COMPLEX DYNAMIC BEHAVIOUR IN A LINEAR THIRD-ORDER CONTINUOUS SYSTEM WITH HYSTERESIS SERIES

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Abstract This paper studies dynamic behaviour in a linear third-order continuous system with hysteresis series. The 1-dimension (1-D) n-scroll chaos and 1-D abnormal chaos can be obtained when one state variable is input to hysteresis series. Complicated limit-circle and 2-D multi-scroll chaos can be obtained when two state variables are input to hysteresis series.

Keywords non-linear systems, hysteresis, chaos, limit-circle

1. Introduction

Some publications deal with dynamic behaviour of hysteresis[1-5]. With a particular class of circuit which includes one nonlinear element, a three segments piecewise, one higher dimensional chaotic circuit can be obtained[1]. The general properties of static and dynamic hysteresis were study by Mauro Parodi[2,3]. An RC operational transconductance amplifier (OTA) hysteresis chaos generator was devised in [4]. Marco Storace presented a hysteresis-based chaotic circuit, the dynamics of the circuit is discussed in detail in [5].

This paper shows complex dynamic behaviours occur in a linear third-order continuous system with hysteresis series. The multi-limit circles and chaos can be obtained by input some state variables to the hysteresis series when proper parameters are chosen.

For all the simulation in this paper, if the initial condition are not written on the figure, then (x_0, y_0, z_0) = (1.2134,0.5793,0.0198).

2. System Model

The linear third-order continuous system can be described as follows:

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = ax + by + cz \end{cases} \qquad (1)$$

where x, y and z are state variables; a, b and c are constant.

The basic hysteresis function when input is within the limitation of $-1<x<1$ is shown in Fig.1. it can be described as equ.(2). Hysteresis series can be described as equ.(3) and is shown in Fig.2.

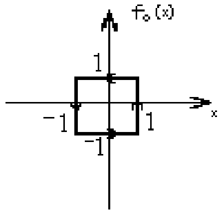


Fig.1 Basic hysteresis function

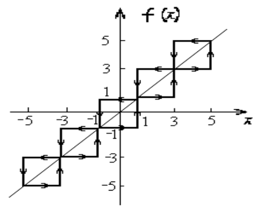


Fig.2 Hysteresis series

$$f_0(x) = \begin{cases} +1 & \dot{x} < 0 \\ -1 & \dot{x} > 0 \end{cases} \qquad (2)$$

$$\phi_k(\zeta) = \sum_{k=-m_2}^{m_1} f_k(\zeta) \quad 2k-1 \le \zeta \le 2k+1 \qquad (3)$$

where within the limit $2k-1<\zeta<2k+1$:

$$f_k(\zeta) = \begin{cases} 2k+1 & \dot{\zeta} < 0 \\ 2k-1 & \dot{\zeta} > 0 \end{cases} \qquad (4)$$

if k=0, equ.(4) takes the form of equ(2).

3. One state variable is input to hysteresis series

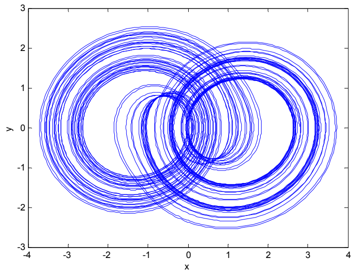
When state variable x is input to hysteresis series $\phi_k(x)$, we assume the state equation takes the form of equ.(5).

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = a(x + y + z) + d\phi_k(x) \end{cases} \qquad (5)$$

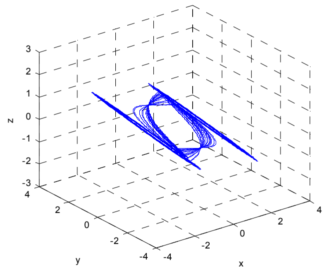
With different k of $\phi_k(x)$ and different coefficient d, equ.(5) will have different dynamic behaviour.

3.1. a=-0.8, d=0.8, 1-D (m_1+m_2+2)-scroll chaos

If a=-0.8, d=0.8, $\phi_k(x)$ takes the form of equ.(3) and equ.(4). 1-D (m_1+m_2+2)-scroll chaos can be obtained. The number of scroll equals to the number of equilibrium points. The (m_1+m_2+2) equilibrium points are: $(-2m_2-1, 0, 0)$, …, (−1, 0, 0), (1, 0, 0), (3, 0, 0), … , $(2m_1+1, 0, 0)$. When m_1=0, m_2=0, equ.(3) corresponding to the basic hysteresis of equ.(2), double-scroll chaos can be obtained, the *x-y* projection and *x-y-z* trajectory are shown in Fig.3. When m_1=1, m_2=0, 3-scroll chaos can be obtained, the *x-y* projection and *x-y-z* trajectory are shown in Fig.4. m_1=2, m_2=2, 6-scroll chaos appears, the *x-y* projection is shown in Fig.5.

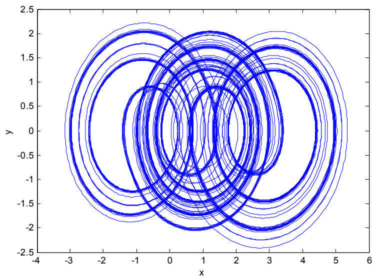


(a) *x-y* projection

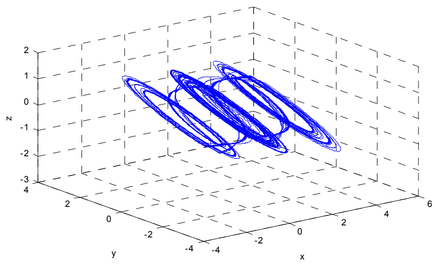


(b) *x-y-z* trajectory

Fig.3 m_1=0, m_2=0, 1-D double-scroll chaos,



(a) *x-y* projection



(b) *x-y-z* trajectory

Fig.4 m_1=1, m_2=0, 1-D 3-scroll chaos

3.2. *a*=-0.8, *d*=1.0, 1-D abnormal chaos

If *a*=-0.8, *d*=1.0, the response trajectories are also chaotic, but the equilibrium points number and position are abnormal depending on the initial conditions. After running for some time, the trajectory is attracted by an equilibrium ponit, and chaotic response around this equilibrium point. m_1=1, m_2=2, the trajectories with different initial conditions are shown in Fig.6.

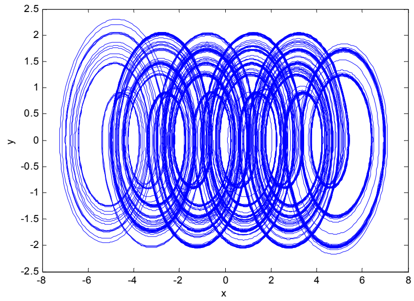
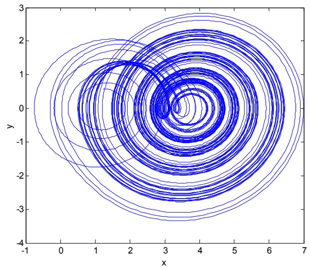
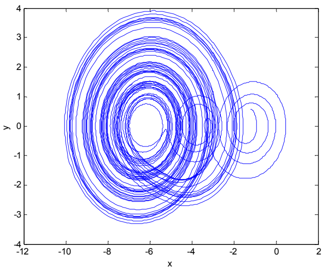


Fig.5 m_1=2, m_2=2, *x-y* projection of 1-D 6-scroll chaos



(a) (x_0, y_0, z_0)=(1.2134, 0.5793, 0.0198)



(b) (x_0, y_0, z_0)=(-1.2134, 0.5793, 0.0198)

Fig.6 m_1=1, m_2=2, *x-y* projection of unsymmetrical chaos

4. Two state variables are input to hysteresis series

When the state variables *x* and *z* are input to hysteresis series which as described in equ.(6).

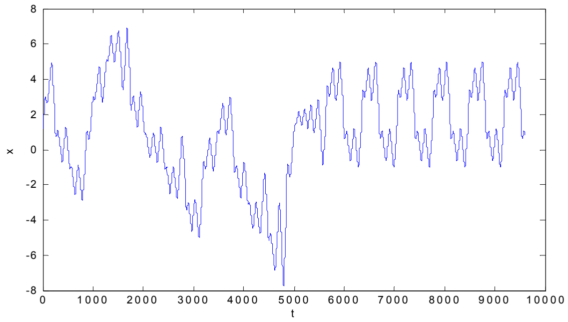
$$\begin{cases} \dot{x} = y + a\phi_{k2}(z) \\ \dot{y} = z \\ \dot{z} = a(x + y + z) + d\phi_{k1}(x) \end{cases} \tag{6}$$

where $\phi_{k_2}(z)$ is the case k_2= -n_2,…, -1,0,1, …,n_1 in equ.(3).

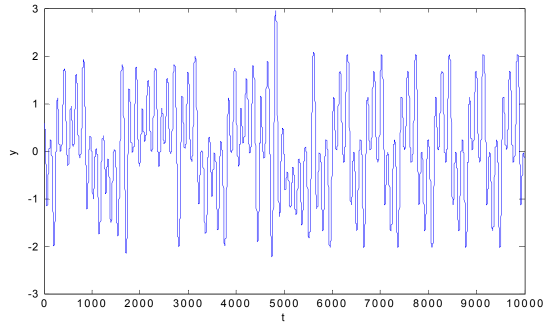
4.1. *a*=-0.8, *d*=0.8, limit-circle appears

If *a*=-0.8, *d*=0.8, $(m_1+m_2+2)\times 2$ limit circles can be obtained. The whole circle-number along *x* and *y* direction is (m_1+m_2+2) and 2, respectively. When m_1=2, m_2=2, (2+2+2)×2=12-circle limit-circle appears, *x-t*, *y-t* and *x-y* projection are shown in Fig.7, respectively. When

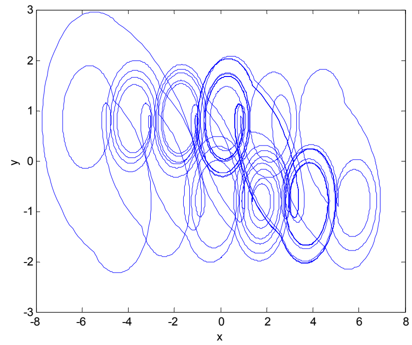
m_1=0, m_2=0, n_1=1, n_2=0 and m_1=2, m_2=2, n_1=1, n_2=1, the trajectories are shown in Fig.8(a) and (b), respectively. We can see that the limit-circle is only part of the trajectories and the circle-number in *y* direction does not increase with the number n_1.



(a) *x-t*

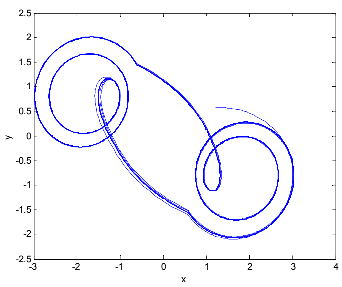


(b) *y-t*

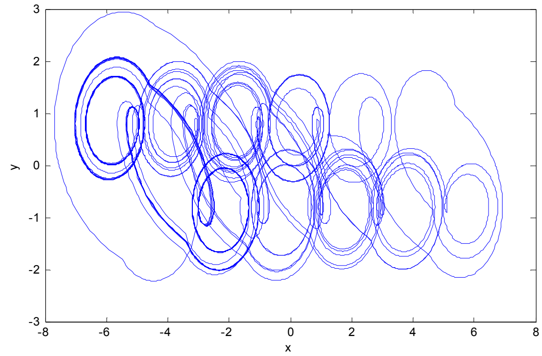


(c) *x-y* projection

Fig.7 12-circle limit-circle



(a) m_1=0, m_2=0, n_1=1, n_2=0



(b) m_1=2, m_2=2, n_1=1, n_2=1

Fig.8 Abnormal limit-circle

4.2. *a*=-0.8, *d*=1.0, chaos appears

4.2.1. (m_1+2)×2-scroll chaos

If m_2=0, k_2=0 (then n_1=n_2=0), $\phi_{k_2}(\zeta)$ takes the form of equ.(2), (m_1+2)×2-scroll chaos appears. The number of equilibrium is (m_1+2)×2. The equilibrium points along direction *x* and *y* is (m_1+2) and 2 respectively. When m_1=0, m_2=0, 2-dimensional 2×2-scroll chaos appears, the *x-y* projection is shown in Fig.9. While m_1=1, m_2=0, 2-D (1+2)×2=6-scroll chaos appears, the *x-y* projection is shown in Fig.10.

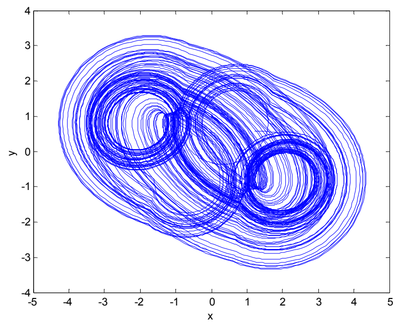


Fig.9 m_1=0, m_2=0, *x-y* projection of 2-D 4-scroll chaos

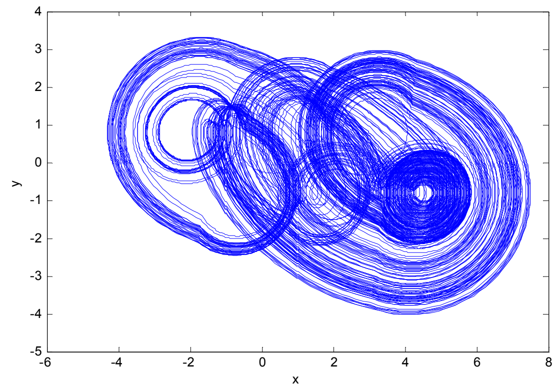


Fig.10 m_1=1*,* m_2=0, *x-y* projection of 2-D 6-scroll chaos

4.2.2. Abnormal-scroll chaos

If m_2=0, k_2=0 (then n_1=n_2=0), we increase m_1 to m_1=2, abnormity-circle chaos appears, the *x-y* projection is shown in Fig.11. Similarly, if m_1=0, we increase m_2 to m_2=2, abnormity-circle chaos appear, m_1=0, m_2=2, the *x-y* projection is shown in Fig.12 which seems upset down of Fig.11.

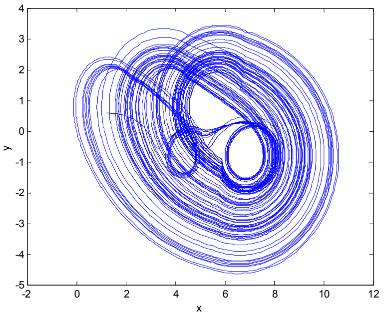


Fig.11 m_1=2*,* m_2=0, *x-y* projection of 2-D abnormal chaos

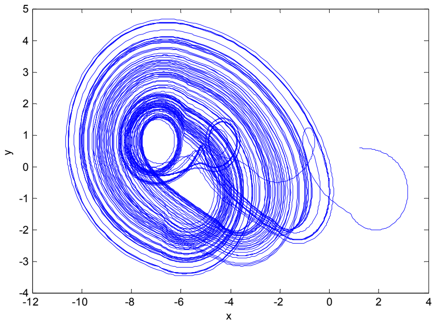


Fig.12 m_1=0*,* m_2=2, *x-y* projection of 2-D abnormal chaos

4.2.3. 2-D multi-scroll chaos

If a=-0.8, d=1.0, k_2≠0, $\phi_{k_2}(\zeta)$ is described as equ.(3) and equ.(4), 2-D multi-chaos appears. m_1=1, m_2=1, n_1=1, n_2=1, multi-scroll chaos is shown in Fig.13. m_1=2, m_2=2, n_1=1, n_2=0, multi-scroll chaos is shown in Fig.14.

From Fig.9 to Fig.14, one can easily found that the chaos trajectory is symmetrical if m_1=m_2 and n_1=n_2.

5. Conclusion

Complex dynamic behaviour can be obtained with the linear third-order continuous system and hysteresis series.

If only state variable x is input to hysteresis series: (1) d=-a=0.8, 1-D (m_1+m_2+2)-scroll chaos can be obtained. The equilibrium points are: (−2m_2−1, 0, 0), …, (−1, 0, 0), (1, 0, 0), (3, 0, 0), … , (2m_1+1, 0, 0). (2) a=-0.8, d=1.0, abnormal chaos can be obtained.

If two state variables are input to hysteresis series: (1) d=-a=0.8, (m_1+m_2+2)×2 limit-circle appears. The whole circle number along x and y direction is (m_1+m_2+2) and 2, respectively, and the limit-circle is only part of the trajectories. (2) a=-0.8, d=1.0, if only one amount is not equals to zero among n_1, n_2, m_1 and m_2 then abnormity-scroll chaos appears. (3) a=-0.8, d=1.0, 2-D multi-chaos can be obtained. (4) If m_1=m_2 and n_1=n_2, chaos trajectory is symmetrical.

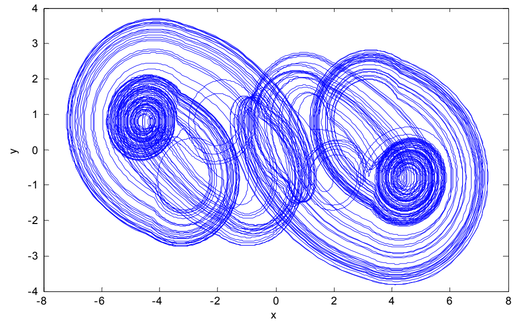


Fig.13 m_1=1, m_2=1, n_1=1, n_2=1 multi-scroll chaos

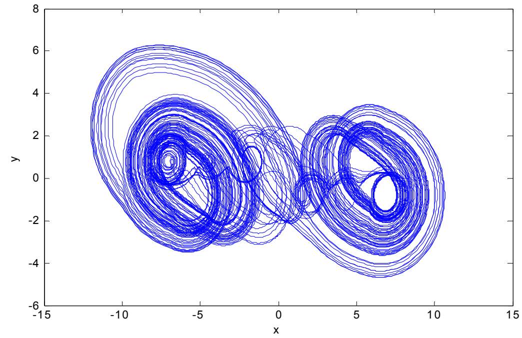


Fig.14 m_1=2, m_2=2, n_1=1, n_2=0 multi-scroll chaos

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