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A Simple and Fractal Method of the TSP Approximation

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Abstract

In this paper, a practical method of creating curves in the Spacefilling Curve method in the Traveling Salesman Problem is proposed and its advantages are also referred. Our deliberate curves which are introduced in our process do not require any advanced mathematical knowledges, thus anyone can easily create and use them. Since in the case of low city density, precise solutions cannot be obtained, some special spacefilling curve with fractal structures are also proposed and discussed.

1. Introduction

The traveling Salesman Problem(TSP) is one of the famous optimization problem to select a minimum way of traveling around all the cities once only. Since there are $(N-1)!/2$ combinations for N cities, it is impossible to obtain a strict solution. In spite of that, from its industrial interest, VLSI circuit design, transportation planning etc., many kinds of approximation methods have been proposed so far. There is one method called "Spacefilling Curve Method" among them, which was proposed in 1982 by Bartholdi & Christfides. In their method, first the special curve which covers all over the area is prepared, and then they connect all cities along the curve. It is well-known that though the accuracy of solution is not very good, the speed of calculation is very high. The concrete algorithm of this method is as follows.

1. We draw a spacefilling curve like Peano curve all over the domain.
2. We replace every city to the nearest point on the curve.
3. From some point on the curve, we travel along the curve and connect cities in sequence.
4. We finish our travel when we return to the starting point.

Since their method has some disadvantages, an improved Spacefilling Curve method is proposed in this paper. Using this method, we can select a proper curve without any complexity and mathematical difficulty, while the speed of calculation is almost the same.

2. Abstract of our method

Though the Spacefilling Curve method is one of the fastest method, the solution cannot be expected to be precise. Moreover it requires some mathematical strictness. Even if we do mathematical hard work, we cannot always obtain reasonable rewards with regard to its accuracy and speed. Thus we make use of TSP approximation method instead of mathematically strict spacefilling curve, and propose a new method. Here we show how to create a new curve.

1. We arrange cities all over the domain.
2. We obtain an approximation solution for them using existing approximation method(This time we use NN method and 2-opt method: Fig.1).
3. Finally we follow and connect all cities along the above curve.

3. Simulation results

Here we show some results of our method for randomly distributed cities. For the comparison, we also show some results of Nearest Neighbor(NN) method. NN method is very popular and typical, and is known for its speed. We compared about total path length of solutions and computational time. Our results are summarized as follows.

1. From a computational time point of view, our method is extremely faster than NN method. In the former case, it increases linearly with numbers of cities, while it does exponentially in the latter(Fig.2).

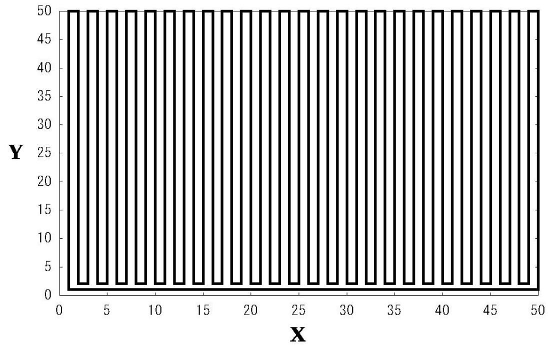


Figure 1: Examples of spacefilling curve in our method.

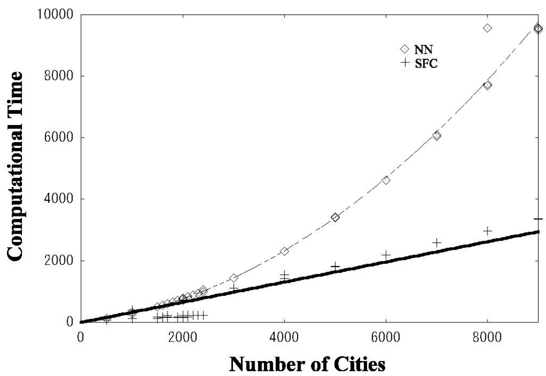


Figure 2: Comparison of computational time.

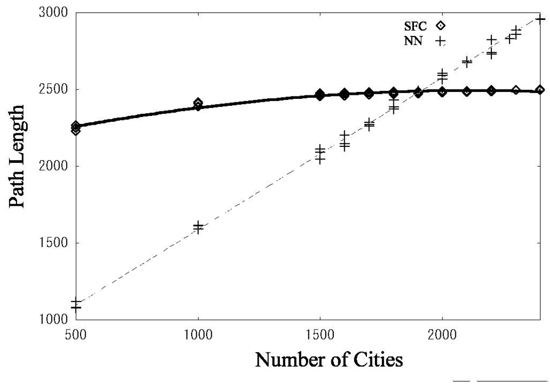


Figure 3: Comparison of path length.

2. From a precision point of view, if the city distribution is dense, we can obtain relatively good solutions, but if not, our results are poor. These quality of solutions depend on the selection of TSP approximation method in creating a new solution. In this paper we selected some better method than NN method.

From these results, an effective usage of our method can be thought that first we use our method for construction, then make use of other improved method for arrangement.

4. TSP solutions with obstacles

If there are some obstacles, we cannot obtain approximation solutions using usual methods, while our method can give some solutions. This is the most excellent advantage against the original Spacefilling Curve method.

Here we show how to create spacefilling curves with obstacles.

1. We distribute imaginary cities all the area except obstacles.
2. We look for an approximation solution for these cities using existing TSP method.
3. We regard above solution as a spacefilling curve.

We show a problem with one obstacle in Fig.4, where the solid line means a spacefilling curve avoiding an obstacle. In Fig.5, we show our result(a) and the one of existing method as a comparison. You might think the latter is better though, that is only a surface impression. The positional distance is certainly short in the latter, but if we think of an obstacle, the former length is 35.5 and the latter is 52.1. Thus ours give the shorter path actually.

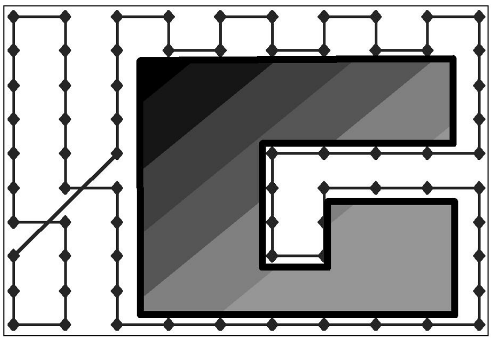
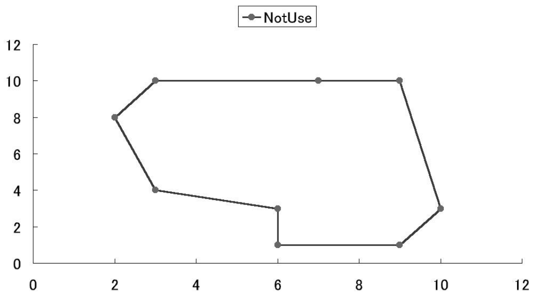
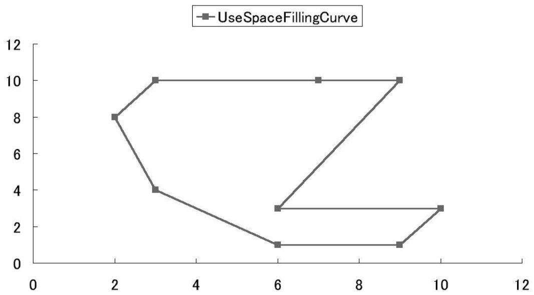


Figure 4: Problem with obstacles.



(a) Using our method.



(b) Using an existing method.

Figure 5: Solutions of obstacle problem.

5. Improved spacefilling curve in the case of low city density

As mentioned in the former section, if the city density is low, our method with imaginary spacefilling curve gives much worse solution than other methods. Now we think of improving how to create imaginary spacefilling curves. In the case of low density, those curves should have the characteristics that they can search well in the local area. In order to achieve the purpose we create imaginary spacefilling curve as follows.

1. We create a spacefilling curve as mentioned in a small area.

2. We align it repeatedly horizontally and vertically.

3. We regard one curve as a city, and obtain a TSP solution.

4. We connect curves as an above rule(Fig.6).

5. We repeat above processes in response to the needs.

Now we show some effects of new spacefilling curves. In Fig.6, imaginary spacefilling curves of 5x5 and their extension are shown. The comparison among NN method, usual our method and improved method with renormalization are also shown in Fig.7. It is clear that the characteristics of our method in the low density develop very much. Of course, the speed of our method is preserved. Therefore we can obtain fast and precise solutions by changing our two methods at some point.

6. Summary

In this paper, we proposed a simplified improved method of Spacefilling Curve method in TSP approximation. Our method does not require complicated mathematical knowledges and is easily to be handled. In order to obtain the better solutions, we created deliberate spacefilling curves using other TSP approximation solutions.

Our method using such curves has advantages as follows.

1. After we create a spacefilling curve, we can obtain a solution at high speed.

2. A relative precise solution can be obtained in the case of high city density.

3. A problem with some obstacles can be also handled.

Though our method had disadvantages in the case of low city density, we also proposed improved spacefilling curve with fractal structures. It might give us the possibilities for large scale TSP.

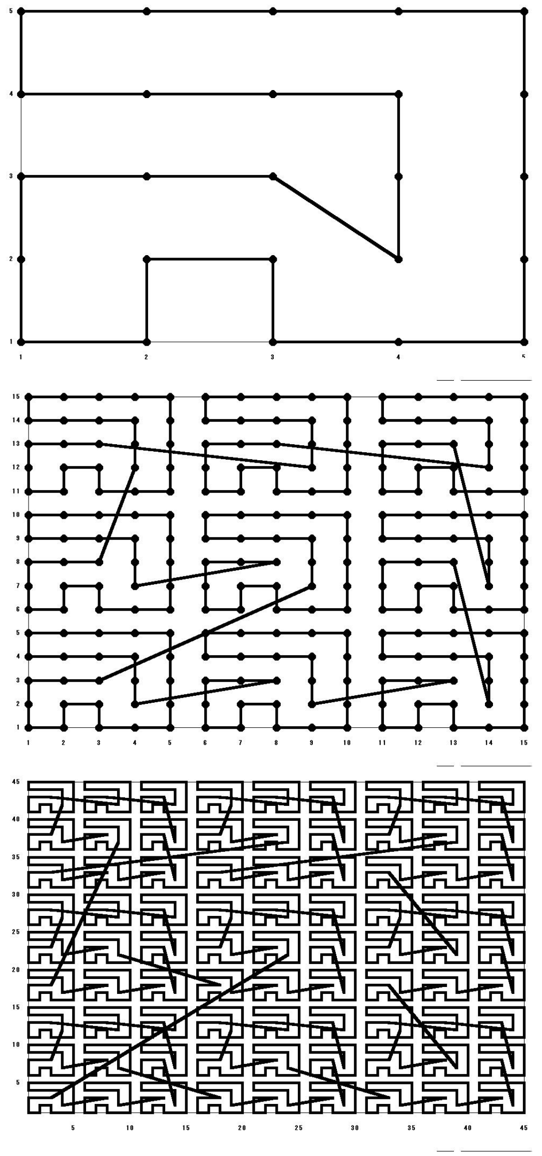


Figure 6: How to make a new spacefilling curve.

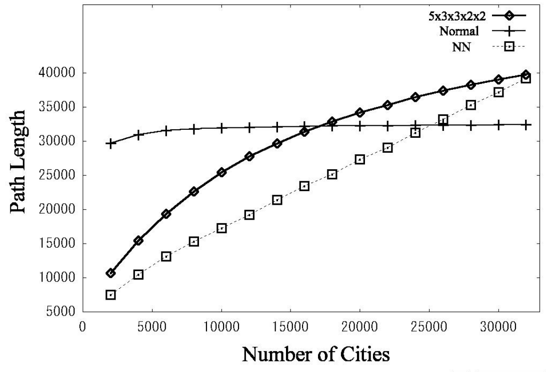


Figure 7: Effects of renormalized spacefilling curves.

References

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