On Variable Step-Size Adaptive Lattice Notch Filter for Detection Sinusoids

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Abstract

In this paper, the new second-order adaptive lattice IIR notch filter is proposed, based on variable step-size algorithm. The proposed adaptive lattice notch filter gives good convergence property, i.e., high convergence rate and low mean square error than the adaptive notch filter is used the direct form structure as based on gradient algorithm. Moreover, it gives a good performance of cancellation of 50-Hz interference in the recording of electrocardiograms signal. Therefore, the estimated parameters have high accuracy. Performance comparison of the proposed algorithm and the other algorithm is made through simulation results.

1. Introduction

Adaptive IIR notch filters (ANF) are very useful in various signal processing application such as the estimation of sine wave components with unknown frequencies from observation time series. For example, of application are in sonar, radar and communication, and the other [1-7].

The plain gradient algorithm (PG) [1] is the LMS-like algorithm which is very simple but work very well. However, this algorithm performs very low convergence speed when the algorithm is far from the optimum. The least mean p-power error criterion algorithm (LMP) [2], if a proper value of *p* is chosen, it will be worked better than the PG algorithm in many practical applications. For example, when we have selected a *p* value equal to 3, the speed of convergence of the algorithm will be improved but the impulse noise robustness will be poorly work. Another disadvantage of this algorithm that is high computational complexity when the value of $p = 3$ is chosen. The new memoryless nonlinear gradient algorithm (MNG) [3], was proposed by Yegi et.al., works better than the PG and LMP algorithms but the performance very low convergence speed when the algorithm is far from the optimum same as the PG algorithm.

This paper presents the new algorithm for a second-order adaptive lattice IIR notch filter. The proposed algorithm provides fast convergence speed, and reduce the mean square error (MSE) and have low variance of filter coefficients. The simulation results for the adaptive detection of a sinusoid with additive white Gaussian noise and canceling 50-Hz interference in the recording of the electrocardiograms (ECG) signal are tested by using the computer programs shows that the proposed algorithm is consistent.

2. Structure and Algorithm Descriptions

2.1 Adaptive IIR Notch Filter

The transfer function of a second-order an adaptive IIR notch filter of the direct form structure can be express as follows:

$$H_d(z) = \frac{1 + az^{-1} + z^{-2}}{1 + \rho a z^{-1} + \rho^2 z^{-2}} \tag{1}$$

where a is the filter coefficient will be estimated by an adaptive algorithm, and should converge to $-2\cos(\omega_o)$ to reject a sinusoidal with frequency ω_o, ρ is the pole construction factor should be close but less than unity to ensure the stability of the filter. The larger the pole radius ρ is, the narrower the bandwidth of the IIR notch filter.

The transfer function of adaptive IIR notch filter of the lattice form structure can be express as follows:

$$H(z) = \frac{1 + 2k_0 z^{-1} + z^{-2}}{1 + k_0(1+\rho)z^{-1} + \rho z^{-2}} \tag{2}$$

Next, we consider a noisy sinusoidal with unknown frequency ω_o, amplitude *A*, and phase ϕ that is uniformly distributed between 0 and $2p$, therefore, the input data are written as

$$x(n) = A\cos(\omega_o n + \phi) + v(n) \tag{3}$$

where $v(n)$ is the zero-mean white noise or impulse noise.

The difference equation of the output signal of the second-order adaptive lattice IIR notch filter can be express as follows:

$$\begin{aligned} y(n) &= u(n) + 2k_0 u(n-1) + u(n-2) \\ u(n) &= x(n) - k_0(1+\rho)u(n-1) - \rho u(n-2) \end{aligned} \tag{4}$$

The gradient signal of the second-order adaptive IIR notch filter can be express as

$$g(n) = \frac{\partial y(n)}{\partial k_0} = 2u(n-1) \tag{5}$$

The PG algorithm used to update the filter coefficient is given by

$$a(n+1) = a(n) - \mu.y(n).g(n) \tag{6}$$

The LMP algorithm used to update of the filter coefficient is respectively shown as follows

$$a(n+1) = a(n) - \mu.p.\operatorname{sgn}[y(n)].y(n)^{p-1} g(n), p: odd \tag{7a}$$

$$a(n+1) = a(n) - \mu.p.y(n)^{p-1} g(n), p: even \tag{7b}$$

The MNG algorithm used to update of the filter coefficient is respectively shown as follows

$$a(n+1) = a(n) - \mu.y(n)\frac{g(n)}{1+\mu(g(n))^2} \tag{8a}$$

$$a(n+1) = a(n) - \mu \cdot y(n) \cdot \frac{g(n)}{1+\varepsilon \cdot (g(n))^2}, \varepsilon \geq \mu \tag{8b}$$

where $\operatorname{sgn}(\cdot)$ is the sign operator, μ is the step-size parameter.

This paper, we have proposed the new algorithm for a second-order adaptive lattice IIR notch filter that gives fast convergence speed and good performance in statistical senses in the next section.

2.2 Variable Step-Size Algorithm for Adaptive Lattice IIR Notch Filter

In the update recursion of the filter coefficient, if we can control the amount overall of the update recursion is large in the initial, the algorithm gives fast convergence speed. And after approach the optimum, the amount overall of the update recursion is small, we ensure that the algorithm gives low MSE. To approach this result, the adaptation step-size is adjusted using the energy of the instantaneous output signal. The new algorithm of notch filter parameter update recursion for adaptive lattice structure IIR notch filter is given by

$$k_0(n+1) = k_0(n) - \mu(n) \cdot y(n) \cdot \operatorname{sgn}\left(\frac{g(n)}{1+|g(n)|}\right) \tag{9}$$

The objective is to ensure large $\mu(n)$ when the algorithm is far from the optimum, $\mu(n)$ decreasing as we approach the optimum. The proposed algorithm achieves this objective by using an estimate of energy of output signal of ANF to control step-size update is given by

$$p(n+1) = \sigma.p(n) + (1-\sigma).y^2(n) \tag{10}$$

In the early stages of adaptation, the signal energy $p(n)$ is large, resulting in a large $\mu(n)$. As we approach the optimum, the signal approach zeros, resulting in a smaller step-size. This provides the fast convergence due to large initial $\mu(n)$ while ensuring low gradient noise due to the small final $\mu(n)$. Thus, the proposed variable step-size update is given by

$$\mu(n+1) = \alpha.\mu(n) + \gamma.p(n) \tag{11}$$

Where $0 < \sigma, \alpha, \gamma < 1$, σ is called exponential weighting parameter or forgetting factor and it is the quality of the estimation. In stationary environment, previous samples contain information that relevant to determining an accurate measure of adaptation state.

3. Simulation Results

In this first simulation, we have used the proposed ANF algorithm estimated a single sinusoidal frequency additive white Gaussian noise. And the angular frequency of the input sinusoidal $\omega_0 = \pi/3$ which are corresponded to the notch parameter $k_0(n)$ = –0.5 and $a(n)$ = –1.5, respectively.

In order to confirms the availability of the proposed algorithm, we used the parameters SNR=10 dB, $p = 3$, A=1, ε =0.08, $a(0) = 0, \phi(0) = 0$,

ρ = 0.9 and μ = 0.02 for the previous algorithms, and the parameter σ = 0.98, α = 0.95, γ = 0.001, $k_0(n)$ = 0, and $\mu(n)$ = 0.02 for the proposed algorithm, respectively.

The Figures 1.1 and 1.2 show the simulation result of the estimated filter coefficient "$k_0(n)$" and "$a(n)$" in Gaussian noise environment, respectively. It is seen that the proposed algorithm given fast convergence speed and low variance better than the results of the PG, LMP, MNG algorithms in Figure 1.2.

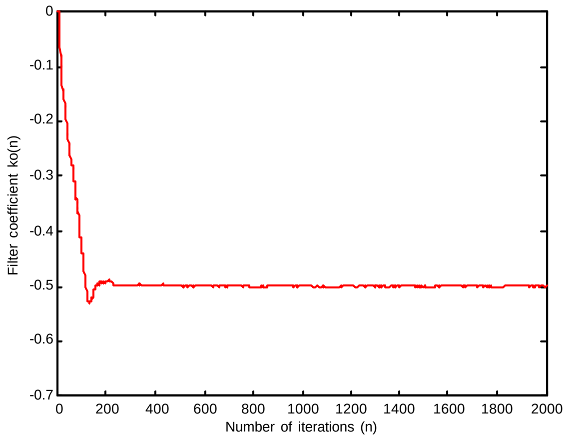


Figure 1.1 Shows the convergence process of the estimated filter coefficient $k_0(n)$ of the proposed algorithm.

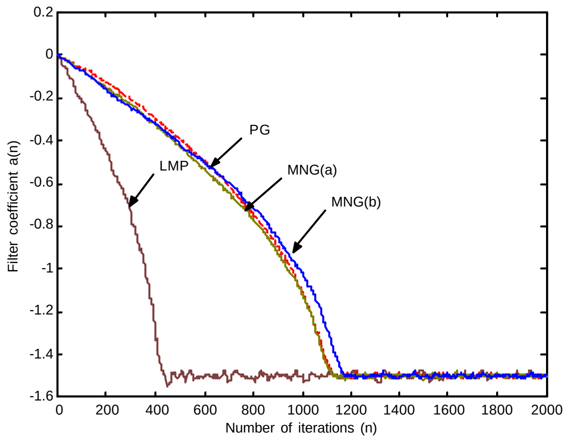


Figure 1.2 Shows the convergence processes of the estimated filter coefficient $a(n)$ of : the MNG, the LMP and the PG algorithms.

In Figure 2 shows an estimated steady-state MSE results derived from the PG, LMP, MNG and the proposed algorithms at the same step-size parameter value. The proposed algorithm have fast convergence speed and good performance in statistical sense such as low steady-state mean square error better than the other algorithms.

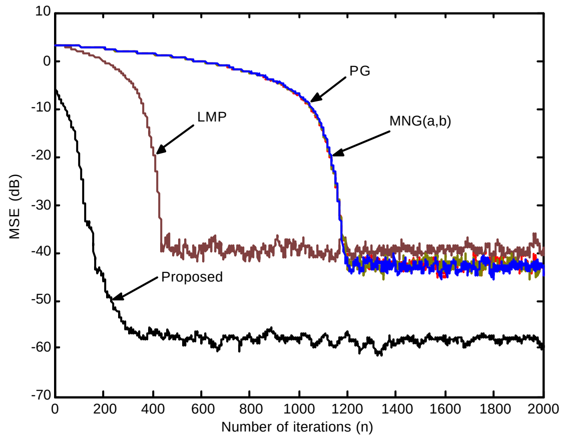
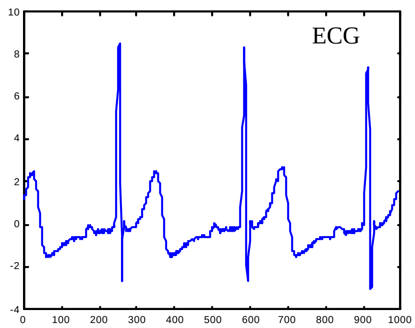
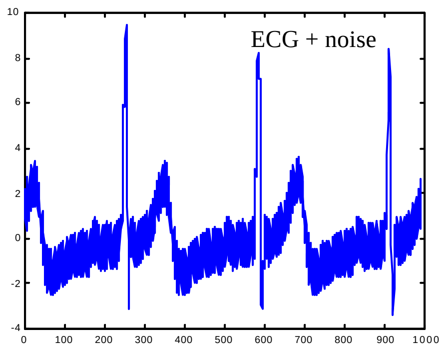


Figure 2 Comparison of an estimated steady-state MSE of : the MNG, the LMP, the PG and the proposed algorithms. Simulation averaging 100 runs.

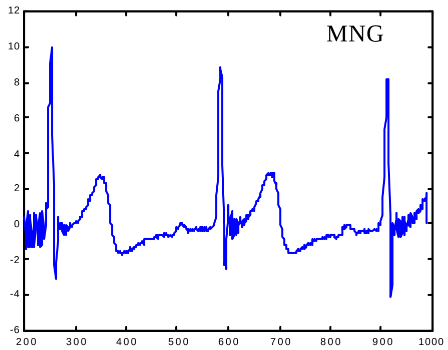
In this second simulation, the method capable of reducing 50-Hz interference in ECG signal is ANF. Figure 3(a) shows an ECG waveform. Figure 3(b) shows an ECG waveform with excessive amount of 50-Hz interference. And Figure 3(c) and 3(d) show the output from an ANF employing the MNG 's and the proposed algorithms with the same step-size parameter values $\mu = \mu(n) = 0.02$, respectively. It is seen that the proposed algorithm given good results in ECG signal.



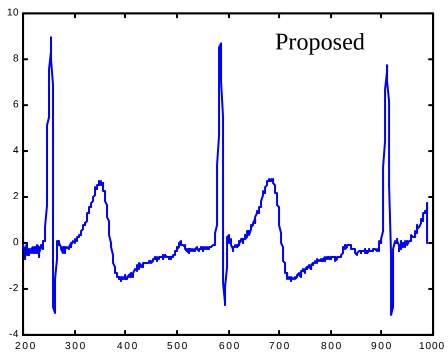
3(a)



3(b)



3(c)



3(d)

Figure 3 Canceling 50-Hz power line interference in electrocardiography. (a) ECG, (b) ECG waveform with 50-Hz interference. (c) and (d) are output of adaptive notch filter employing the MNG 's and the proposed algorithms at the same step-size parameter.

4. Conclusions

This paper we have proposed the second-order adaptive lattice IIR notch filter based on variable step-size algorithm. The new algorithm performs fast convergence speed, low steady state mean square error and low variance of the filter coefficient when compared with other algorithms that which is used the direct form structure as based on gradient algorithm. Therefore, the estimated parameters of the new algorithm have high accuracy.

5. References

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