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Binary-Quantized Diffusion Systems and Their Filtering Effect on Sigma-Delta Modulated Signals

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1. Introduction

In recent years, progress in microfabrication technology and thin film forming technology for semiconductor devices has led to the development of new devices that employ quantum effects, such as quantum-dot cellular arrays using tunnel effect and inter-electron Coulomb force[1]. When this kind of nanoscale signal processing system is constructed, the use of simple logical operator circuits arranged regularly makes fabrication of the system less complex than the use of many different arithmetic modules in conventional way. In addition, such a system can be expected to be more resistant to small partial defects of its own.

Achieving filter processing using this kind of cellular array having uniform structure is analogous to producing an equivalent model of a distributed parameter system. In addition, if simple logical operator circuits are used as the elements that are arranged regularly, single-bit signals are better suited than multi-bit signals for internal signal representation of the filter. The AD/DA conversion technology that permits signals of binary single-bit form to be obtained includes sigma-delta ($\Sigma\Delta$) modulation. If this modulated signal is employed as the form of signal inside the uniform structure, no interface is needed between the modulation section and the signal processing section which performs the filter processing. This allows the entire system to be simplified[2].

In the present paper, we consider to construct binary-quantized discrete-time distributed parameter systems that correspond to linear diffusion systems. We first attempt to build the system using one kind of elements that add $\Sigma\Delta$ modulated signals and quantize the sum in the form of single-bit signal. We then investigate whether the system can be used as a lowpass network for single-bit $\Sigma\Delta$ modulated signals in the way that CR networks are used as the constituent elements of analog filters.

2. Network Element

In this section, we explain the principle of operation and the relation to the cellular automata of the network element, i.e. the noise-dispersion-type bit-stream adder.

This noise-dispersion-type bit-stream adder which is a variant of the adder described in reference[3] has two inputs and two outputs related as follows:

$$\begin{aligned} L(n), R(n) &= \frac{1}{2}(x(n)+y(n)) \\ &\qquad \text{if } x(n)=y(n) \\ L(n) &= q(n) \\ R(n) &= -q(n) \\ &\qquad \text{if } x(n)\neq y(n) \end{aligned} \tag{1}$$

where $x(n)$ and $y(n)$ (n is the time index) are the bit-stream input signals, and $L(n)$ and $R(n)$ are the bit-stream output signals, each of which has a value of $+1$ or -1. Variable $q(n)$ is an internal state changing as follows:

$$q(n) = \begin{cases} q(n-1) & \text{if } x(n)=y(n) \\ -q(n-1) & \text{if } x(n)\neq y(n) \end{cases} \tag{2}$$

The noise added to the output by this operation is expressed in terms of the internal state $q(n)$ as follows:

$$e_q(n) = \frac{1}{2}\left\{q(n)-q(n-1)\right\} \tag{3}$$

The resulting equivalent circuit for the noise-dispersion-type bit-stream adder is shown in Fig. 1.

The action of this noise-dispersion-type bit-stream adder can also be expressed in cellular automata form. Whether two "+1" or "−1" particles (input signals) undergo a Cross or Reflect operation is determined by themselves and the internal state $q(n)$. Accordingly, circuits that perform operations equivalent to Eqs. (1) and (2) can be formed by a simple configuration consisting of one-bit memory elements and switches.

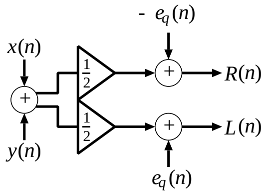

Figure 1: Equivalent circuit of the noise-dispersion-type bit-stream adder.

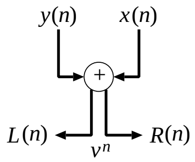

Figure 2: Symbol of a noise-dispersion-type bit-stream adder.

We define $\overline{x(n)}$ or $\overline{x^n}$ as the local time average of a signal x around discrete time n. Letting p be the probability of Cross, from the standardized condition the probability of Reflect is $1-p$. The input/output relationships of the noise-dispersion-type bit-stream adder are given by

$$\begin{aligned} \overline{L(n)} &= p\overline{x(n)} + (1-p)\overline{y(n)} \\ \overline{R(n)} &= (1-p)\overline{x(n)} + p\overline{y(n)} \end{aligned} \tag{4}$$

In the noise-dispersion-type bit-stream adder that is acting according to Eqs. (1) and (2) p becomes $\frac{1}{2}$.

Hereafter, the noise-dispersion-type bit-stream adder will simply be referred to as an adder and will be indicated by the symbol shown in Fig. 2. For the sake of convenience, the variable v^n shown in Fig. 2 is used to indicate an output value that is distinct from the output signal, and is defined as

$$v^n = \frac{1}{2}(R(n) + L(n)) \tag{5}$$

3. One-Dimensional Diffusion Network

In this section, we describe a method of combining the adders in **2.** into a network that has lowpass characteristics corresponding to an analog CR network.

3.1. Principle of Operation

Consider the addition of adjacent variables in a one-dimensional (1D) diffusion network as shown in Fig. 3. The subsequent analysis relates to the local time average of the bit-stream signals in the network. Let i be the position index. In Fig. 3, the results of additions are

$$\overline{R_i(n+1)} = p^2\overline{v_{i-1}^n} + (1-p)\overline{v_i^n} + p(1-p)\overline{v_{i+1}^n} \tag{6}$$

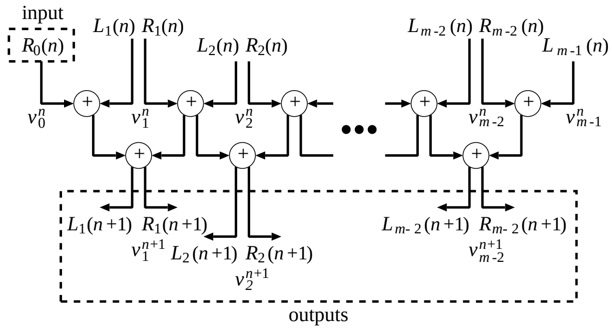

Figure 3: One-dimensional (1D) diffusion network.

$$\overline{L_i(n+1)} = p(1-p)\overline{v_{i-1}^n} + (1-p)\overline{v_i^n} + p^2\overline{v_{i+1}^n} \tag{7}$$

Accordingly, we obtain

$$\begin{aligned} \overline{v_i^{n+1}} &= \frac{1}{2}\left(\overline{R_i(n+1)} + \overline{L_i(n+1)}\right) \\ &= \overline{v_i^n} + \frac{p}{2}\left(\overline{v_{i-1}^n} - 2\overline{v_i^n} + \overline{v_{i+1}^n}\right) \end{aligned} \tag{8}$$

Equation (8) is equivalent to a discrete one-dimensional diffusion equation,

$$u_i^{n+1} = u_i^n + \frac{k\Delta t}{(\Delta x)^2}\left(u_{i-1}^n - 2u_i^n + u_{i+1}^n\right) \tag{9}$$

Δx, Δt:the space and time intervals of discretization

The term $\frac{k\Delta t}{(\Delta x)^2}$ which contains the diffusion coefficient $k(> 0)$ corresponds to $\frac{p}{2}$ in Eq. (8). This shows that diffusion speed can be controlled by the Cross/Reflect ratio, accordingly the lowpass characteristics can also be varied. However, in the subsequent discussion, we will continue to configure the system with $p = \frac{1}{2}$.

3.2. Transient Response

In order to investigate the transient characteristics of the 1D diffusion network, we compare the bit-stream output signal obtained from the network with a numerical analysis of the discrete one-dimensional diffusion equation (9). The input signals to the 1D diffusion network and the diffusion equation (9) are given at $R_0(n)$ and u_0^n, respectively. The output signals are $R_i(n+1)$ and $u_i^{n+1}(i = 1, 2, 3, \ldots, m-2)$. The boundary conditions on $L_{m-1}(n)$ and u_{m-1}^n are

$$\begin{aligned} L_{m-1}(n+1) &= L_{m-3}(n+1) \\ u_{m-1}^{n+1} &= u_{m-3}^{n+1} \end{aligned} \tag{10}$$

and the initial condition is assumed to be

$$\begin{aligned} R_i(0) &= -L_i(0) \\ u_i^0 &= 0 \\ &(i = 0, 1, 2, \ldots, m-1) \end{aligned} \tag{11}$$

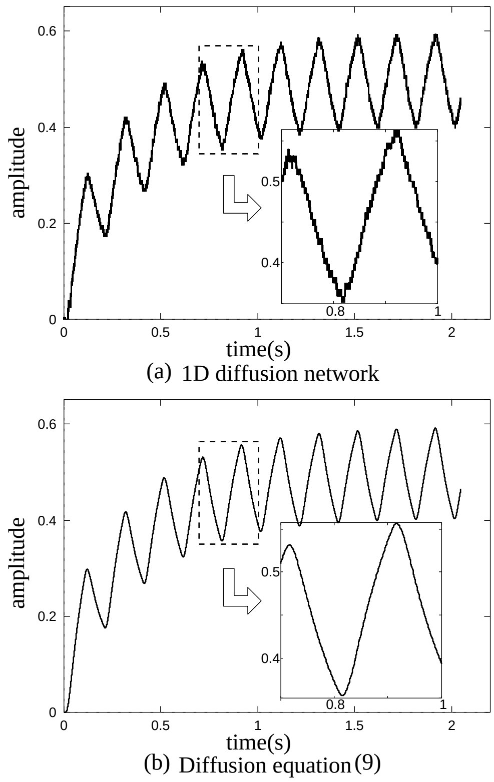

Figure 4: Transient response of the 1D diffusion network and the diffusion equation (9).

In the comparison, m is taken to be 100. We take the diffusion coefficient k in the diffusion equation (9) to be 1, the space interval of discretization to be $\Delta x = \frac{1}{100}$ and the time interval to be $\Delta t = \frac{1}{40000}$ so that Eq. (9) is equivalent to Eq. (8) with $p = \frac{1}{2}$. The input signal to the 1D diffusion network is formed by taking the signal,

$$u(t) = \begin{cases} 1 & (0 \leq t < 0.1, 0.2 \leq t < 0.3 \ldots) \\ 0 & (0.1 \leq t < 0.2, 0.3 \leq t < 0.4 \ldots) \end{cases} \tag{12}$$

and converting it to a bit-stream signal by first order $\Sigma\Delta$ modulation. The signal expressed by Eq. (12) is directly applied to the diffusion equation (9). The output signals at R_{50} and u_{50} are shown in Figs. 4(a) and (b), respectively. The bit-stream output signal from the 1D diffusion network is averaged by a decimation number of 256. Then, the output error of the 1D diffusion network in contrast to the diffusion equation (9) is 1.0[%]. The error is defined as the average obtained by dividing the absolute value of the difference between the two values by the maximum value of u_{50}. Based on the above results, the 1D diffusion network and the diffusion equation (9) are found to behave almost identically.

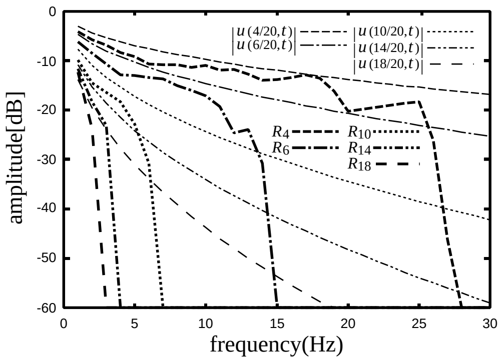

Figure 5: Frequency responses of the 1D diffusion network and comparison with the analytical solutions to a continuous one-dimensional diffusion equation.

3.3. Frequency Response

Since the 1D diffusion network has shown a transient response that is almost identical to that of the diffusion equation (9), the 1D diffusion network is expected to have lowpass frequency characteristic. In 3.3, we compare the frequency responses of the 1D diffusion network and those of a continuous one-dimensional diffusion equation.

In the simulation of the 1D diffusion network, we set $m = 20$. The boundary and the initial conditions are as given by Eqs. (10) and (11), respectively. The $\Sigma\Delta$ modulation circuit at the input and the adders are assumed to operate at a frequency of $f_s = 1024$[Hz]. The frequency responses obtained at R_4, R_6, R_{10}, R_{14} and R_{18} are shown in Fig. 5. The dependence of the lowpass characteristics on position can be found from this figure.

The solution to a continuous one-dimensional diffusion equation with a diffusion coefficient k is given by

$$\begin{aligned} u(x,t) &= f_c(x)\cos wt + f_s(x)\sin wt \\ f_c(x) &= \exp(-\sqrt{\frac{w}{2k}}x)\cos\sqrt{\frac{w}{2k}}x \\ f_s(x) &= \exp(-\sqrt{\frac{w}{2k}}x)\sin\sqrt{\frac{w}{2k}}x \end{aligned} \tag{13}$$

when $\cos wt$ is applied at $x = 0$. The solution with $k = 1$ is also shown in Fig. 5. This figure reveals that the 1D duffusion network is an adequate approximation of a continuous one-dimensional diffusion equation.

3.4. Noise Analysis

Because of the reverse actions of the state q inside the adder organizing the 1D diffusion network, noise given by Eq. (3) is added to the output of the adder. In 3.4, we describe a noise analysis of the 1D diffusion network.

Replacing the adders in Fig. 3 by their equivalent circuits in Fig. 1, we obtain the relation between outputs R and the states Q of the adders in the Z-domain as follows:

$$\begin{aligned} &R = H(Z_t, Z_s)Q \\ &H(Z_t, Z_s) \\ &\quad = \left\{ \left(1+Z_s^{-1}\right)\left(Z_s^{-\frac{1}{2}}+2-Z_s^{\frac{1}{2}}\right) \right. \\ &\quad\quad \left. +\left(4Z_t-1-Z_s^{-1}\right)\left(Z_s^{-\frac{1}{2}}-2-Z_s^{\frac{1}{2}}\right)\right\} \\ &\quad\quad \times\left(1-Z_t^{-1}\right)\left\{\left(4Z_t-1-Z_s\right)\left(4Z_t-1-Z_s^{-1}\right)\right. \\ &\quad\quad \left. -\left(1+Z_s^{-1}\right)\left(1+Z_s\right)\right\}^{-1} \end{aligned} \tag{14}$$

The Z variables in this equation are

$$\begin{aligned} Z_t &= \exp(jw_tT_t) = \exp(jw_t/f_{st}) \\ Z_s &= \exp(jw_sT_s) = \exp(jw_s/f_{ss}) \end{aligned} \tag{15}$$

where f_{st} and f_{ss} are the temporal and spatial sampling frequencies, w_t and w_s are the temporal and spatial angular frequencies.

The spectral density (SD) of Q, $S_Q(w_t)$, is obtained from the discrete Fourier transform of its autocorrelation function. If $q(n)$ is assumed to be a randomly varying binary signal, we have

$$S_Q\left(2\pi\frac{k}{f_{st}}\right) = \sqrt{T_t} \tag{16}$$

Accordingly, the noise spectral density in the output signal R of the 1D diffusion network is the product $H\left(w_t/f_{st}, w_s/f_{ss}\right)S_Q\left(2\pi k/f_{st}\right)$ of Eqs. (14) and (16). The noise spectral density for the case of $f_{st} = 1024$[Hz], $f_{ss} = 128$[cycles per unit length] is shown in Fig. 6(a). The noise spectral density obtained using a two-dimensional fast Fourier transform (FFT) is shown in Fig. 6(b) for the case of $f_s = 1024$[Hz], $m = 128$. In both Figs. 6(a) and (b), the spectral density increases as the time and space frequencies increase; the noise spectral density distributions in these two cases approximately agree. The ordinates are normalized by $(2\pi)^2$ in Fig. 6(a) and by $f_{st} \times f_{ss}$ in Fig. 6(b).

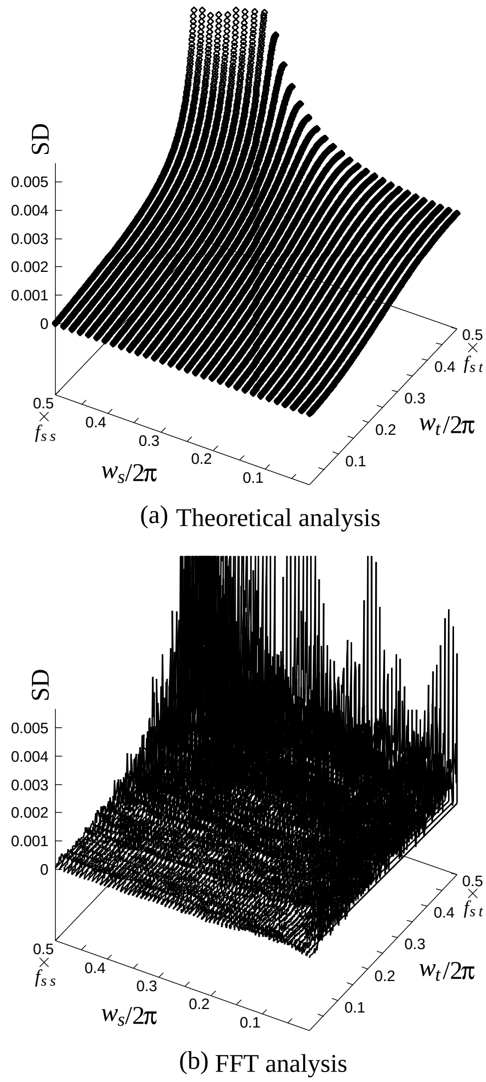

Figure 6: Spectral density of the output noise from the 1D diffusion network.

4. Conclusions

In the present paper, we have proposed a method of constructing binary-quantized discrete-time distributed parameter systems that correspond to linear diffusion systems and confirmed that the proposed networks function as lowpass networks for $\Sigma\Delta$ modulated signals.

Acknowledgements

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