave in Coupled Nonlinear Control Cells

3.1. Spatially discrete reaction diffusion field

Coupling the proposed nonlinear control cells diffusively to their neighbors in some spatial lattice is a discrete analogue of a reaction diffusion system. Since the problem occurs on excitable wave propagation to a diagonal direction, not a square lattice but a hexagonal lattice was used (Fig.4). The state equations are,

$$\begin{aligned}
\dot{x}_1^{i,j} &= x_2^{i,j} \\
\dot{x}_2^{i,j} &= -p_k x_1^{i,j} - c \cdot x_2^{i,j} + x_3^{i,j} + q_k + u_2^{i,j} \\
&\quad + D_1 (x_1^{i+1,j-1} + x_1^{i+1,j} + x_1^{i,j-1} \\
&\quad + x_1^{i,j+1} + x_1^{i-1,j-1} + x_1^{i-1,j} - 6 x_1^{i,j}) \quad , \\
\varepsilon \dot{x}_3^{i,j} &= b_f \cdot x_1^{i,j} - a_f \cdot x_3^{i,j} + u_3^{i,j} \\
&\quad + D_2 (x_3^{i+1,j-1} + x_3^{i+1,j} + x_3^{i,j-1} \\
&\quad + x_3^{i,j+1} + x_3^{i-1,j-1} + x_3^{i-1,j} - 6 x_3^{i,j}) \\
i &= 1 \cdots N,\ j = 1 \cdots M ,
\end{aligned} \tag{7}$$

where D_2 and D_3 are coupling constants. This system has the ability to generate rich spatiotemporal patterns such as Turing pattern and excitable wave propagation. In order to examine the dependence of the propagation velocity v of excitable wave as the coupling constant D_1 varied (D_3 is assigned zero), numerical simulations of the spatially one-dimensional coupling cells were carried out with the parameters in the section 2.3 (Fig.5). The results were fitted to the relation [3],

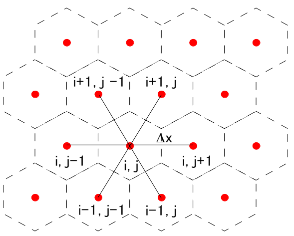


Fig.4 Hexagonal lattice

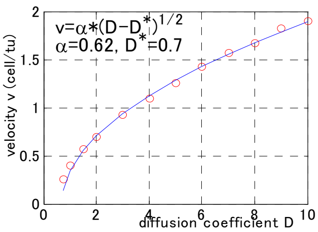


Fig.5 Excitable wave Velocity

$$v = \gamma \sqrt{D - D^*} \ , \tag{8}$$

where γ is a constant, D^* is a critical value at which propagation failure occurs. The critical coupling constant $D^* \approx 0.7$ was obtained.

3.2. Design of excitable wave propagation

Information operations with excitable-chemical systems has been studied, and it is shown that elemental logic devices such as OR, NOT and AND can be created by designing the suitable geometrical shape of an excitable field [2]. Compared with a chemical system, the proposed system has advantages: the characteristics of cell dynamics can design freely and the dynamic configuration of excitable field shape is possible by switching power supplies to the cells. When the power supply is stopped, the cell becomes passive.

Since the diffusively coupling exists without changing, the passive cell prevents the excitations of its neighbors. Therefore, the group of excitable cells is required so that the excitable wave is propagating in a system. In order to estimate the size of the required cells, propagation routes of 1-, 2-, 3-, 5-, 10- excitable cell widths were assigned in the 60×100 passive cells, and excitable wave propagations were examined (parameters were same as section 2.3). Results are shown in Fig.6. The top panel is the mask, which determines the shape of the discrete excitable field. The white area means excitable, and the black area means passive. Even if the width of the propagation route was one cell, the excitation wave propagation was succeeded. This is because the feedback gain parameter p_i is sufficiently larger than the dimensionless spring constant. With decreasing p_i, the required propagation route width increases. The width of a passive cell wall required for preventing the interference of the excitable waves was also examined. In the propagation routes over the three-cell width, excitable wave propagates beyond a passive cell wall of one-cell width. Excitable waves could not propagate beyond a 2-cell width passive cell wall.

The numerical simulation was carried out using the propagation routes of various angles in order to examine directional dependence of the propagation. By the use of hexagonal lattice, propagations succeeded regardless of the angle of the propagation route.

3.3. Application

3.3.1. Emergence of sensing function

The behavior of homogeneous one-dimensional coupled excitable cells in applying the external input is shown in Fig.8. The excitable cells in which the external inputs are applied are transformed into the oscillatory cells. The period of oscillation changes by the value of the external input. In order to emphasize this change, the time constant parameter ε is modified as $\varepsilon = \varepsilon_0(1+\alpha u_1)$. Even if there is the dispersion of the external inputs, the oscillatory cells are entrained by the local interactions. This oscillation generates excitable waves that propagate at constant speed. Therefore, it is possible to know the value of the input by the pulse interval at the edge. Fig.8 shows the relation between the external force and the pulse interval.

The detecting device that encodes the value of external input to the pulse interval and signal transmission line emerged from the homogenous system.

3.3.2. Autonomous conveyor utilizing an excitable wave

Applying the property of excitable wave that propagate at constant speed without deforming its shape, a system which carries the object of constant quantity in constant interval can be realized. Utilizing the proposed cells, only by arranging the propagation route, the object is autonomously carried to the assigned point. One example is shown in fig.9. The top right panel is the mask that has two input port and one output port. Since the asymmetric shape passive area is assigned at the junction (top left pane), excitable waves propagate only from input port to output port. By exchanging the mask, it is possible to set desired transport route.

4. Conclusion

The design method of discrete reaction diffusion field by spatially distributed piecewise linear control cells and the control method of excitable wave propagations in a two-dimensional coupling system are proposed. As a first step of the application, a sensing function emergence and an autonomous conveying system are demonstrated.

The geometric shapes of reaction diffusion field for information processing has been proposed [2], therefore, it is possible to construct the information processing function by assigning the similar shape. The speed does not lower even if multiple systems emerge, since the information processing is carried out parallel and simultaneously. By further study, it will become possible to emerge the more complex and more intelligent autonomous distributed control system at the desired point of the system.

References

[1] T.Ohta, "Physics of Non-equilibrium Systems", Shokabo, 2000 (in Japanese)

[2] I. Motoike and K. Yoshikawa, "Information operations with an excitable field", Phys.Rew.E, Vol.59, pp.5354-5360, 1999

[3] A.P.Muñuzuri, et al.,"Spatiotemplral structures in discretely-coupled arrays of nonlinear circuits: Review", Int.J.Bifurcation and Chaos, pp.17-50, 1995

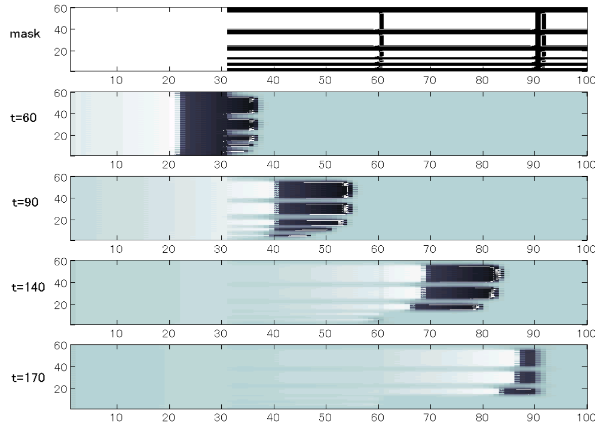


Fig.6 Effect of passive cell on the excitable wave propagation

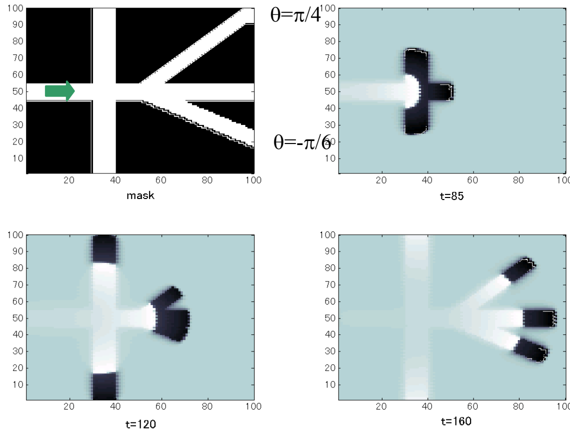


Fig.7 Directional dependence of the propagation

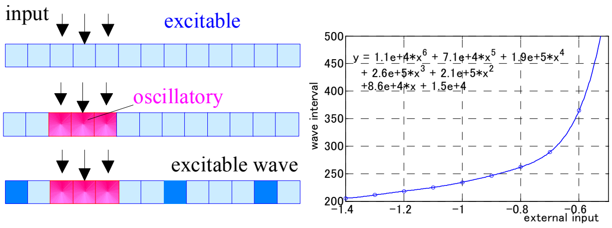


Fig.8 Emergence of sensing function

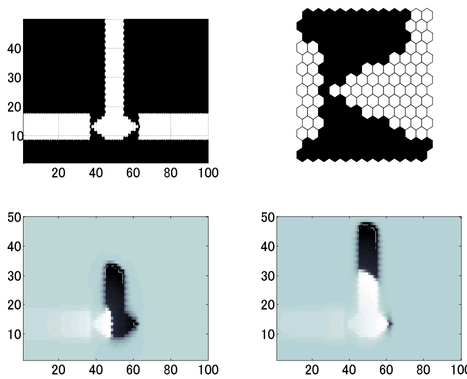


Fig.9 Autonomous conveyor utilizing an excitable wave