International Symposium on Nonlinear Theory and its Applications
Xi'an, PRC, October 7-11, 2002

Combinatorial Resonances in A Coupled Duffing's Circuit with Asymmetry

Yue MA and Hiroshi KAWAKAMI

The faculty of engineering, The University of Tokushima, JAPAN
Minami-joshanjima 2-1, Tokushima, JAPAN, 770-8506
Tel: +81-88-656-7465, Fax: +81-88-656-9060
mayue (kawakami) @ee.tokushima-u.ac.jp

1. Introduction

A nonlinear resonance is one of the typical phenomena in nonlinear circuits with a periodic external force. A circuit containing a saturable core [1] described by a Duffing's equation can exhibit a typical nonlinear resonance. Meanwhile, the system of coupled oscillators has attracted a great deal of attention recent years, because it can exhibit a great variety of fundamental dynamical phenomena. In our research, we consider a coupled system consisted of two Duffing's circuits shown in Fig.1. We conclude that a combinatorial resonance occurs if subsystems are coupled on a linear and weakly condition.

A coupled system may exhibit some kinds of symmetrical property, which are determined by periodic external forces and parameters of each oscillator. According to our previous researches [2], symmetry of this coupled Duffing's circuit has been clarified. Combinatorial resonant solutions were discussed when periodic external forces are in-phase and anti-phase respectively. But in real systems, symmetries are not always satisfied. Both a time delay of external force and element's difference between subsystems can make the coupled system asymmetry. As a result, combinatorial resonant solutions will lose their symmetry also. In this paper, we study the bifurcation problem of combinatorial resonances in coupled Duffing's circuit when symmetry is broken. Four perturbations of symmetry are discussed. Comparing with symmetrical cases, we can understand combinatorial resonances of coupled oscillators thoroughly.

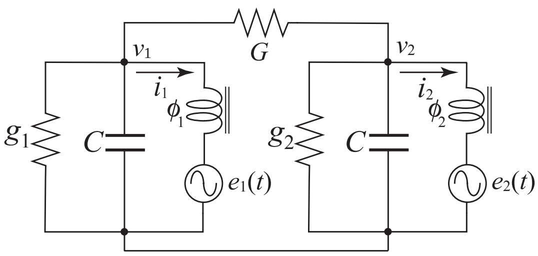

Figure.1 The coupled Duffing's circuit.

2. Circuit Equations

Let's consider the coupled Duffing's circuit shown in Fig.1. With the notations in the figure, we can describe the circuit equations as follows:

$$\begin{cases} \dfrac{d\phi_1}{dt} = v_1 + e_1(t) \\ \dfrac{d\phi_2}{dt} = v_2 + e_2(t) \\ C\dfrac{dv_1}{dt} + g_1 v_1 + i_1 + G(v_1 - v_2) = 0 \\ C\dfrac{dv_2}{dt} + g_2 v_2 + i_2 + G(v_2 - v_1) = 0 \end{cases} \tag{1}$$

In this circuit, we assume the characteristic of nonlinear inductors as following cubic functions:

$$\begin{aligned} i_1 &= f(\phi_1) = f_1\phi_1 + f_3\phi_1^3 \\ i_2 &= f(\phi_2) = f_1\phi_2 + f_3\phi_2^3 \end{aligned} \tag{2}$$

State variables and coefficients of Eqs.(1) are normalized as:

$$\begin{aligned} &\phi_1 = x_1,\ \phi_2 = x_2,\ v_1 = y_1,\ v_2 = y_2 \\ &c_1 = \frac{f_1}{C},\ c_3 = \frac{f_3}{C},\ k_1 = \frac{g_1}{C},\ k_2 = \frac{g_2}{C},\ \delta = \frac{G}{C} \end{aligned} \tag{3}$$

The external forces e_1 and e_2 described below are sinusoidal voltage forces with a phase-shift θ:

$$e_1(t) = B\sin(\omega t), \quad e_2(t) = B\sin(\omega t + \theta) \tag{4}$$

Substitution yields

$$\begin{cases} \dfrac{dx_1}{dt} = y_1 + B\sin(\omega t) \\ \dfrac{dx_2}{dt} = y_2 + B\sin(\omega t + \theta) \\ \dfrac{dy_1}{dt} = -c_1x_1 - c_3x_1^3 - k_1y_1 - \delta(y_1 - y_2) \\ \dfrac{dy_2}{dt} = -c_1x_2 - c_3x_2^3 - k_2y_2 - \delta(y_2 - y_1) \end{cases} \tag{5}$$

where we fix some system parameter as:

$$c_1 = 0\,,\quad c_3 = 1\,,\quad \omega = 1 \tag{6}$$

3. Combinatorial Resonance and Symmetry

In a system of m-coupled oscillators, each oscillator may exhibit many nonlinear resonant solutions. We assume that numbers of solutions are $n_1, n_2, \cdots, n_m$. If oscillators are coupled by linear elements, and on a weak coupling condition (the weakest case is uncoupled condition), we can conclude that the global behavior of coupled system is just the complete combination of solutions occurred in each oscillator. We call it combinatorial resonance. The amount of combinatorial resonant solutions in this weak linear coupled system is the product of $n_1, n_2, \cdots,$ and n_m.

A system described by a Duffing's equation, as you know, can exhibit 3 periodic solutions: nonresonant, resonant and unstable solutions. So, in this coupled Duffing's circuit, it can exhibit 9 combinatorial solutions on weak coupling condition. Increasing the intensity of coupling, we observe that some of combinatorial resonant solutions bifurcate so that the amount of solutions changes.

Let's observe the system equations Eqs.(5). If $k_1 = k_2$, we conclude that symmetry of coupled Duffing's circuit is determined by external forces. It exhibits complete or inversion symmetry when phase-shift θ is 0 (in-phase) and π (anti-phase) respectively (see Fig.2). In this paper, we investigate several asymmetrical cases by changing θ or k. Comparing with symmetrical cases, we analyze the bifurcation problem and combinatorial resonance when symmetry of coupled oscillators is broken down.

4. Bifurcation and Combinatorial Resonance

Firstly, let's review results of symmetrical cases briefly. In Eqs.(5), we fix parameter: $k_1 = k_2 = 0.2$, and draw the bifurcation diagrams in (B, δ) plane when external forces are in-phase and anti-phase respectively. As shown in Fig.3(c)(d), a pair of solutions branch out by D-type of

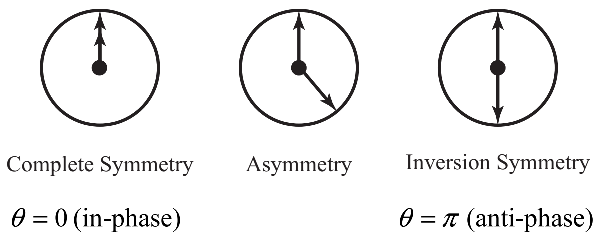

Figure.2 Symmetry and phase-shift.

branching. Because they own the same bifurcation structure and amplitude, we draw them by one bold curve. Then we discuss several asymmetrical cases.

4.1 External forces depart from in-phase

We fix parameters as:

$$k_1 = 0.2,\quad k_2 = 0.2,\quad \theta = 10^\circ \tag{7}$$

On this condition, the coupled system is an asymmetrical case that external forces depart from in-phase slightly. In this case a bifurcation diagram in (B, δ) plane is shown in Fig.4(a). As you can see, D-type of branching turns to tangent bifurcation. The symmetrical solution pair mentioned before loses its symmetry. Two solutions separate and occur tangent bifurcations of their own. These phenomena can be explained clearly by one parameter bifurcation diagrams Fig.4(b),(c). Fig.4(b) and (c) are drawn by changing B along line L1 ($\delta = 0.1$) and L2 ($\delta = 0.05$) respectively. Note that, the resonant curve is separated into two parts on L1, whereas it is connected on L2. This difference appears when parameter across the region R, which is enlarged in Fig.4(a).

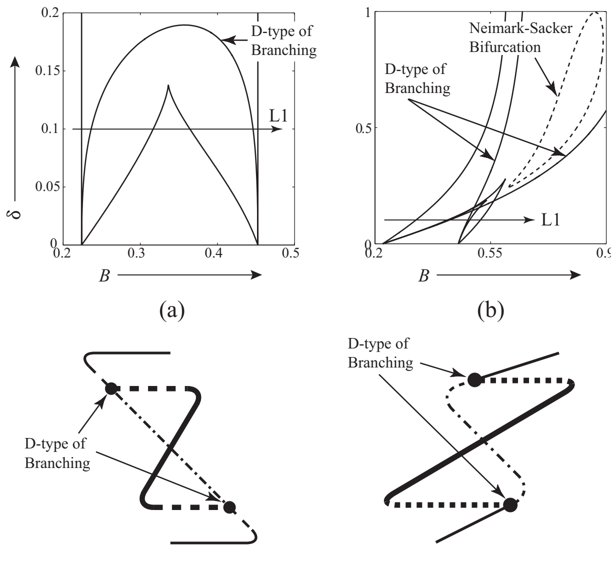

Figure.3 Bifurcation diagrams of complete symmetrical case (a) and inversion symmetrical case (b); below ones are one parameter bifurcation diagrams changing B along L1 (δ= 0.1). (c) corresponds to (a), while (d) corresponds to (b). In (c) and (d), solid curves represent stable solutions; dash curves and dash-dot curves represent 1-dimensional and 2-dimensional unstable solutions respectively.

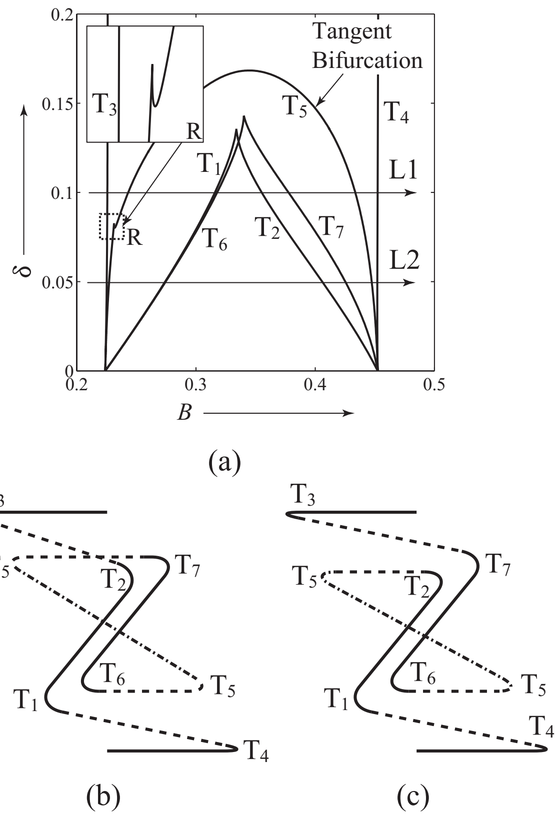

Figure.4 Bifurcation diagram (a) and one parameter bifurcation diagrams along L1 (b) and L2 (c).

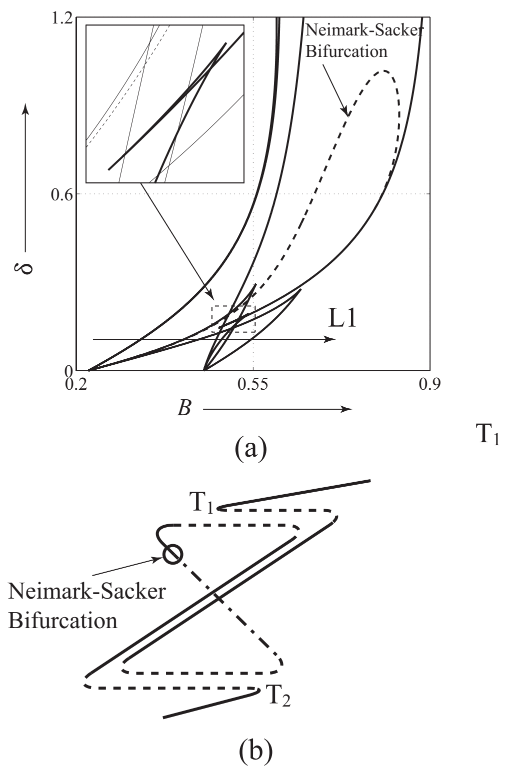

Figure.5 Bifurcation diagram (a) and one parameter bifurcation diagram (b) along L1.

4.2 External forces depart from anti-phase

Next we consider an asymmetrical case that the system depart from inversion symmetry slightly by fixing parameters as:

$$k_1 = 0.2, \quad k_2 = 0.2, \quad \theta = 170° \tag{8}$$

We obtain the bifurcation diagram as Fig.5(a). Compareing with Fig.3(b), Neimark-Sacker bifurcation is tangent to tangent bifurcation curves instead of a close curve. We also draw a one parameter bifurcation diagram along L1 ($\delta = 0.1$) in Fig.5(a). Note that, in this case D-type of branchings shown in Fig.3(d) correspond to the tangent bifurcations of stable solutions (see Fig.5(b)), but in previous case they correspond to the tangent bifurcations of 1-dimensional and 2-dimensional unstable solutions (compare Fig.4(c) with Fig.3(c)).

4.3 $k_1 \neq k_2$ and in-phase forces condition

In this case, external forces are in-phase, but we break the balance of resistors in Duffing's circuits. We choose parameters in Eqs.(5) as:

$$k_1 = 0.1, \quad k_2 = 0.2, \quad \theta = 0° \tag{9}$$

Then we have the bifurcation diagram as Fig.6(a), and the one parameter bifurcation diagram along L1 ($\delta = 0.1$) as Fig.6(b). In Fig.6(a), tangent bifurcation T1 and T2 are far away from T6 and T7, specially, when $\delta = 0$, they don't degenerate to two points as previous cases. The stable solution between T1 and T2 is the couple of resonant solution in left Duffing's circuit and nonresonant solution in right one. The stable solution between T6 and T7 is contrary; which is coupled by resonant solution in right oscillator and nonresonant solution in left one. Moreover, see Fig.6(b), the resonant curve is separated into two parts, which is similar to Fig.4(b). If we also fix $k_2 = 0.2$ and $\delta = 0.08$, we obtain a bifurcation diagram in (B, k_1) plane as Fig.7. Note that, at $k_1 = k_2 = 0.2$, T1 intersects T6 and T2 intersects T7, i.e., degenerate bifurcations are observed as system becomes symmetrical.

4.4 $k_1 \neq k_2$ and anti-phase forces condition

We fix parameter as following,

$$k_1 = 0.1, \quad k_2 = 0.2, \quad \theta = 180° \tag{10}$$

In this case, we only draw bifurcation diagram as shown in Fig.8. Similar to Fig.6(a), they can't converge to two points decreasing δ to 0. There are two Neimark-Sacker

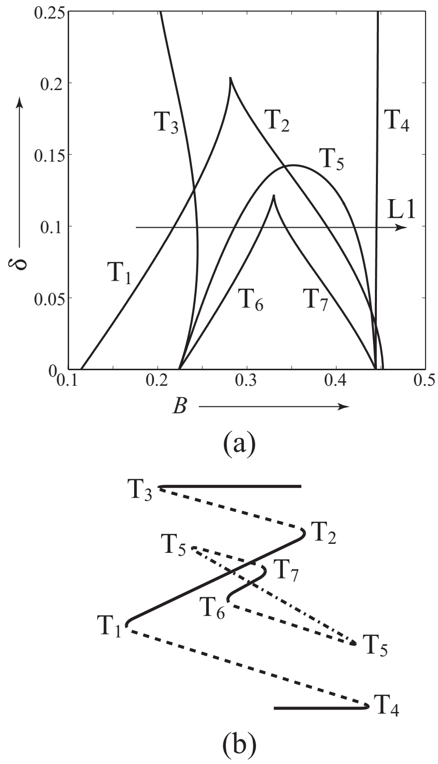

Figure.6 Bifurcation diagram (a) and one parameter bifurcation diagram (b) alone L1.

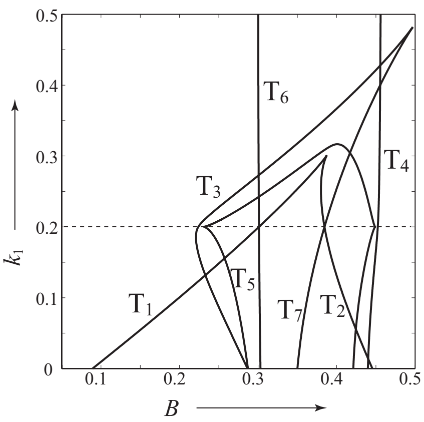

Figure.7 Bifurcation diagram in (B, k_1) plane when parameters satisfy Eq.(9).

bifurcation curves in Fig.8. N1 is a close curve like Fig.3(b), and N2 is tangent to tangent bifurcation curves like Fig.5(a). Two quasi-periodic solutions can join together when the parameters are given in the center region of N1 and N2.

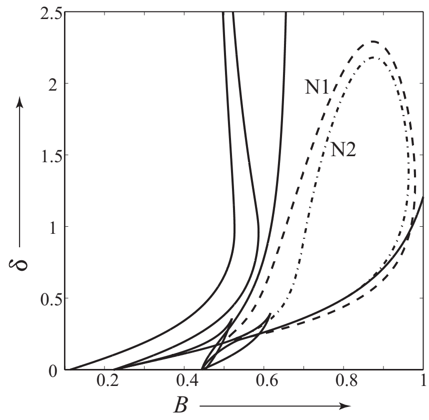

Figure.8 Bifurcation diagram in (B, k_1) plane when parameters satisfy Eq.(10).

5. Concluding Remarks

In this paper, we considered the bifurcations of combinatorial resonant solutions of coupled Duffing's circuit on several asymmetrical parameter conditions. By using bifurcation diagrams we showed that a D-type of branching of periodic solution becomes a tangent bifurcation when the symmetry of the system is broken. The results may be applied to the more general coupled systems, such as a ring or a ladder connected Duffing's circuit.

References

[1] C. Hayashi: "Nonlinear Oscillations in Physical Systems," Princeton Univ. Press, 1985.

[2] Y. Ma and H. Kawakami: "Combinatorial Resonances in Coupled Duffing's Circuits," IEICE Trans. Fundamentals, Vol.E85-A, No.3, pp.648-654, 2002/3.