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A Numerical Analysis of Migration and Economic Growth

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1. INTRODUCTION

Azegami [2] (2001) explores the long-run economic growth and labor migration in the process of economic development with a dual economy model where the traditional sector and the modern sector coexist. According to the ordinary dual economy assumption, each sector is represented by agriculture in a rural region and manufacturing in an urban region, respectively. Since labor market is not segmented, migration between the two regions may happen with a change in the economic environment. In equilibrium, migration continues during transition in the case of small open economy while the labor allocation between both sectors is constant over time in the case of closed economy. It is natural that the equilibrium of the closed economy is balanced growth equilibrium because of AK technology assumption. Further, along the equilibrium path in the small open economy, the agricultural products gradually decline to zero and the balanced growth is attained. After the economy specializes in manufacturing, GDP grows at a constant rate. The balanced growth rate is exactly larger in the small open economy than in the closed economy since the former assumption is less restricted than the latter. Therefore, overall welfare in the small open economy is necessarily better off than in the closed economy if the difference between the two growth rates is significant. These are the main results in Azegami [2] (2001), but there are two remaining questions. One is that whether instability during transition has negative effects on economic welfare, or how long the economy must stay in transition. The other is that whether enhancing agricultural productivity has positive or negative effects on economic welfare in the small open economy, while high agricultural productivity is welfare-improving in the closed economy. This paper gives a numerical example about these questions. In the next section the model of Azegami [2] (2001) is introduced and is modified a little. The numerical results are summarized in the section 3 and a brief comment on the results is in the section 4.

2. THE MODEL

2.1 Statement of the model

Notations:

x_M : products of manufacturing;

x_A : products of agriculture;

n: input of labor;

k: individual firm's stock of capital;

K : economy-wide stock of capital;

A : agricultural productivity;

r : interest rate;

p : relative price of agricultural commodity;

w : wage rate in terms of manufacturing commodity;

δ : rate of depreciation;

c_M : consumption of manufacturing commodity;

c_A : consumption of agricultural commodity;

ρ : subjective discount rate

c: expenditure on consumption.

Let an economy consist of two production parts, which are manufacturing and agricultural sector, indexed by M and A, respectively. A representative firm produces $x_M(t)$ units of manufacturing commodity as of time t, which are either consumed or invested. Its production technology is defined by a Cobb-Douglas function,

$$x_M(t) = k(t)^{\alpha} n(t)^{1-\alpha} K(t)^{1-\alpha}, \quad 0 < \alpha < 1, \quad \text{(1-a)}$$

where K is a proxy for knowledge so that it is exogenous to each firm. Normalizing the number of firms to be unity implies $k = K$, and thus aggregate production function can be expressed as,

$$x_M(t) = n(t)^{1-\alpha} k(t). \quad \text{(1-b)}$$

The number of households is also normalized to be unity and the members of the household are all workers, who inelastically supply one unit of labor service as of t. Some of them are employed by firms and the others cultivate their own land to produce the agricultural commodity. The representative household decides its labor allocation in order to maximize the labor income that consists of wage income from firms and sales of agricultural commodity. The production function of agricultural commodity is defined as follows. By normalizing the number of workers to be unity, the labor supply to agricultural sector is expressed as $1-n$. The production technology of agricultural sector is assumed to be linear;

$$x_A(t) = A(1-n(t)). \quad (2)$$

r, p, and w are exogenous to the firm and the household. Noting that the individual firm's production function is Eq.(1-a), the standard profit maximization behavior induces Eqs.(3) and (4),

$$r(t) = \alpha n(t)^{1-\alpha} - \delta, \quad (3)$$

$$w(t) = (1-\alpha) n(t)^{-\alpha} k(t). \quad (4)$$

Under Assumption 1 in Azegami [2] (2001), the wage rate is equalized to the marginal productivity of labor in terms of manufacturing commodity, that is,

$$w(t) = p(t)A. \quad (5)$$

Household's instantaneous utility function is simply given by $\ln c_M(t) + \ln c_A(t)$. The agricultural good is all consumed while some amount of the manufacturing commodity is saved in the form of capital. The household shares the stock of capital and receives income from interest. Therefore household's total income amounts to $rk + wn + pA(1-n)$, while it is reduced to $rk + pA$ with Eqs.(4) and (5). Therefore its flow budget constraint is $\dot{k}(t) = r(t)k(t) + p(t)A - c_M(t) - p(t)c_A(t)$, so that the intertemporal utility maximization problem is formulated as follows.

$$\max_{c_M(t), c_A(t), k(t)} \int_0^{\infty} \{\ln c_M(t) + \ln c_A(t)\} e^{-\rho t} dt$$

s.t. $\dot{k}(t) = r(t)k(t) + p(t)A - c_M(t) - p(t)c_A(t)$.

The necessary conditions are Eqs.(6) and (7), and the transversality condition is Eq.(8).

$$1/c_M(t) = 1/(p(t)c_A(t)) \tag{6}$$

$$\dot{c}_M(t)/c_M(t) = r(t) - \rho \tag{7}$$

$$\lim_{t\to\infty} k(t)/(c_M(t)e^{\rho t}) = 0 \tag{8}$$

Finally, we introduce another variable to make the numerical analysis easy to understand, although the equilibrium can be illustrated with the variables of capital stock and manufacturing good. If $c(t)$ denotes the consumption expenditure in terms of manufacturing commodity, that is, $c(t) \equiv c_M(t) + p(t)c_A(t)$, then Eqs.(9) and (10) are straightforward from the necessary condition Eq.(6).

$$c_M(t) = c(t)/2 \tag{9}$$

$$c_A(t) = c(t)/(2p(t)) \tag{10}$$

Thus Eq.(11) is straightforward from Eq.(7).

$$\dot{c}(t)/c(t) = r(t) - \rho \tag{11}$$

In the next subsection, we will define the dynamic system of the economy with the variables k and c.

2.2 Dynamic system and equilibrium

This subsection illustrates the dynamic system of the economy and the equilibrium. The equilibrium is defined as the solutions of the dynamic system of the economy that maximize the welfare and satisfy the transversality condition, and market clearing conditions or balance of current account. We consider two polar situations; a closed economy and a small open economy.

2.2.1 Closed economy

In the closed economy both manufacturing and agricultural market must equilibrate. Their market clearing conditions are Eqs.(12) and (13), respectively.

$$p(t)x_A(t) = p(t)c_A(t) \tag{12}$$

$$x_M(t) = c_M(t) + \dot{k}(t) + \delta k(t) \tag{13}$$

Using Eqs.(2), (4), (5), (6), (9), and (12), we can induce the Eq.(14) with respect to c, k, and n.

$$n(t)^{\alpha} = 2(1-\alpha)(1-n(t))c(t)/k(t) \tag{14}$$

This is solved for the labor share of manufacturing sector as a function of the ratio of consumption expenditure to the stock of capital, so that we let $n(c/k)$ denote the *solution* of Eq.(14). Substituting both Eqs.(1-b) and (9) into Eq.(13) induces the differential equation with respect to the stock of capital.

$$\dot{k}(t)/k(t) = n(c(t)/k(t))^{1-\alpha} - (c(t)/k(t))/2 - \delta \tag{15}$$

Finally the differential equation with respect to consumption expenditure is induced from Eqs.(3) and (7), and thus the dynamic system of this economy is closed.

$$\dot{c}(t)/c(t) = \alpha n(c(t)/k(t))^{1-\alpha} - \delta - \rho \tag{16}$$

For any initial value of capital stock there exists a balanced growth path (BGP), along which the stock of capital and the consumption expenditure grow at an equivalent rate and the transversality condition is satisfied. If another assumption about parameters is added, we can illustrate the equilibrium path is unique and it is the BGP. Two characteristics of the BGP are mentioned. One is that along the BGP the labor allocation does not change over time so that the rate of economic growth and the amount of agricultural products are constant. The other is that both the labor allocation and the rate of growth on the BGP depend upon neither the initial level of capital stock nor the agricultural productivity.

2.2.2 Small open economy

We configure the small open economy in which both manufacturing and agricultural commodity are tradable while the labor is immobile across countries and international borrowing and lending are not permitted. Since the economy is too small to affect the international commodity markets, the relative price of agricultural commodity is exogenously given and we assume that it is constant over time; $p(t) = p$ for all $t \geq 0$. The agents can freely import and export each good at that price so that the industry may specialize in either sector. Thus the labor allocation is directly lead from Eq.(5) as the function with respect to the stock of capital but it is divided into two parts.

$$n(t) = [(1-\alpha)/(pA)]^{\frac{1}{\alpha}} k(t)^{\frac{1}{\alpha}}, \text{ for } 0 \leq k(t) \leq \bar{k} \tag{18}$$

$$n(t) = 1, \text{ for } \bar{k} < k(t),$$

where $\bar{k} \equiv pA/(1-\alpha)$. When the stock of capital exceeds the critical level $\bar{k}$, all the labor is employed by the firm and the agricultural sector vanishes. The dynamic system of the small open economy must be expressed as two couples of differential equations.

- 2-sector economy

The dynamic system is induced in the same way of the last subsection except that the balance of current account replaces the market clearing conditions. Noting that the balance of current account is $x_M - c_M - \dot{k} - \delta k = p(c_A - x_A)$, the dynamic system for $k \leq \bar{k}$ is Eqs.(19) and (20).

$$\dot{k}(t) = \alpha p A [(1-\alpha)/(pA)]^{\frac{1}{\alpha}} k(t)^{\frac{1}{\alpha}} /(1-\alpha) - \delta k(t) - c(t) + pA, \quad 0 \leq k(t) \leq \bar{k} \tag{19}$$

$$\dot{c}(t) = \left\{ \alpha [(1-\alpha)/(pA)]^{\frac{1-\alpha}{\alpha}} k(t)^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right\} c(t), \quad 0 \leq k(t) \leq \bar{k} \tag{20}$$

- 1-sector economy

The dynamic system of the economy when the industry specializes in manufacturing is given by Eqs.(21) and (22).

$$\dot{k}(t) = (1-\delta)k(t) - c(t), \quad \bar{k} \leq k \tag{21}$$

$$\dot{c}(t) = (\alpha - \delta - \rho)c(t), \quad \bar{k} \leq k \tag{22}$$

The dynamic system of the 1-sector economy has also a balanced growth path so that we call it 1-BGP.

Although it is not easy to specify the equilibrium of the small open economy, Azegami [2] (2001) shows that for a given initial level of capital stock there exists a path along which the economy switches from the 2-sector economy to the 1-sectro economy and it satisfies the conditions for equilibrium. Therefore the equilibrium in the small open economy is a path that reaches the 1-BGP and the 1-BGP itself. On the 1-BGP, the rate of economic growth is very simple, $\alpha - \delta - \rho$. It is higher than the rate of economic growth in the closed economy because $0 \leq n \leq 1$. This result is not surprising because the closed economy is considered as a restricted version of the small open economy. Azegami [2] (2001) points out that in the small open economy the agents may bear the recession before it reaches the 1-BGP. Since the fixed point in the small open economy is unstable, the transition to the balanced growth is rather complex. If the difference of the two growth rates between closed and small open economy is significant, the welfare in the small open economy necessarily overtakes that in the closed economy. But the period when the economy is on transition may be long and the lower initial stock of capital prolongs it. Since these quantitative arguments are not covered with Azegami [2] (2001), we will show some numerical results in the subsequent section.

3. NUMERICAL RESULTS

Our numerical analysis is based on the parameters listed in Table 1. All the parameters except for the agricultural productivity and the relative price are standard. We may not have to deal with the agricultural productivity and the relative price apparently because the qualitative effects of them on the economy are almost the same. Three different levels of agricultural productivity will be used to examine the effect of the change of that parameter.

Table 1

Production values for benchmark economy

Production parameters	α=0.3; δ=0.1; A=1.0
Preference parameters	ρ=0.04
International commodity market	p=1.0

3.1 Equilibrium path in the small open economy

We will point out two features of the equilibrium of the small open economy. First, the equilibrium is not defined for rather a low level of initial stock of capital ($k(0) < 0.6$ in the benchmark case). Next is that there exist two or more patterns of choice of consumption or savings at starting point in the case where the initial level of capital stock is given near the fixed point. In the benchmark case when the level of capital stock is 1.02 at time zero, the household can choose the initial level of consumption expenditure (savings) among four patterns. If the lowest level is chosen, the economy can reach the 1-BGP in at least 2.3 years. Although economic welfare in this choice is the best among the four cases, the agents are forced to keep extremely high rate of savings (about 20%). One interesting fact, however, is that the economy is not necessarily better off with higher initial saving rate, e.g. in the case of 10% saving rate welfare is worse off than in the case of 3%. Economic welfare depends negatively on the length of transition in the small open economy.

3.2 Agricultural productivity and welfare

The agricultural productivity has no effect on the rate of balanced growth in a small open economy as well as in a closed economy. But it is clear that an increase in agricultural productivity improves welfare in the closed economy because it raises agricultural consumption without any loss of manufacturing products. On the other hand, it seems that the agricultural sector continues to exist for more years with high productivity, that is, the economy has to be in transition for longer period.

The agricultural productivity, A, is 1 in the benchmark case. In order to show the effect of agricultural productivity on welfare, we examine another two levels of it, 0.95 and 1.05. Computation demonstrates a positive link between agricultural productivity and welfare. Although high productivity exactly lengthens the period of transition, the economy restarts with higher level of consumption after the stock of capital attains to the critical level. Since the rate of economic growth on the 1-BGP is constant and independent from agricultural productivity, the difference of welfare increases at its rate. Welfare loss during transition caused by the low agricultural productivity is necessarily offset by welfare gain after specialization. The positive link between agricultural productivity and welfare prevails in this model.

3.3 Small open vs. closed economy

If the initial level of capital stock is identical, overall welfare in the small open economy is better off than that in the closed economy because the balanced growth rate in the former case is exactly greater than that in the latter. In the short run, however, does openness necessarily improve economic welfare? The two economies during the first several decades are compared with the numerical computation and Table 2 summarize the results. The initial level of capital stock is set at 1. In the closed economy the manufacturing labor share is 47.7% and the rate of economic growth is 3.9%. On the other hand two levels of initial expenditure on consumption are considered in the case of small open economy. If the lower level is chosen (the saving rate is 19.2%), the manufacturing labor share is 30.5% at time zero and it rises consistently with time. The industry specializes in manufacturing in 2.5 years and GDP grows at 16% after that. During this period welfare is worse off than that in the closed economy but this welfare loss is offset in following 13.5 years. If the higher level of initial expenditure on consumption is chosen (the saving rate is 3.1%), however, while the manufacturing labor share at time zero is the same, 30.5%, the transition period is over 7.7years and the welfare loss during this period is offset in at least 18 years. These results illustrate that openness necessarily improves economic welfare but forces the agents to be patient during transition. Moreover, if the initial level of expenditure on consumption is not appropriately chosen, the transition period may be prolonged.

Table 2

(a) closed economy

initial consumption expenditure	0.874
saving rate (%)	13.2
manufacturing labor share (%)	47.7
balanced growth rate (%)	3.87

(b) open economy

initial consumption expenditure	c(0)=0.913	c(0)=1.096
saving rate (%)	19-26	3-26
manufacturing labor share (%)	30-100	18-100
balanced growth rate (%)	16	16
Wclose=Wopen (years)	16	28

4. CONCLUDING REMARKS

In this paper, we have summarized the dual economy model explored by Azegami [2] (2001) and numerically examined its analytical results. In the bench mark case, manufacturing labor share and the rate of economic growth in the closed economy is 47.7% and 3.9%, respectively, while the balanced growth rate in the small open economy is 16%. Given an identical initial stock of capital, welfare in the small open economy is better off than that in the closed economy due to the significant difference between their growth rates. Azegami [2] (2001) emphasizes that the small open economy is unstable during transition and that the agents may experience a recession where the stock of capital and GDP declines, even though they could choose the equilibrium path. In this study, however, we find that the agents can avoid a recession with an appropriate choice of the initial level of expenditure on consumption. Since a choice of consumption expenditure determines the rate of savings, the appropriate choice in the bench mark case implies that the initial saving rate must be 19%. The high rate of savings leads the economy to the balanced growth path in only 2 years. Although the agents initially save at most 3% of their income if the other level is chosen, they experience a recession and the transition continues for over 7 years. In any case the level of welfare in the small open economy overtakes that in the closed economy within 16 or 28 years.

Another result of this study is that agricultural productivity has a positive effect on welfare in the small open economy. In the closed economy an increase in agricultural productivity necessarily improves welfare because it raises consumption of agricultural good without any loss of manufacturing products. In the small open economy, however, it raises the critical level of capital and it may prolong the transition period. As the qualitative analysis is difficult, we have numerically examined the welfare effects of agricultural productivity. Our result is that even though an increase in agricultural productivity prolongs the transition period, it improves the overall welfare through a rise of agricultural consumption. High agricultural productivity postpones industrialization but its welfare effect is positive in our model.

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