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# Oscillatory Activities in A Neuronal Network Model

Hirofumi Nagashino† and Yohsuke Kinouchi‡

† Department of Radiologic Science and Engineering
School of Health Sciences
The University of Tokushima
3-18-15 Kuramoto, Tokushima, Japan
Phone:+81-88-633-9025, Fax:+81-88-633-9025
Email: nagasino@medsci.tokushima-u.ac.jp

‡ Department of Electrical and Electronic Engineering
Faculty of Engineering
The University of Tokushima
2-1 Minami-Josanjima, Tokushima, Japan
Phone:+81-88-656-7475, Fax:+81-88-656-7475
Email: kinouchi@ee.tokushima-u.ac.jp

**Abstract** We analyzed the oscillatory activities in a neuronal network model as the basis of synchrony of the activities in the brain. The model consists of two groups of neurons that are interconnected. One group is composed of an excitatory and an inhibitory neuron which are expressed by Hodgkin-Huxley equations. The network shows different phase-locked oscillations depending on the structure and intensity of interconnection between groups or coupling of neurons in the group, or the value of transmission delay. The oscillations include various periodic solutions in which the two groups oscillate not only in in-phase or anti-phase but also in continuously changing phase difference with the parameters of coupling and transmission delay.

## 1. Introduction

The synchronization of the neural activities in different regions in the brain is one of the important features of information processing in the central nervous system[1]. To clarify the fundamental mechanism of the scynchronization [2], it is very significant to elucidate the phase-locked activities of the neuronal networks with different coupling configurations [3]. We analyzed the oscillatory activities in a neuronal network model which consists of two groups of neurons to investigate the basic characteristics of synchronization. The neurons are coupled with synaptic delay. Each group consists of excitatory and inhibitory neurons which are interconnected. In the simulation they are represented by an excitatory neuron and an inhibitory neuron for simplicity of analysis. The neurons are formulated by Hodgkin-Huxley equation[4].

In the present paper, we demonstrate that oscillatory activities of not only the in-phase mode and anti-phase mode but also the periodic solutions in which the phase difference of the oscillation in two groups of the neurons continuously changes with the coupling strength and latency.

We have so far shown the results of the analysis of the characteristics of a network with one type of mutual symmetrical coupling composed of two identical groups of neurons[5]. In the present paper we show the relationship between the emerging oscillatory modes, and coupling parameters and initial conditions in the networks with four types of symmetrical coupling of two nonidentical groups of neurons.

## 2. A Neuronal Network Model

The dynamics of the membrane potentials of the neurons is formulated as

$$C_M \frac{d\mathbf{V_k}}{dt} = \mathbf{I_{ext,k}} + \mathbf{I_{ion,k}} + \sum_{j=1}^{2} \mathbf{W_{kj}} \mathbf{I_{syn,j}} \qquad (k = 1, 2), \tag{1}$$

where

$$\mathbf{V_k} = \begin{pmatrix} V_{ek} \\ V_{ik} \end{pmatrix}, \;\; \mathbf{I_{ext,k}} = \begin{pmatrix} I_{ext,ek} \\ I_{ext,ik} \end{pmatrix},$$

$$\mathbf{I_{ion,k}} = \begin{pmatrix} I_{ion,ek} \\ I_{ik} \end{pmatrix}, \;\; \mathbf{I_{syn,k}} = \begin{pmatrix} I_{syn,ek} \\ I_{syn,ik} \end{pmatrix}.$$

In the above, $I_{ext,rk}$, $I_{ion,rk}$ and $I_{syn,rk}$, $(r = e, i)$ denote the external input current, the ion currents of the neuron and the synaptic current from the corresponding neuron in the network, respectively. The subscripts $e$ and $i$ denote excitatory and inhibitory neurons respectively. The ion currents $I_{ion,rk}$ are formulated conventionally. Some formulations have been proposed for synaptic coupling [6], [7], [8]. The synaptic current $I_{syn,rk}$ is expressed here as

$$\begin{aligned} I_{syn}(V) &= \sum_J A_{syn}(J) w \{\exp(\frac{-t + t_s(J) + \tau_d}{\tau_+}) \\ &\quad - \exp(\frac{-t + t_s(J) + \tau_d}{\tau_-})\}, \end{aligned} \tag{2}$$

where $w$ is the coupling coefficient between the corresponding neurons and the element of the coupling matrix

$$\mathbf{W_{kj}} = \begin{pmatrix} a_{kj} & -c_{kj} \\ b_{kj} & -d_{kj} \end{pmatrix}.$$

The $J$-th instant when the membrane potential increases and exceeds -30mV is denoted by $t_s(J)$ and the synaptic delay is denoted by $\tau_d$. If the current of a previous firing is larger than 0.01, it is added to the present current and otherwise neglected. The factor $A_{syn}(J)$ is expressed as

$$A_{syn}(J) = \begin{cases} 1 & (if \;\; t \geq t_s(J) + \tau_d) \\ 0 & (else) \end{cases}. \qquad (3)$$

In the present simulations the values of the external input current $I_{ext,rk}$ and parameters $C_M$, $\tau_+$ and $\tau_-$ are set as

$$I_{ext,rk} = 10[\text{mA}] \;\; (r = e, i, \; k = 1, 2),$$
$$C_M = 1[\mu\text{F}/cm^2], \; \tau_+ = 3[\text{ms}], \; \tau_- = 1[ms].$$

## 3. Results

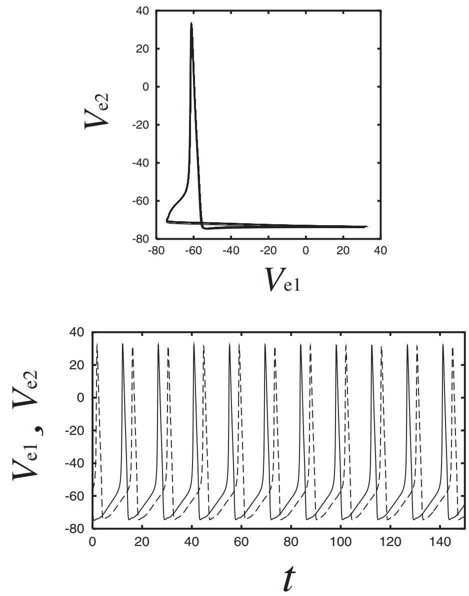
We analyzed the symmetrical networks of two identical groups and asymmetrical networks in that the groups are nonidentical. When the corresponding coupling coefficients in two groups are equal, we made the coupling coefficients between the neurons in the groups denoted as $a_{11} = a_{22} = a_0$, $b_{11} = b_{22} = b_0$, $c_{11} = c_{22} = c_0$, and $d_{11} = d_{22} = d_0$. The bidirectional coupling coefficients between groups are equal and denoted as $a_{12} = a_{21} = a_1$, $b_{12} = b_{21} = b_1$, $c_{12} = c_{21} = c_1$, and $d_{12} = d_{21} = d_1$. The coupling coefficients inside the group are set as

$$a_0 = 10, \; b_0 = 9, \; c_0 = 10, \; d_0 = 0$$

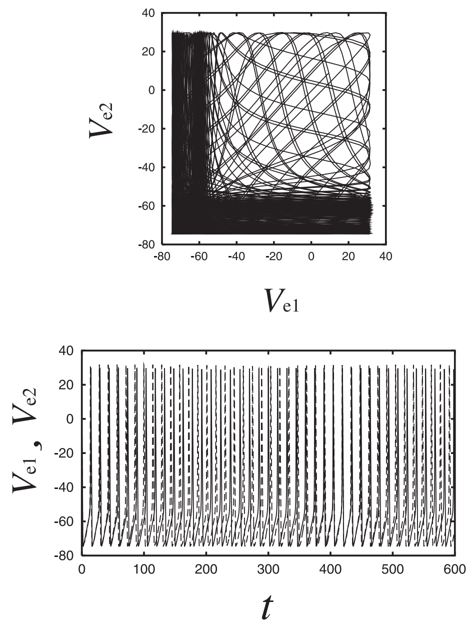
when they are fixed. By the coupling denoted by $a_{11}$ and $a_{22}$ it is assumed that a number of excitatory neurons are interconnected to act synchronously.

We investigated the following four configurations of mutual coupling between the groups;
(1) EE coupling: both excitatory units are coupled bidirectionally, $a_1 \neq 0$, $b_1 = c_1 = d_1 = 0$;
(2) IE coupling: the excitatory unit excites the inhibitory unit of the each other group, $b_1 \neq 0$, $a_1 = c_1 = d_1 = 0$;
(3) EI coupling: the inhibitory unit inhibits the excitatory unit of the each other group, $c_1 \neq 0$, $a_1 = b_1 = d_1 = 0$;
(4) II coupling: both inhibitory units are coupled bidirectionally, $d_1 \neq 0$, $a_1 = b_1 = c_1 = 0$.

The networks show a variety of phase-locked oscillations depending on the structure and intensity of interconnection between groups or coupling of neurons in the




(a)




(b)

Figure 1: Examples of phase portraits and time series of oscillatory modes, (a) intermediate-phase mode, (b) varying phase mode .

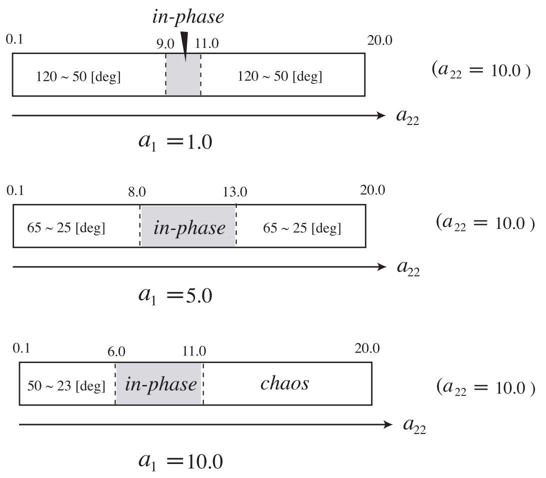



Figure 2: The regions of different oscillatory modes in EE-coupling, $\tau_d = 1$, $a_{11} = 10$.

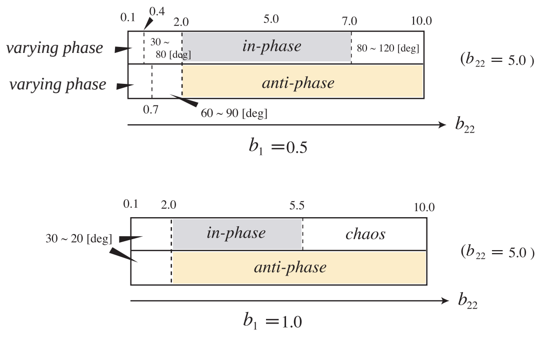



Figure 3: The regions of different oscillatory modes in IE-coupling, $\tau_d = 1$, $b_{11} = 5$.

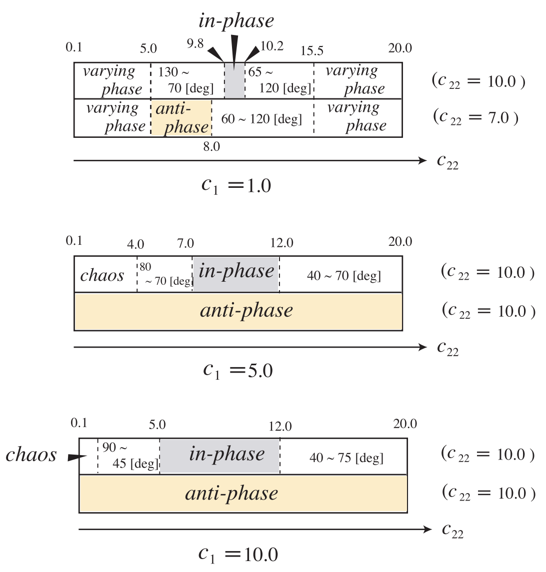



Figure 4: The regions of different oscillatory modes in EI-coupling, $\tau_d = 1$, $c_{11} = 10$.

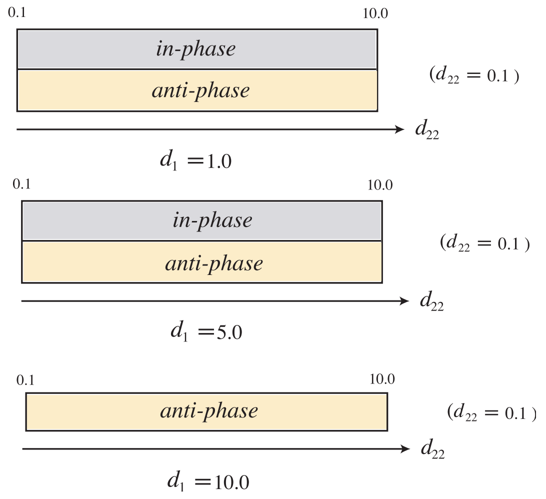



Figure 5: The regions of different oscillatory modes in EI-coupling, $\tau_d = 1$, $d_{11} = 0$.

group, or the value of transmission latency. As in a number of oscillatory systems composed of two components, the present model generates in-phase mode, in which the phase difference of oscillation in two neuron groups is 0 degrees, and anti-phase mode, in which the phase difference is 180 degrees. In addition a remarkable oscillatory mode exists in the present model. In that solution the two groups oscillate in continuously changing phase difference with the parameters of coupling and latency. The mode is referred to as intermediate-phase mode. Chaotic oscillations and quasiperiodic oscillations are also observed. In the latter the phase difference of the oscillation gradually changes with time. It is referred to as varying phase mode. Figure 1 shows examples of phase portraits and time series of intermediate-phase mode and varying phase mode.

We obtained the following results of the analysis of asymmetrical networks, which are illustrated in Figures 2, 3, 4 and 5. When the illustration consists of two rows, the upper row shows the ranges of the periodic solutions that emerge with the initial conditions where the in-phase mode would be likely to occur, while the lower shows those with the initial conditions where the anti-phase mode would be likely to occur. When the illustration consists of one row, the same solutions occur with the initial conditions for the in-phase mode and those for the anti-phase mode. The parameter value shown in the right end shows the one at which the analysis is started.
(1) EE-coupling
In the symmetrical network in-phase mode occurs in large range of the parameter $a_0$. In the asymmetrical network in which $a_{11}$ and $a_{22}$ are different, the region of $a_{22}$ in which in-phase mode occurs become small and

there exits the oscillatory mode in which the phase difference variance from about 23 degree to 120 degree is observed with the change of the coupling coefficient $a_{22}$. Chaotic oscillations also emerge with the larger values of $a_{22}$ and $a_1$.

(2) IE-coupling

In the symmetrical network anti-phase mode exists in large range of parameters $b_0$ and $b_1$. In the asymmetrical network in which $b_{11}$ and $b_{22}$ are different, anti-phase mode exists as well in large range of parameter $b_{22}$. In-phase mode also occurs with relatively large range of $b_{22}$. In-phase mode, intermediate-phase mode and chaotic oscillations coexit with anti-phase mode in the respective regions of $b_{22}$. Which mode occurs depends on the initial conditions of the membrane potentials. Varying phase mode emerges with the small values of $b_{22}$.

(3) EI-coupling

In the symmetrical network in-phase mode and intermediate-phase mode coexist with anti-phase mode in large range of parameters $c_0$, $c_1$ and $\tau_d$. mode exits. In the asymmetrical network in which $c_{11}$ and $c_{22}$ are different, the mode coexistence is same as the case of the symmetrical network. The region of in-phase mode becomes smaller and that of intermediate-phase mode becomes larger. Varying phase mode emerges with the small values of $b_{22}$ or large values of $b_{22}$, in other words, when the asymmetricity is large. Chaotic activities are also observed with small values of $c_{22}$. It coexists with anti-phase mode. Which mode occurs depends on the initial conditions of the membrane potentials.

(4) II-coupling

In the symmetrical network anti-phase mode occurs in large range of parameters $d_0$, $d_1$ and $\tau_d$. In the asymmetrical network in which $d_{11}$ and $d_{22}$ are different, anti-phase mode occurs in the large region of $d_{22}$. In addition in-phase mode occurs with relatively large range of $d_1$ and $d_{22}$. It coexists with anti-phase mode for the large range of the parameters.

### 4. Conclusions

We analyzed the oscillatory activities in a neuronal network model to elucidate the basis of synchrony of the activities in the brain. The model consists of two groups of neurons that are interconnected.

The results of the analysis of some asymmetrical networks reveal that the model shows different phase-locked oscillations depending on the structure and intensity of interconnection between groups or coupling of neurons in the group, or the value of transmission latency. The rich variety of the results suggests that it is necessary to make further analysis of the networks with different coupling configurations and different asymmetricities.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