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DETECTING DETERMINISM IN TIME SERIES DATA: WHEN SHOULD WE BOTHER TO BUILD MODELS?

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Abstract

Nonlinear modeling routines are often applied in an effort to extract underlying determinism from time series data. The best of these methods perform well for short noisy time series *when there is determinism* in the underlying system. We show that nonlinear modeling does not distinguish between a static nonlinear transformation of linearly filtered noise and dynamic nonlinearity. To relieve this problem we recommend that surrogate data methods should be applied prior to nonlinear modeling, and the results of that analysis used to guide model selection.

1. Introduction

Nonlinear dynamics and chaos provide an appealing hypothesis: even apparently stochastic time series data may be generated by simple deterministic dynamics. Furthermore, reconstruction theorems provide a methodology to reconstruct dynamics that are topologically equivalent to the underlying system from a single observed time series [8]. However, these results only apply when the system under study is known to be deterministic: something that can rarely be confirmed *a priori*.

Dynamic invariants quantify determinism in a system and can be estimated from a time series. But such estimates are only valid if determinism is present. It is for this reason that the method of surrogate data [9] was proposed to exclude certain trivial possibilities. Surrogate techniques may be applied to test that an observed time series is not: (i) independent and identically distributed noise; (ii) linearly filtered noise; and (iii) a monotonic nonlinear transformation of linearly filtered noise. In this paper we argue that, from a modeling perspective the three standard surrogate tests may be used to determine (respectively): whether a dynamic model is required; whether a ARMA (linear) process is sufficient to model the data; and whether observed nonlinearities are purely static (that is, all dynamics are linear).

A new surrogate technique, the *pseudo periodic surrogate* (PPS) method, has been proposed [7]. The PPS test allows one to test (in time series that exhibit periodic structure) against the null hypothesis of a noisy periodic orbit [5]. We propose that this test may be applied to any time series data to test whether a local linear or noisy state dependent model is sufficient to capture the observed dynamics, or whether a more complex modeling regime is advisable.

In this communication we concentrate on two examples which we feel are representative of the general situation. At the very least, these two examples serve as a counter example to the commonly held view that deterministic nonlinear dynamics in a model of the data implies deterministic nonlinear dynamics in the data. In Section 2 we show that a static nonlinearity may be mistaken for deterministic nonlinear dynamics and that standard linear surrogates may be applied to exclude this possibility. Section 3 demonstrates that the PPS algorithm may be applied to noisy time series data to test against the hypothesis of conditional heteroskedastic origin.

2. Static Rescaling or Dynamic Nonlinearity?

The most general surrogate hypothesis test proposed by Theiler [9] is that the data is generated as a static nonlinear transformation of linearly filtered noise and for real time series data this is often the most realistic. Real data, such as that in Figure 1 are often not Gaussian and usually contain evidence of linear correlation.

We consider the data exhibited in Figure 1 as typical. From this data we generate a surrogate time series that follows the same power spectrum and probability distribution as the data, but is in all other respects a non-

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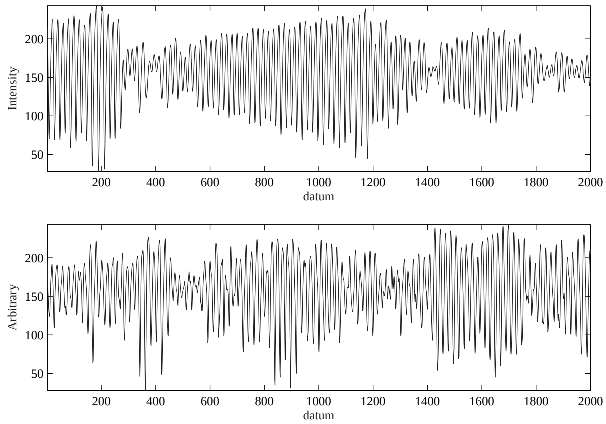

Figure 1: The top panel depicts 2000 time series points from a nonlinear laser exhibiting chaotic dynamics. The bottom plot is a static nonlinear transformation of a linear noise process.

linear transformation of linearly filtered noise. We have repeated this calculation with both the algorithm suggested by Theiler [9] and an iterative version [3]. While the iterative version provides a surrogate that looks more like the data it is statistically not as "typical" as the standard relalisation [5]. For this reason the computation was repeated a third time by generating a parametric estimate of the static nonlinear transformation and the linear noise process. All three sets of results are comparable. We intend to demonstrate a linear stochastic process, subject to a static nonlinear transformation will generate nonlinear models with spurious deterministic dynamics. For illustrative purposes, we generate stochastic process that appear like the true chaotic data in Figure 1.

From both the original data and the linear noise surrogate we constructed a nonlinear model [4]. To avoid over-fitting, this algorithm utilises *minimum description length* (MDL) as a model selection criteria and has been shown to have a wide application and provide good results in a variety of situations [4]. Figure 2 depicts the behaviour of both models in the absence of noise. As expected the model of the laser data exhibits chaotic dynamics. Contrary to expectation, the model of the statically transformed linear noise time series also exhibits similar behaviour.

In an effort to avoid spurious identification of chaos in data consistent with only static nonlinearity we suggest that surrogate data methods should be applied. In Figure 3 we compare both data sets to 50 surrogates generated by Theiler's "algorithm 2" [9] to test whether these data may be explained as a static nonlinear trans-

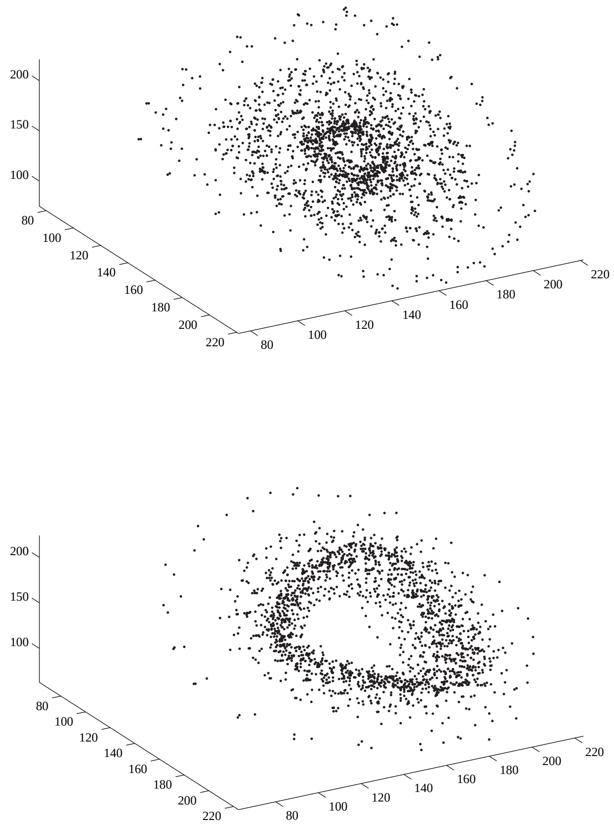

Figure 2: Asymptotic dynamics exhibited by the model of the laser time series (upper panel) and for a model of a linear noise process (lower panel) illustrated in Figure 1.

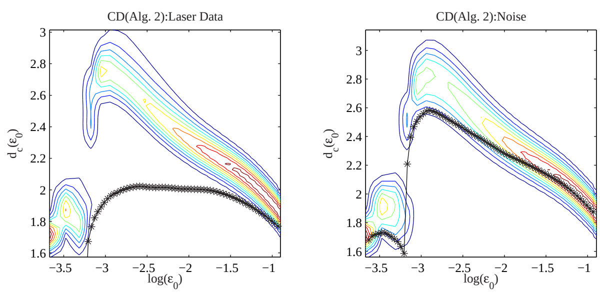

Figure 3: The left and right panels depict (respectively) the comparison of correlation dimension estimate for the data set (heavy line) and 50 linear noise surrogates (contour plot) for the original laser data and the linear noise data of Figure 1.

formation of linearly filtered noise. To compare data and surrogates in a qualitative manner we compute correlation dimension [1]. If the correlation dimension for the data is distinct from the 50 surrogates then the data may be taken as atypical and the system that generated the surrogates is rejected as the likely origin of the data. If not, then the system generated the surrogates may not be rejected. This analysis shows that the linear noise time series is correctly identified as linear noise. We suggest that analysis such as this should be applied prior to modeling to prevent the misidentification of deterministic nonlinearity in time series models.

The reason for this mis-identification of deterministic dynamics can be seen when one examines the form of the model. The modeling algorithm described in [4, 2] fits a function f of the form

$$y_t \quad = \quad f(y_{t-1}, y_{t-2}, y_{t-3}, \ldots, y_{t-d}) + e_t \qquad (1)$$

(where $e_t \sim N(0, \sigma^2)$) to the system dynamics. However, for the linear noise process in Figure 1, this is not appropriate. The static nonlinear transformation $g : \mathbf{R} \longmapsto \mathbf{R}$ means that (1) is inadequate and the dynamics can only be fitted by a function of the form

$$y_t \quad = \quad g\left(f(g^{-1}(y_{t-1}), g^{-1}(y_{t-2}), \ldots, g^{-1}(y_{t-d})) + e_t\right).$$

While the observation function g is invertible (as it is assumed to be monotonic), it is unknown. Taken's theorem [8] still applies: the y_t are topologically equivalent to the underlying dynamics. Unfortunately, the transformation g places a rather severe burden on the nonlinear model fitting algorithms that are typically employed. Furthermore, although the oft cited but rather generic statements that either neural networks or radial basis functions can be employed to fit an arbitrary nonlinear function are still true, the nonlinearity of the problem has been severely compounded. Finally, the noise term now enters the problem nonlinearly and the usual least means squares fit can no longer be applied directly.

3. Non-Stationary or Stochastic?

We have shown that nonlinear modeling routines can misidentify static nonlinearity as deterministic dynamics. In this Section we consider another possible source of spurious determinism in nonlinear modeling routines and suggest that the new PPS algorithm may be employed to avoid such mis-identification.

Strictly linear noise processes of the form (1) for linear f can be easily identified by nonlinear modeling routines. However, intermittent variation in otherwise noisy time series can lead nonlinear models to identify nonlinear deterministic "structure". Simple linear statistical models have also been developed to explain such structure. For

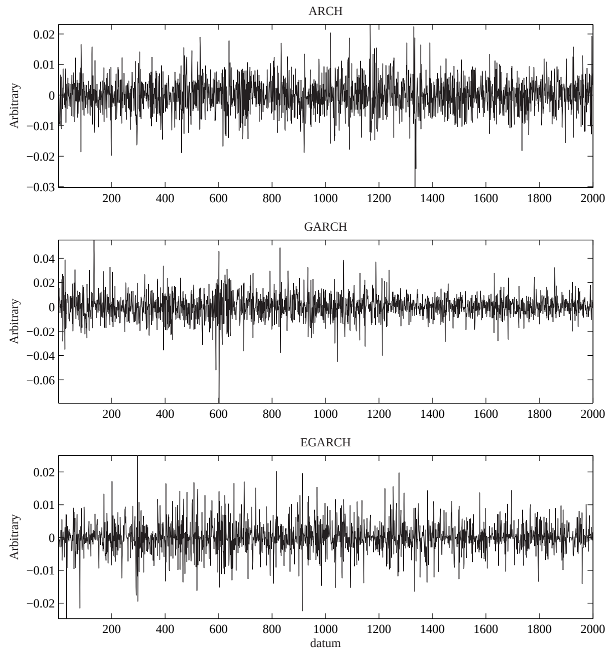

Figure 4: The three panels depict typical conditional heteroskedastic processes from equations 2, 3 and 4 respectively.

example, conditional heteroskedastic processes are used in financial time series analysis to explain time dependent changes in variance. In this Section we describe a test for conditional heteroskedasticity utilising the PPS algorithm.

An autoregressive conditional heteroskedastic (ARCH) model makes the magnitude of the noise perturbations a function of the current state. For example

$$\begin{aligned} x_t &= \sigma_t u_t \\ \sigma_t^2 &= \alpha + \phi x_{t-1}^2 \end{aligned} \qquad (2)$$

is a first order ARCH process (we set $\alpha = 1$ and $\phi = 0.5$). Equation 2 may be extended to form a generalized ARCH (GARCH) in which the noise level is also a function of the past noise level, for example

$$\sigma_t^2 \quad = \quad \alpha + \phi x_{t-1}^2 + \xi \sigma_{t-1}^2. \qquad (3)$$

We simulate (3) with $\alpha = 1$, $\phi = 0.1$ and $\xi = 0.8$. Finally, an exponential GARCH (EGARCH) process introduces a static nonlinear transformation of the noise level. In our computations we consider the EGARCH process defined by

$$\log \sigma_t^2 \quad = \quad \alpha + \phi \left| \frac{x_{t-1}}{\sigma_{t-1}} \right| + 2\xi \log \sigma_{t-1} + \gamma \frac{x_{t-1}}{\sigma_{t-1}} \quad (4)$$

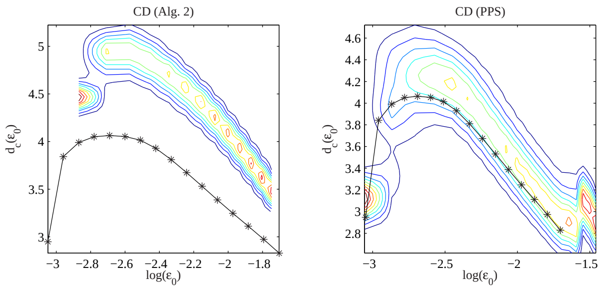

Figure 5: Comparison of correlation dimension estimate for data (heavy line) and either 50 linear noise surrogates (contour plot, left panels) or PPS data (contour plot, right panels) for the the process characterised by equation 4. Results for equations 2 and 3 are equivalent (not shown here).

($\alpha = 1$, $\phi = 0.1$, $\xi = 0.8$ and $\gamma = 0.1$). Figure 4 depicts typical realisations of each of these processes.

The time series in Figure 4 exhibit "burstiness": they appear noisy, but the level of that noise changes with time. This could either be evidence of nonlinear deterministic patterns, or simply conditional heteroskedasticity. In [5] we argue that the PPS algorithm mimics the local dynamics of a system but destroys fine deterministic structure. Conditional heteroskedasticity is essentially state dependent noise. Hence, provided the data is properly embedded one should generate surrogates that preserve the burstiness of the data.

Figure 5 shows the result of the application of the PPS algorithm for conditional heteroskedastic processes such as these. The linear surrogate algorithms (algorithm 2 is illustrated) provide clear evidence that this data was not consistent with the null hypothesis (of nonlinear monotonic transformation of linearly filtered noise). However, the PPS algorithm was unable to reject these data. The PPS algorithm is able to mimic the state dependent noise behaviour of these data, and as we show in [5], it can differentiate between nonlinearity and conditional heteroskedasticity.

4. Conclusion

We have argued that surrogate tests should be applied to an observed time series, prior to modeling, to determine the best form of model to build. We applied a complex nonlinear modeling algorithm with an information theoretic model selection criteria, MDL, to confirm these results. The application of MDL methods ensures that this modeling algorithm builds the simplest model that will adequately describe the dynamics. For data consistent with the three standard surrogate hypotheses this is a purely linear model (perhaps subject to a static nonlinear transformation). For data inconsistent with the three standard surrogate hypotheses a fully nonlinear model is required.

With several well known test systems and experimental time series we showed that application of surrogate data testing can prevent the misidentification of static nonlinearity as a nonlinear dynamical system. Furthermore, PPS surrogate testing is able to identify when conditional heteroskedastic or other state dependent noise process are adequate to describe observed data.

Surrogate techniques may therefore be used to determine whether a complex nonlinear model is justified by the data. These methods should be applied to time series data to ensure that (for example) complex nonlinear dynamics exhibited by models of the data are not simply artifacts of static nonlinear transformations, as demonstrated in Section 2.

In a related paper [6] we have applied these methods to time series of financial indicators. We find that the financial time series are not consistent with linear noise processes or conditional heteroskedasticity.

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