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An Idea of Errorless Coding and Reproduction of Arbitrarily Given Multiple Cyclic Binary Patterns by Means of Totalistic Rule of Cellular Automata

Teruhiko TAMURA†, Jousuke KUROIWA‡ and Shigetoshi NARA*

†Department of Electrical and Electronic Engineering
Okayama University, Okayama, 700-8530, Japan
teruhiko@chaos.elec.okayama-u.ac.jp
nara@elec.okayama-u.ac.jp

‡Department of Human and Artificial Intelligent Systems
Fukui University, Fukui, 910-8507, Japan
jou@i.his.fukui-u.ac.jp

Abstract

Recently, we have proposed a novel method which enables compressive coding and errorless reproduction of digital sound data by means of rule sequences of one dimensional cellular automata with two states and three neighbors. In this paper, we apply our idea to describing arbitrarily given multiple cycle patterns with use of totalistic rule of cellular automata with two states and many neighbors. This method leads to coding of the cycle patterns and potentially works as compressive coder with noise removing dynamics.

1. Introduction

Recently, by authors, a novel method which enables compressive coding and errorless reproduction of digital sound data with use of one dimensional cellular automata with two states and three neighbors (referred as the 1-2-3 CA) was proposed [1, 2]. The basic idea is corresponding to "rule dynamics" suggested by Aizawa and Nagai [3]. Based on our theoretical consideration and computer experiments, we have discovered that only two rules in all the $2^8 = 256$ rules are sufficient for compressive coding and errorless reproduction of the data [4]. Furthermore, it was shown [5] that, even if starting from any initial bit-pattern, the rule sequences determined to code the given sound data can reproduce the original data. It means that the rule sequences can works as a generator of "attractor dynamics".

Our idea to reproduce digital sound data by means of rule sequences of CA can be applied to various digital data, for an instance, animation data. Therefore our purposes of this paper are (i) to extend the idea for applying to coding and reproducing of arbitrarily given multiple cycle patterns which are rather long data strings represented by bit-patterns, and (ii) to investigate noise robustness of this new coding method potentially working as compressive coder with noise removing dynamics. This consideration is the first step in realizing coding of animation data.

2. Describing Method

2.1. Totalistic Rule

The compressive and errorless description of digital sound data with use of 1-2-3 CA has been successfully reported by the authors [4]. However, it is difficult to code and reproduce big binary data, for an instance two-dimensional animation data, by means of the 1-2-3 CA because they have varieties in time development of various local patterns which lead to contradictory updating rule sequences. In order to overcome this problem, it is a plausible strategy to introduce rules with use of many neighbors. But the problem is that many neighbors give a large number of rules, for an instance R neighbors gives 2^{2^R} rules in CA. This situation leads to difficulty in choosing an appropriate rule sequence which can reproduce the given multiple sequences of cycle patterns. Therefore, the new method is necessary in order to code arbitrarily given multiple cycle patterns.

We employ "totalistic rule for each cell" given by,

$$a_i^{t+1} = f_i\left(\sum_{j\in G_i} a_j^t\right) \tag{1}$$

where a_i^t (= 0 or 1) indicates the state of the ith site in one-dimensional cell array at time step t, the function $f_i(\cdot)$ takes 0 or 1, and G_i specifies the configuration of the neighbors of the ith site.

In this paper, the two methods of taking neighboring configuration are employed as shown below : (i) Symmetric neighboring configuration which, for an instance r neighbors on both right and left side from the ith cell, contains the ith cell itself and $2r$ neighboring cells, so that the total number of neighbors is $R = 2r + 1$. (ii) Asymmetric neighboring configuration which contains R neighbors. It should be noted that, in employing the both two neighboring configurations, the total number of neighbors of R have to be adjusted by the following procedure in order to reproduce the given multiple cycle patterns effectively.

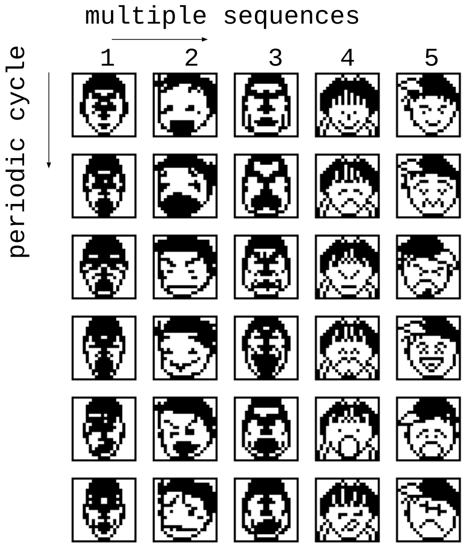

Figure 1: An example of multiple cycle patterns employed in the computer experiments. Each pattern consists of 20×20 pixels.

2.2. Strategy for Finding Rule Dynamics

Now, we propose the algorithm to determine the totalistic rule for each cell as follows, in other words, we present the method how to determine neighboring configuration and the function $f_i(\cdot)$ (0 or 1) depending on the vale of the summation $\sum_{j \in G_i} a_j^t$.

Method:

1. For each cell, assign the total number of neighbors R and the configuration of the neighbors, where the initial value of R is 1 on starting of this procedure.
2. Specify the value of the function $f_i(\cdot)$ which must reproduce all the multiple cycle patterns without error.
3. If errorless reproduction is not possible, then change the configuration of the neighbors. And continue the procedure 1 and 2 until obtaining the erroless reproduction. It should be noted that this procedure is not necessary in the case of the symmetric neighboring configuration .
4. If errorless reproduction is not possible for all the possible configurations, then increase the number of neighbors R and continue the procedure 1 and 2 until obtaining the erroless reproduction.

3. Computer Experiments

3.1. Experimental Conditions

In this paper, we employ the multiple cycle patterns as shown in Figure 1. Each pattern is represented by one-dimensional 400 bit-patterns obtained by

Table 1: Maximum, minimum and averaged total number of neighbors R.

	max	min	average
symmetric	627	1	178.06
asymmetric	129	1	52.19

the raster scanning. These type of the multiple cycle patterns can be easily embedded in neural network models (NN), and the system reveals attractor dynamics [6]. The main different point is as follows:

- In NN, the patterns are recorded in synaptic connections among neurons, which take "real value". If we employ the pseudo inverse method to determine them [6]
- In CA, the patterns are recorded in totalistic rule with binary value.

We will give our results of errorless reproduction of the multiple cycle patterns by dividing into two cases, (i) symmetric neighbor configuration and (ii) asymmetric neighbor configuration. In computer experiments, we employ the periodic boundary conditions when we update the edge cells.

3.2. Symmetric Neighbor

In Table 1 maximum, minimum and averaged total number of neighbors R in errorless description with use of totalistic rules for each cell for the multiple cyclic patterns are given. In Figure 2, at each cell site, the assignment of 0 or 1 to the function of the totalistic rule of $f_i(x)$ in Equation (1), where x can takes from 0 to R (R is the total number of neighbors), is depicted. One can observe that in describing the multiple cyclic patterns, specific value of 0 or 1 is not assigned to almost values of x in the function. Thus, we do not exhaust all the possible cases of the totalistic rule to reproduce the multiple cyclic patterns.

Let us investigate the noise robustness of our method. If noise is introduced in initial pattern or during updating of patterns, the updating of patterns sometimes falls into the function value of $f_i(x)$ which is not assigned specific value of 0 or 1 in complete pattern sequences. Then, we need to determine the rule, which the value of 0 or 1 should be employed. There would be many choices because a large number of values of $f_i(\sum_{j \in G_i} a_j^t)$ are not assigned. In this paper, we employ the following interpolated extension of the totalistic rule:

Interpolated extension: If to the value of x in the function, specific value of 0 or 1 is not assigned and also to the value of $x-1$ or $x+1$, specific value is

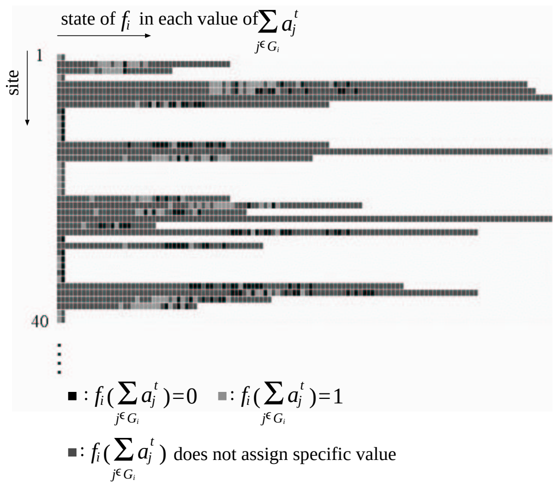

Figure 2: Totalistic rule of symmetric neighboring configuration for each cell used in complete reproduction of the multiple cycle patterns. The rules of only initial 40 sites are shown.

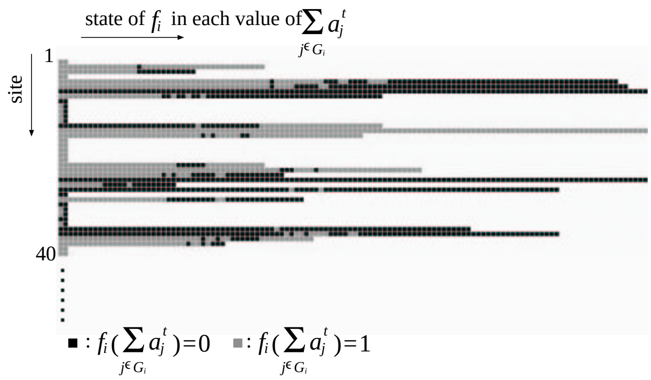

Figure 3: Interpolated totalistic rule symmetric neighboring configuration for each cell used in complete reproduction of the multiple cycle patterns. The rules of only initial 40 sites are shown.

assigned, one should assigns the same value of $x-1$ or $x+1$ to that of x. This procedure continues until specific value are assigned to all the possible values of x.

In Figure 3, the totalistic rule in the interpolated scheme is shown. Figure 4 shows pattern dynamics given by the interpolated extension of the totalistic rule with symmetric neighboring configuration, starting from an initial condition including the noise (five pixels inverted). One can observe that the time development is different from the original pattern sequence. However the pattern dynamics are not random and seems to be itinerant in the space near the original pattern. If a large deviation from the original pattern could occur, this method is weak in the sense of the noise removing functions.

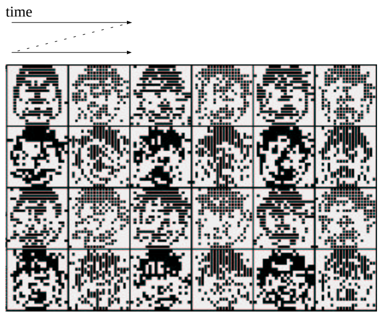

Figure 4: Pattern dynamics given by the interpolated extension of the totalistic rule with symmetric neighboring configuration, starting from an initial condition including the noise (five pixels inverted).

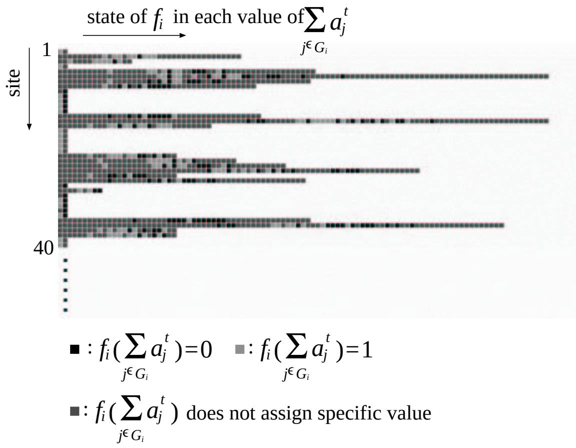

Figure 5: Totalistic rule of asymmetric neighboring configuration for each cell used in complete reproduction of the multiple cycle patterns. The rules of only initial 40 sites are shown.

3.3. Asymmetric Neighbor

From Table1, one can observe that the asymmetric neighboring configuration is much better in obtaining an effective totalistic rule than the symmetric one. In Figure 5 and Figure 6, the totalistic rule and the totalistic rule in the interpolated scheme are shown, respectively. In addition, Figure 7 shows pattern dynamics given by the interpolated extension of the totalistic rule with asymmetric neighboring configuration, starting from an initial condition including the noise (five pixels inverted). One can observe that the updating of patterns converges to the original pattern sequence.

It means that this interpolation scheme makes the updating be “an attractor dynamics” in the existence of noise.

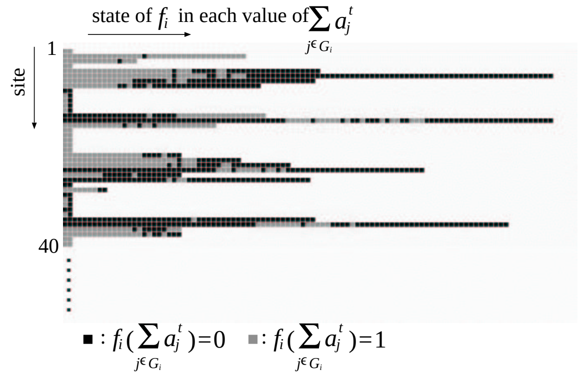

Figure 6: Interpolated totalistic rule of asymmetric neighboring configuration for each cell used in complete reproduction of the multiple cycle patterns. The rules of only initial 40 sites are shown.

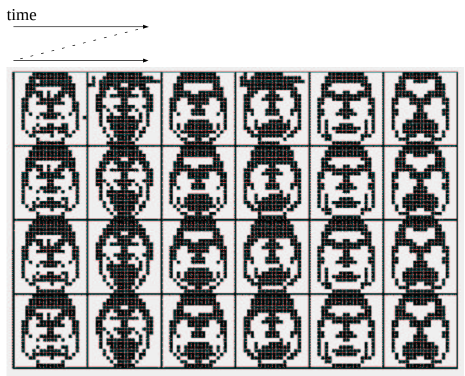

Figure 7: Pattern dynamics given by the interpolated extension of the totalistic rule with asymmetric neighboring configuration, starting from an initial condition including the noise (five pixels inverted).

4. Conclusions

In this paper, we apply our method [1, 2, 4] to describe arbitrarily given multiple cycle patterns with use of totalistic rule of cellular automata with two states and many neighbors. Main results are as follows:

- In both cases, symmetric neighboring configuration and asymmetric neighboring configuration, we succeeded that errorless and completely reproducible description of the pattern is possible by means of local rules assigned to each cell.
- The asymmetric neighboring configuration in the totalistic rule is much better in obtaining an effective totalistic rule than the symmetric one.
- By introducing the interpolated extension of totalistic rules in the case of neighboring configuration, we can succeed in the erroless reproduction of the original five multiple cycles even if one starts from a perturbed pattern such with a certain number of inverted pixels.

These computer experiments and considerations suggest that the interpolated assignment of totalistic rules makes the updating of patterns potentially be "an attractor dynamics" under the existence of noise, while in this report only the perturbed case with five inverted pixels was investigated as a preliminary evaluation. The attractor dynamics can be observed in reproduction of digital sound data by means of rule sequence of 1-2-3 CA [5].

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