# A New EM_Based Algorithm for the Tracking of the Maneuver Target in Cluttered Environment

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**Abstract** The tracking of the maneuver target in cluttered environment is a difficult problem in surveillance systems study. In this paper, the maneuver input is described by using a finite-state Markov chain, and the Expectation Maximization (EM) algorithm is adopted to solve so-called "incomplete data problem". The contributions in this paper are mainly in determining the maneuver input sequence optimally, and separating the measurement sequences coming from target efficiently by means of the EM algorithm. Furthermore, the more precise state estimates of the target can be obtained according to the procedure. The simulation results indicate that the performance of the new algorithm is better than the existing ones.

**Keywords** EM algorithm, Viterbi algorithm, maneuver input sequence, measurement sequence

## 1. Introduction[1]

An important and difficult problem in surveillance systems is the tracking of the maneuver target in cluttered environments. The IMM-PDA algorithm can provide a solution to track the maneuver target in cluttered noise environment [3]. But the approach uses N filters running in parallel at every sampling time. In each filter, the state is updated using all measurements within the validation gate. The will result in the high computational cost and the low estimation accuracy because of the false measurement and the incorrect models.

To simplify, the maneuver input sequence is considered as the cause of the target maneuver, and other system matrices are described as constant in this paper. The maneuver input sequence is regarded as a discrete-time Markov chain with known initial probability distribution and transition probabilities matrix.

The aim of the paper is to seek the optimal maneuver input and measurement sequences based on the principle of maximum a posterior (MAP). Consequently, the filter estimates can be obtained

The Expectation Maximization (EM) algorithm [1,2] is an efficient means of finding the ML or MAP estimate of the parameters of an underlying distribution from a given data set when the data is incomplete or has missing values. It is an iterative scheme. Each cycle consists of two steps. At first step (E-step), the cost function is obtained by means of calculating the log-expectation of the joint probability density function of the complete data and parameters. At second step (M-step), the maximization computation is efficiently implemented by using the extended Viterbi algorithm [4]. The principle of EM algorithm assures that the joint probability density of parameters increases at each iteration. The two steps are repeated as necessary.

The paper is organized as follows. Section 2 formulates the problem. In section 3, the procedure of seeking MAP estimation of the maneuver input and measurement sequences by using EM algorithm is introduced. The simulation results are illustrated in section 4. In section 5, the conclusions and discussion are given.

## 2. Problem Formulation

Firstly, it is supposed that there is one single target in the surveillance region. The dynamic equation is modeled as

$$x(k+1) = F(k)x(k) + Gv(k) + Ju(k) \qquad (1)$$

If measurement $z_j(k)$ originates from the target, it satisfies the measurement equation

$$z_j(k) = H(k)x(k) + w(k) \qquad (2)$$

$j = 1,2,\ldots,m(k)$; (the notation $m(k)$ indicates the number of measurements at scan $k$ )

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If some measurement is a false alarm, it is considered as a random variable of uniformity distribution within the validation gate.

Let $x(k)$ be the state information of the target at scan $k$. The state history up to and including scan K can be expressed as follows

$$X^K \stackrel{\Delta}{=} \{x(i)\}_{i=1}^K \qquad (3)$$

The total set of measurements up to and including scan $K$ is given by below

$$Z^K \stackrel{\Delta}{=} \{Z(i)\}_{i=2}^K \qquad (4)$$

Where $Z(i)$ denote the total set of measurements at scan $i$, and

$$Z(i) = \{z_j(i)\}_{j=1}^{m(i)} \qquad (5)$$

$F, G, J$ and $H$ are system matrices. The process noise $v(k)$ and measurement noise $w(k)$ are mutually uncorrelated zero-mean white Gaussian process with known covariance matrices $Q$ and $R$ respectively.

Let $u(k)$ be an unknown input sequence which is described as a discrete time Markov chain. It is assumed that there are a total of $r$ possible maneuver inputs $u(k) \in U = \{u_1, u_2, ..., u_r\}$

The initial probabilities of input $u_i$ are supposed known.

$$\rho_i = P(u(1) = u_i), i = 1,2,...r \qquad (6)$$

The switching from input $u_i$ to $u_j$ is governed by a finite-state Markov chain with transition probabilities

$$\pi_{ij} = P(u(k+1) = u_j \mid u(k) = u_i) \qquad (7)$$
$$i, j = 1,2,..., r$$

The input sequence up to and including scan K is denoted

$$U^K \stackrel{\Delta}{=} \{u(i)\}_{i=1}^K \qquad (8)$$

The sequence of measurements up to and including scan $K$ which originates from the target is defined as

$$\Phi^K \stackrel{\Delta}{=} \{\Phi_i\}_{i=2}^K \qquad (9)$$

$\Phi_i \in \{0,1,2,..., m(i)\}$ is defined as follows

$$\begin{cases} \Phi_i = j & \text{if measurement } z_j(\text{i}) \text{ originats from the traget at scan i} \\ \Phi_i = 0 & \text{if all measurements are false alarms at scan i} \end{cases} \qquad (10)$$

According to the above nomenclature, the parameters to estimate in this paper are defined as:

$$\theta \stackrel{\Delta}{=} \{\Phi^K, U^K\} \qquad (11)$$

If a batch of measurements $Z^K$ has been received, the aim of the paper is to maximize the joint probability density function $p(\theta \mid Z^K)$ over all $r^K \times \prod_{i=1}^K (m(i)+1)$ possibilities. An off-line algorithm is introduced in this paper.

**3. Sequences Estimation by Using Em Algorithm (in MAP Sense)**

The EM algorithm is implemented by the following iteration

$$\begin{cases} & Q(\theta, \theta_p) \\ E\text{ step} & = E_{\text{miss data}}(\log p(\text{complete data}, \theta) \mid \\ & \theta_p, \text{observe data}) \\ \text{M step} & \theta_{p+1} = \arg\max_\theta Q(\theta, \theta_p) \end{cases} \qquad (12)$$

In this paper, the measurement set $Z^K$ is considered as the “observe data”, which is also called the “incomplete data”. The “miss data” consists of the state history of the target $X^K$. The integration of the “observe data” and the “miss data” forms the “complete data”.

This paper aims to seek the optimal maneuver input sequence which plays function on the system dynamic equation and the measurement sequence which originates from the target by using EM algorithm. Therefore, the state estimate of the target can be obtained by means of the two sequences.

**3.1 E step**

$$\begin{aligned} & Q(\theta, \theta_p) \\ & = E_{miss\ data}\{\ln p(\text{complete data}, \theta) \mid \\ & \theta_p, \text{observe data}\} \\ & = E_{X^K}\{\ln p(U^K, \Phi^K, X^K, Z^K\} \\ & \mid U_p^K, \Phi_p^K, Z^K\} \\ & = E_{X^K}\{\ln\{p(u(1)) \times p(x(1)) \times \\ & \prod_{i=2}^K (p(u(i) \mid u(i-1)) \\ & \times p(\Phi_i \mid \Phi_{i-1}, u(i-1), x(i-1)) \\ & \times p(x(i) \mid x(i-1), u(i-1)) \\ & \times p(z(i) \mid x(i)))\} \mid U_p^K, \Phi_p^K, Z^K\} \end{aligned} \qquad (13)$$

The details of the inference about the function $Q(\theta, \theta_p)$

aren't described here.

### 3.2 M step

$$\begin{aligned}&\theta_{p+1}\\&=\arg\max_{\theta} Q(\theta,\theta_p)\\&=\arg\min_{\theta}(-Q(\theta,\theta_p))\end{aligned} \tag{14}$$

The function to minimize is as follows.

$$J(U^K,\Phi^K) \tag{15}$$

$$\overset{\Delta}{=} -E_{X^K}\{\ln p(U^K,\Phi^K,X^K,Z^K\}\,|\,U_p^K,\Phi_p^K,Z^K\}$$

Thus, the problem is changed into a minimum problem. It is well known that the Viterbi algorithm gives us a recursive procedure for finding the single path with the smallest path metric. In this paper, because we intend to optimize both input sequence and measurement sequence, it is infeasible by Using Viterbi algorithm directly. The idea in literature [6] is adopted in our optimal procedure. In each scan k, the 2-tuple of measurement and input forms the node used in trellis. Figure 1 illustrates the idea. Where, the maneuver input sets consist of two elements $\{u_1,u_2\}$, two measurements are received at each scan. The notation $z_0$ indicates miss detection.

The scan time $k$ represents the $k^{th}$ stage. The notation $\cos t(k,i,j)$ means the cost connected node $i$ at stage $k$ to node $j$ at stage $k+1$. To define it, another notations are introduced. The notation $node\_\Phi(k,i)$ means the measurement index at node $i$ at stage $k$, and $node\_u(k,i)$ means the input at node $i$ at stage $k$.

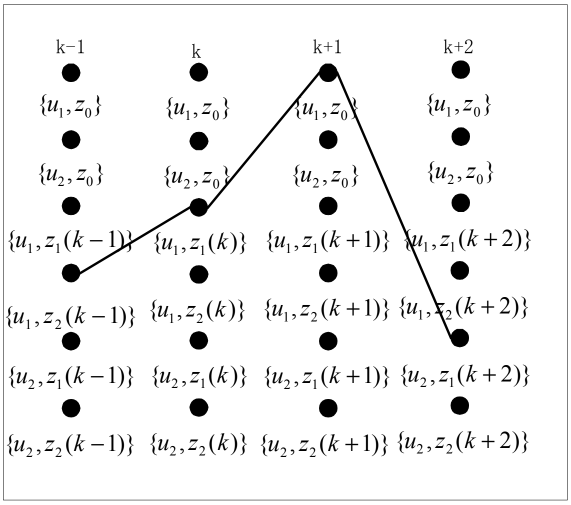


Figure 1:Viterbi trellis adopted in our method

Thus,

$$\begin{aligned}&\cos t(k,i,j)\\&=-E_{X^K}\{\ln p(node\_u(k+1,j)\,|\\&node\_u(k,i))\\&\times p(node\_\Phi(k+1,j)\,|\,node\_\Phi(k,i),\\&node\_u(k,i),x(k))\\&\times p(x(k+1)\,|\,x(k),node\_u(k,i))\\&\times p(z_{node\_\Phi(k+1,j)}(k+1)\,|\,x(k+1))\,|\\&U_p^K,\Phi_p^K,Z^K\}\end{aligned} \tag{16}$$

If $k=1$, the initial term

$$-E_{X^K}\{\ln p(node\_u(1,i))\,|\,U_p^K,\Phi_p^K,Z^K\}$$

should be added.

### 3.3 Simulation steps

1.Initialization:l=1,given initial parameter value $\Phi_1^K,U_1^K$, a little positive $\varepsilon$

2. Recursion：for l= 2, 3, …

(1) Expectation Step

According to the formula (8) and (9), compute the cost function of the nodes which lie the two adjacent stage.

(2) Maximization Step

Seek the optimal measurement and model sequence $\Phi_{l+1}^K,U_{l+1}^K$ by using Viterbi algorithm

3.Termination

When l=q+1

If $\left\|\Phi_{q+1}^K-\Phi_q^K\right\|\le\varepsilon, and \left\|U_{q+1}^K-U_q^K\right\|\le\varepsilon$

$$\Phi^*=\Phi_q^K,U^*=U_q^K \tag{17}$$

## 4. Simulation Results

In this section, the relative performance of IMM-PDA and the method presented in this paper are provided.

$$x(k+1)=\begin{bmatrix}1&T&0&0\\0&1&0&0\\0&0&1&T\\0&0&0&1\end{bmatrix}x(k)+$$

$$\begin{bmatrix}T^2/2&0\\T&0\\0&T^2/2\\0&T\end{bmatrix}(v(k)+u(k)) \tag{18}$$

$$z(k)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}x(k)+w(k) \qquad (19)$$

Here, the sampling period $T$ =1

the initial state of the target $x0$ =[10 2.5 18 4.5],the process noise covariance $Q=4I$ ,the measurement noise covariance $R=2.5I$ , the maneuver input is considered as the cause of the target maneuver, and $u(k)\in U=\{u_1,u_2,u_3\}$ ,

$u_1=\begin{bmatrix}-3\\-2\end{bmatrix}, u_2=\begin{bmatrix}1\\-1\end{bmatrix}, u_3=\begin{bmatrix}2\\3\end{bmatrix}$, the initial probability of input $\rho=\{\frac{1}{3},\frac{1}{3},\frac{1}{3}\}$ ,the transition probabilities of maneuver input $\pi=\begin{bmatrix}0.75 & 0.25 & 0\\ 0.125 & 0.75 & 0.125\\ 0 & 0.25 & 0.75\end{bmatrix}$ , the actual input at every scan

$$u(k)=\begin{cases}u_1 & 0\le k<5\\ u_2 & 5\le k<10\\ u_3 & 10\le k<15\\ u_2 & 15\le k<20\\ u_3 & 20\le k<30\end{cases} \qquad (20)$$

The figures (2-3) illustrate the simulation results by IMM-PDA and the EM_based method presented in this paper.

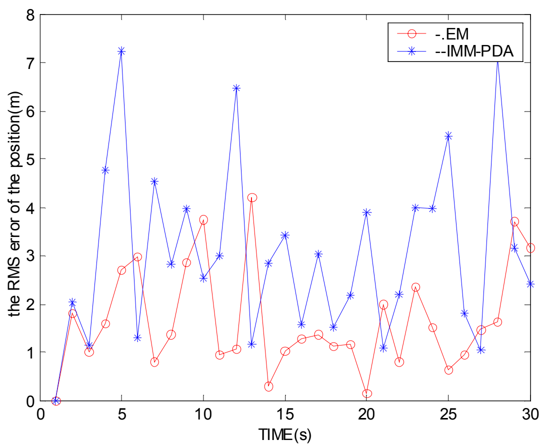


Figure 2:RMS error of position of the target

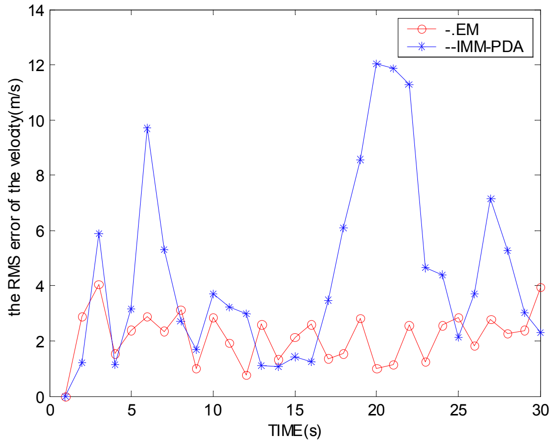


Figure 3:RMS error of velocity of the target

## 5. Conclusions

The simulation results show that the method presented in this paper can be used to seek the optimal measurement and input sequences successfully. Our method performs better than the contrastive method. In addition, there are some deficiencies to deserve to further research in this paper. For example, the method is limitative when maneuver input is not in the input set. As the model set and the measurement set augment, the mass of the 2_tuple combination will result in the high computational cost. As a result, the Viterbi search will go slow. The above facets mentioned are expected to improve in the further research.

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