QUASI-PERIODIC AND CHAOTIC OSCILLATIONS OF A MAGNETICALLY SUSPENDED ROTOR

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Abstract

This work reports on a numerical investigation on the non-linear response of a rigid rotor in active magnetic bearings. The mathematical model of the rotor-bearing system used in this study incorporates non-linearity arising from geometric coupling of the magnetic actuators. The response of the rotor with the variation of the unbalance parameter, U, exhibited a rich variety of dynamical behavior including synchronous oscillation, quasi-periodicity, phase-locking and chaos. Chaotic oscillations, which appeared via the torus breakdown route, were observed in the range $0.130 \le U \le 0.369$. For $U \ge 370$, no stable solution of the rotor response was found for values of initial conditions used in this study. The value of rotor unbalance force where non-synchronous and chaotic oscillations were observed in this study, albeit being higher than the specified unbalance level for rotating machinery, may possibly occur with eroded rotors or in the event of a partial or entire blade failure.

1. Introduction

The use of active magnetic bearings as an alternative to rolling-element and fluid-film bearings in rotating machinery applications has been increasing in recent years. Their utilization are found in a wide class of rotating machinery ranging from turbo-molecular pumps to industrial steam turbines. The main advantage of the active magnetic bearings over the conventional bearings is their higher mechanical efficiency due to reduced friction losses since no contact occurs between the rotor and the bearing stator during operation of the machine. The magnetic bearings, however, exhibits highly non-linear characteristics that can be detrimental to the performance of the rotating machinery supported by them. There are several sources of non-linearity in an active magnetic bearing system, of which the most prominent is the relationship between the forces generated in the electromagnetic actuator and the coil current and the air gap between the rotor and the stator. The force is proportional to the current squared and inversely proportional to the gap squared. Cross-coupling between the electromagnetic forces acting in two orthogonal directions is also a source of non-linearity in a magnetic bearing system. There are three causes of cross-coupling i.e. geometry of the actuators, eddy-current effect and gyroscopic effect. The air gap at a point on a magnetic pole is actually not constant over the entire pole area due to the geometrical curvature of the pole. This results in a normal force, which is perpendicular to the principal force, which in turn causes geometric coupling between these forces. Eddy currents are induced in a solid (non-laminated) magnetic core when the shaft rotates. Their effective lift is reduced and their drag cause coupling between the two orthogonal directions. Gyroscopic effect, which is more pronounced in rotors with large or overhung discs, causes the motion in the two orthogonal directions to be coupled. Other sources of non-linearity in a magnetic bearing system include hysteresis of the magnetic core material, noise from the sensors, time delay of the digital controller and the current slew-rate limitation.

Hebbale & Taylor investigated the non-linearities in magnetic bearings due to the effects of coordinate coupling that arose from eddy currents during shaft rotation [1]. By applying the center manifold theory, they found that the equilibrium position could lose stability due to a sub-critical Hopf bifurcation as the rotor speed is varied. The effects of gyroscopic motion on the non-linear response of a rigid rotor supported by magnetic bearings were investigated by Mohamed & Emad [2]. As the rotor speed was varied, the system was found to undergo a Hopf bifurcation at certain values of speed parameter. The resulting periodic solution was found to be unstable. A non-linear feedback control of quadratic order was used to control the Hopf bifurcation occurring in the system. The effects of coordinate coupling due to the geometry of the magnetic pole arrangement on the non-linear response of a rigid rotor mounted in magnetic bearings were examined by Virgin *et al.* [3]. Since the magnetic forces were modeled based on flux control with a linear feedback law, the only non-linearities in the system were due to the geometric coupling. Multiple co-existing solutions were found at primary resonance of the system. Their work also revealed the existence of fractal boundaries separating stable and unstable regions in the response of the rotor. The harmonic balance method was used to obtain the unstable solutions. The effects of geometric coupling on the non-linear response of a magnetic bearing supported rotor were also investigated by Chinta *et al*. [4]. As opposed to the work of Virgin *et al*. [3] who modeled the magnetic forces based on flux control, their modeled were instead based on current control. Therefore, in addition to the effect of geometric coupling, the magnetic forces also contributed to the non-linearities of the system, as they were non-linear functions of rotor position and coil current. Numerical integration of the equations of motion of the system revealed the occurrence of quasi-periodic and period-2 motion. This work was further extended by Chinta & Palazzolo [5] to take into account the effect of rotor weight on the response of the rotor. The trigonometric collocation method was used to find the stable and unstable periodic solutions of the rotor motion due to the effects of geometric coupling and rotor weight. Regimes of non-linear behavior such as jump phenomena and sub-harmonic motions were observed in the response of the rotor. Steinschaden & Springer [6] used the numerical integration and numerical continuation methods to investigate the unbalance response of a rigid rotor in magnetic bearings. The non-linearity considered were only the magnetic forces due to non-linear force to displacement and non-linear force to coil current relationship. Their simulation revealed the occurrence of symmetry-breaking and period-doubling bifurcations, and response of global instability for some combination of the system parameters. This work was further extended to include an improved version of the mathematical model of the magnetic force [7]. In particular, the non-linear saturation effects of the magnetic materials as well as limitations of power amplifier and of the control current were included in this model. Effects of coil inductance combined with underlying current controller were also included. They observed the occurrence of symmetry breaking and quasi-periodic solutions in the response of the rotor.

In the work presented herein, the influence of rotor unbalance force on the non-linear response of a rigid rotor in active magnetic bearings is numerically investigated. Non-linearity arising from geometrical coupling of the magnetic actuators is incorporated into the mathematical model of the rotor-bearing system. Direct numerical integration of the equations of motion representing the rotor-bearing system was carried out using the MATLAB software package. Poincare maps and power spectrum plots are used to illustrate the results obtained from the numerical simulation.

2. Mathematical Model

The sources of non-linearity considered in this work are due to the relationship between the force, current coil and air gap; and the cross-coupling effect due to the geometry of the electromagnetic actuator. In the derivation of the equations of motion of a rigid rotor in active magnetic bearings, the following assumptions hold:

(i) the rotor is rigid and symmetric,
(ii) unbalance in the rotor is purely static and located in its mid-span,
(iii) rotor motion in the axial direction is neglected,
(iv) gyroscopic effect is neglected,
(v) leakage of magnetic flux is neglected,
(vi) fringing effect i.e. the spreading of magnetic flux in the air gap is neglected,
(vii) the magnetic iron is operating below saturation level.

The motion of the rotor-magnetic bearing system can be described by the displacements x and y of the geometric center of the journal. With the external forces acting on the journal that include the magnetic bearing forces, gravity and unbalance forces, the equations of motion of the journal center taking into account the effect of geometrical coordinate coupling can be expressed in non-dimensional form as follows

$$\begin{aligned}\ddot{x} &= F_{x,+} - F_{x,-} + \alpha x\left(F_{y,+} + F_{y,-}\right) + U\Omega^2 \cos\Omega\tau \\ \ddot{y} &= F_{y,+} - F_{y,-} + \alpha y\left(F_{x,+} + F_{x,-}\right) + U\Omega^2 \sin\Omega\tau - W\end{aligned} \tag{1}$$

where

$$F_{x,+} = \left(\frac{1}{4(P-1)}\right)\left[\frac{(1 - Px - D\dot{x})}{(1-x)}\right]^2$$

$$F_{x,-} = \left(\frac{1}{4(P-1)}\right)\left[\frac{(1 + Px + D\dot{x})}{(1+x)}\right]^2$$

$$F_{y,+} = \left(\frac{1}{4(P-1)}\right)\left[\frac{(1 - Py - D\dot{y})}{(1-y)}\right]^2$$

$$F_{y,-} = \left(\frac{1}{4(P-1)}\right)\left[\frac{(1 + Py + D\dot{y})}{(1+y)}\right]^2$$

The unbalance parameter, U, which is a measure of the rotor unbalance, is defined as the ratio of the eccentricity (u) of the rotor center of gravity (G) from its geometric center of rotation, to the nominal clearance of the magnetic bearing. The gravity parameter, W, represents the unidirectional static force acting on the rotor. α is the geometric coupling parameter, which is the ratio of the attractive, on-axis force between each magnet and the shaft to the normal, off-axis force. Ω, the speed parameter, is the ratio of the rotor operating speed (ω) to the system linear natural frequency (ω_n). P and D are respectively the non-dimensional proportional and derivative feedback gains of the controller. τ is the non-dimensional time.

3. Results and Discussion

Direct numerical integration of Eq. (1) was carried out using the MATLAB software package. This package utilizes a variable-step continuous solver based on an explicit Runge-Kutta (4,5) formula, the Dormand-Prince pair. It is a one-step solver, that is, in computing $x(t_n)$, it needs only the solution at the immediately preceding time point $x(t_{n-1})$. The values of the following design and operating parameters were fixed; Ω=1.0, P=1.1, D=0.03, α=0.24 and W=0.03. The unbalance parameter U was varied from 0 to 0.369 at intervals of 0.001 in order to investigate the effect of increasing the rotor unbalance force on the response of the magnetically supported rotor. The results of the numerical simulation are illustrated using Poincare maps and power spectrum plots. Poincare map is obtained by sampling the trajectory of the rotor whirl orbit at constant interval of the forcing period of $T = 2\pi$. The power spectrum, which exhibits the frequency contents of the rotor response, is determined from the Fourier transformation of the time series of the rotor response in the X-direction.

For small values of the unbalance parameter, U, the rotor response was synchronous, and such motion was observed to persist up to a value of U=0.114. Quasi-periodic motion of the rotor, which appeared due to a secondary Hopf bifurcation of the synchronous rotor response, was first observed at U=0.115, and was found to exist up to U=0.125. The Poincare map and power spectrum plot of the quasi-periodic motion at U=0.116 is shown in Fig. 1. The smooth continuous closed curve of the torus is evident of a two-period quasi-periodic motion. As the value of U was increased to 0.118, wrinkling of the torus was observed. The distortion of the torus became more significant as the value of U was further increased. Fig. 2 illustrates the Poincare map and power spectrum plot of the rotor response for U=0.123. The breaking-up of the continuous curve of the torus was first observed at U=0.124, followed by phase-locked oscillations, which occurred at U=0.126. Quasi-periodic oscillations reappeared at U=0.127, followed again by phase-locked oscillations for the range $0.128 \le U \le 0.129$. The emergence of the chaotic attractor was observed immediately after the phase-locked oscillations. Chaotic oscillations of the rotor were found to exist in the range $0.130 \le U \le 0.369$. Fig. 3 shows the chaotic response of the rotor for U=0.140. For $U \ge 370$, no stable solution of the rotor response was found for values of initial conditions used in this study. The transition to chaos in the rotor-magnetic bearing system was via the torus breakdown route, which involved the wrinkling and breaking-up of the torus. Quasi-periodic and phase-locked oscillations were also observed to appear and disappear alternately prior to the emergence of chaos.

4. Summary

This work reports on a numerical investigation on the non-linear response of a rigid rotor in active magnetic bearings. The response of the rotor, with the variation of the unbalance parameter, U, displayed a rich variety of dynamical behavior including synchronous oscillation, quasi-periodicity, phase-locking and chaos. The transition to chaos was found to be via the torus breakdown route. The occurrence of non-synchronous and chaotic oscillations in rotating machinery is undesirable and should be avoided as they introduce cyclic stresses in the rotor, which in turn may rapidly induce fatigue failure. The value of rotor unbalance force where non-synchronous and chaotic oscillations were observed in this study, albeit being higher than the specified unbalance level for rotating machinery, may possibly occur with eroded rotors or due to a partial or entire blade failure.

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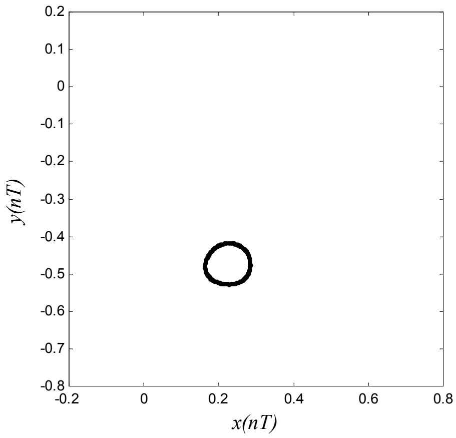

Fig.1a Poincare map of rotor response for U =0.116.

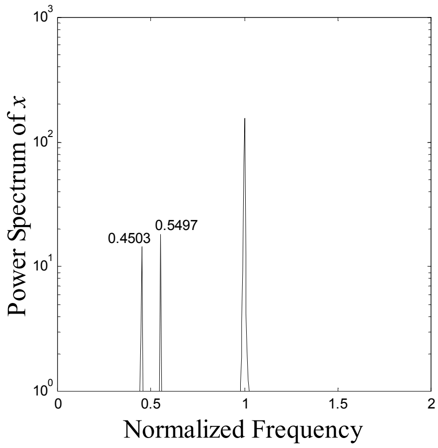

Fig. 1b Power spectrum of rotor response for U =0.116.

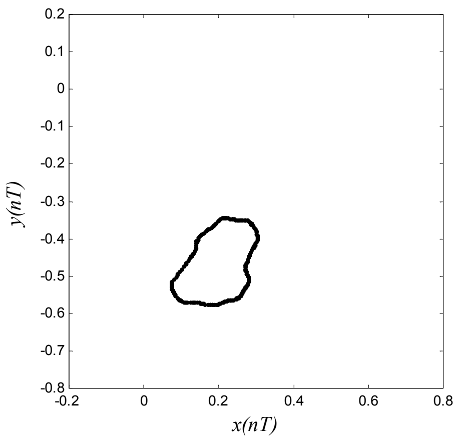

Fig.2a Poincare map of rotor response for U =0.123.

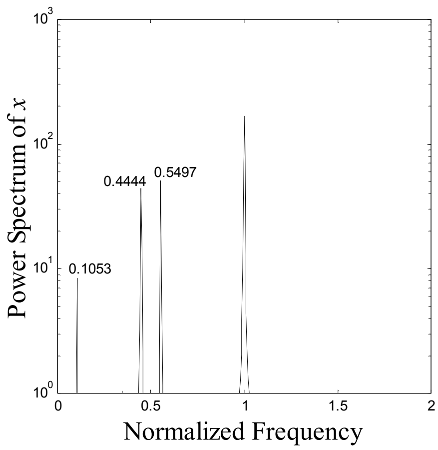

Fig. 2b Power spectrum of rotor response for U =0.123.

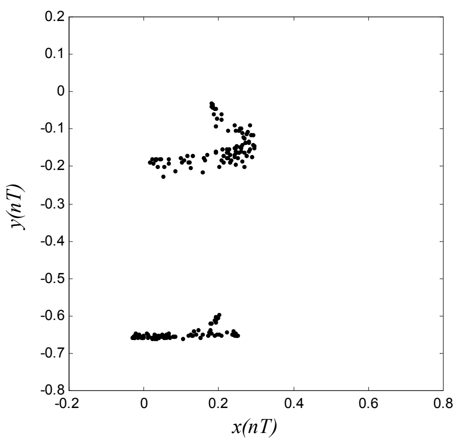

Fig.3a Poincare map of rotor response for U =0.14.

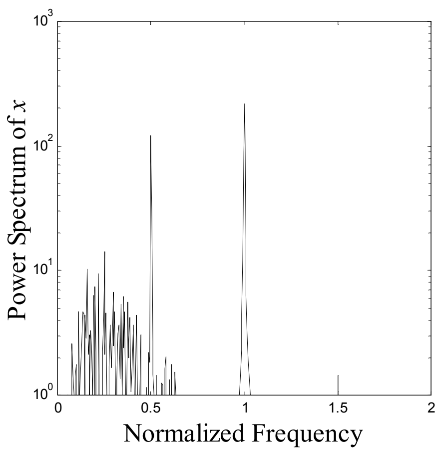

Fig. 3b Power spectrum of rotor response for U =0.14.