Lyapunov Spectrum Estimation of the Chaotic System that the Numerical Jacobian is Unidentified Though the Dynamics is Clear.

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Abstract - This paper propose a method for the Lyapunov spectrum estimation based on the time series data. The conventional method is required to find any data sets that exist near an orbital point of the system, so the much processing time is needed. Then, we try to intentionally generate the data sets near the orbital point of the system that the numerical Jacobian is unidentified though the dynamics is clear. By this step, the processing time of the Lyapunov spectrum estimation based on the time series data can be greatly reduced.

Keywords: Lyapunov spectrum estimation, Time series data, Intentional generation of the data.

1. Introduction

Various methods for estimating the Lyapunov spectrum of the chaotic system have been proposed in the field of nonlinear theory. The typical method is the one that is based on the numerical Jacobian obtained from the dynamics of the system, another is the method that estimates from the time series data[1,2] generated by the chaotic system. In particular, the method based on the time series data is effective to estimate the Lyapunov spectrum of the system that has unknown dynamics and the Jacobian. However, the method is needed to find any data sets that exist near an orbital point of the system from extremely a lot of data. In addition, the methods for embedding the time series data into the vector and for deciding the dimension of vector are difficult problems to obtain the reliable results, so trial-and-error experiments and much computation time are required, generally.

In our study, the private communication with chaos realized by 16-bit fixed-point computation has been examined [3-5]. In the system, the overflow and the round-off properties that arise in the fixed-point computation are precisely expressed in both the modulation and demodulation systems[4,5], so the recovery error on the demodulation side does not occur. However, when the chaotic property of the system is investigated by Lyapunov spectrum, the derivative of the round-off function cannot be estimated[5]. Therefore, the numerical Jacobian is unidentified though the dynamics of chaotic system is clear; the trustworthy numerical result of the Lyapunov spectrum cannot be obtained.

In this study, we examine the Lyapunov spectrum estimation for the system that the Jacobian cannot be identified though the dynamics is clear. The examined method is the one that applies the conventional time series analysis based on the least-squire method[1,2]. In this case, the problem about the embedding into the vector can be avoided because the dynamical system is clear.

Besides, the conventional method is needed to find any data sets that exist near an orbital point of the system, so the much processing time is spent. Then, we try to intentionally generate the data sets near the orbital point of the system. By this step, the processing time can be greatly reduced. However, there is the case that the generated data sets might not exist on the orbit of the system, so the estimated results of Lyapunov spectrum should be carefully verified by comparing with the real value.

In this paper, we first investigate the reliability of the proposed method that generates data sets near the orbital point using Henon map. In this investigation, several methods are verified about the method of arranging the data sets intentionally generated. the estimation time and accuracy of the proposed method are compared with the conventional method based on the time series data.

Next, the proposed method is adopted to the Lyapunov spectrum estimation of the private communication system[5] included in the round-off function, the derivative of the round-off function is identified.

2. Example of the incalculable Jacobian

In out study, the private communication system with chaos is examined. This system is designed as a system that is suitable to realize using fixed-point computation such as Digital Signal Processor(DSP)[3-5]. The modulator is shown as

$$x_1(n) = s(n) - g(f(x_1(n-1))) + a\, f(x_3(n-1)) + q\ , \qquad (1)$$

$$x_2(n) = f(x_1(n-1)) - b\, f(x_2(n-1)) - g\, f(x_3(n-1))\ , \qquad (2)$$

$$x_3(n) = x_2(n-1)\ , \qquad (3)$$

where $s(n)$ is the input signal, $x_1(n)$, $x_2(n)$ and $x_3(n)$ are the internal state variables, a, q, b and g are parameters and $g(x)$ is the nonlinear function.

$$g(x) = \begin{cases} kx + s & : x \le -d \\ \dfrac{kd - s}{d} x & : -d < x < d \\ kx - s & : x \ge d \end{cases} \qquad (4)$$

Furthermore, the nonlinear function $f(x)$ is the one that combines the overflow function $O(x)$ and the round-off function $R(x)$ attending on the fixed-point computation[4,5] shown in Figure 1.

$$f(x)=O(R(x))=R(O(x)) \qquad (5)$$

By including this nonlinear function to the modem system, the proposed private communication system can keep the perfect recovery characteristic.

Besides, when the Lyapunov spectrum of the private communication system is investigate, the derivative of the nonlinear function $f(x)$ is required in the Jacobian $\mathbf{J}$.

$$\mathbf{J} = \begin{bmatrix} -\dfrac{\partial g(f(x_1))}{\partial x_1} & 0 & a\dfrac{\partial f(x_3)}{\partial x_3} \\ \dfrac{\partial f(x_1)}{\partial x_1} & -b\dfrac{\partial f(x_2)}{\partial x_2} & -g\dfrac{\partial f(x_3)}{\partial x_3} \\ 0 & 1 & 0 \end{bmatrix} \qquad (6)$$

It is thought that the derivative of the round-off function $R(x)$ becomes 0 based on the shape shown in Figure 1(c), so the numerical result of the Lyapunov spectrum cannot be estimated though the dynamics is clear.

3. Lyapunov spectrum based on the time series data

Figure 2 shows the concept of the Lyapunov spectrum estimation from the time series data[1,2].

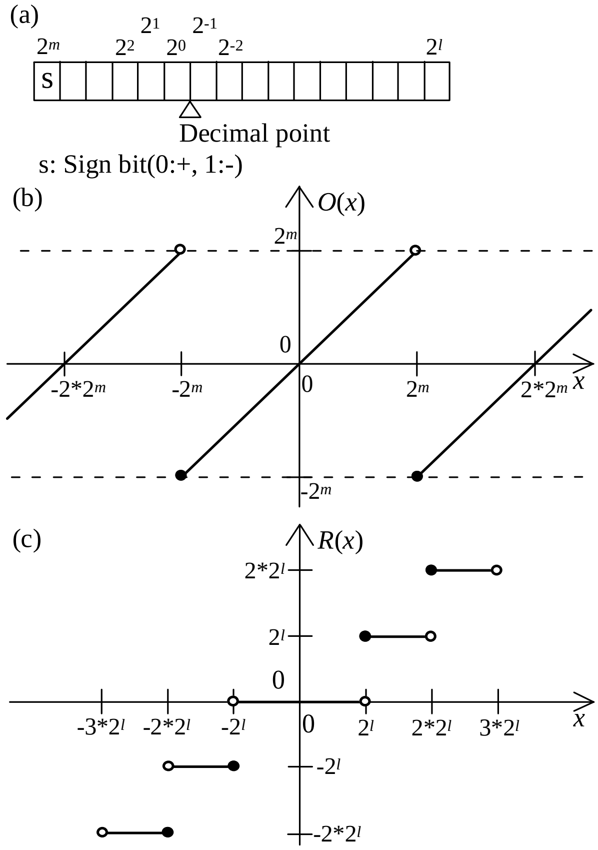

Figure 1: Relation between the fixed-point computation and the nonlinear functions. (a) Format of the computation, (b) Overflow function $O(x)$, (c) Round-off function $R(x)$.

A point $\mathbf{x}_c(n)$=[$x_1(n)$, $x_2(n)$,, $x_d(n)$] that is an orbital point of the attractor is supposed. A small ball of radius e centered at the point $\mathbf{x}_c(n)$ is considered, and points {$\mathbf{x}_n(k)$: k=1,2,,M} included in the small ball are found from the generated data set by the chaotic system. When the point $\mathbf{x}_c(n+1)$ after a time interval of $\mathbf{x}_c(n)$ is considered, the neighboring point {$\mathbf{x}_n(k)$} also proceed to {$\mathbf{x}_{n+1}(k)$}. The Lyapunov spectrum [l_1, l_2,, l_d] is estimated by the degree of the expansion and contraction of the radius of the ball that includes {$\mathbf{x}_{n+1}(k)$}. When the displacement vectors $\mathbf{y}_n(k)$=[$y_1(k)$, $y_2(k)$,, $y_d(k)$] and $\mathbf{z}_n(k)$=[$z_1(k)$, $z_2(k)$,, $z_d(k)$] are calculated by

$$\mathbf{y}_n(k)=\mathbf{x}_n(k)-\mathbf{x}_c(n) \qquad : k=1,2,,M \qquad (7)$$

$$\mathbf{z}_n(k)=\mathbf{x}_{n+1}(k)-\mathbf{x}_c(n+1) \quad : k=1,2,,M. \qquad (8)$$

If the radius e is small enough, the relation between $\mathbf{y}_n(k)$, $\mathbf{z}_n(k)$ and the Jacobian $\mathbf{Jt}(n)$ is approximated as

$$\mathbf{z}_n(k)=\mathbf{Jt}(n)\mathbf{y}_n(k) \qquad : k=1,2,,M, \qquad (9)$$

so the Jacobian $\mathbf{Jt}(n)$ is estimated by least square method as follows:

$$\mathbf{Jt}(\mathrm{n})=\mathbf{C}\mathbf{V}^{-1} \qquad (10)$$

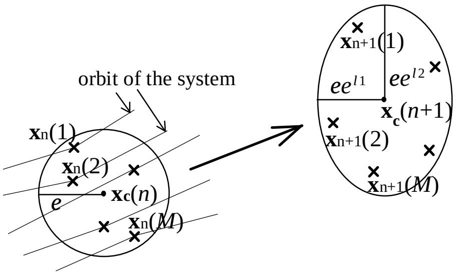

Figure 2: Concept of the Lyapunov spectrum estimation based on the time series data.

$$c_{ij} = \frac{1}{M}\sum_{l=1}^{M} z_i(l)y_j(l) \quad (11)$$

$$v_{ij} = \frac{1}{M}\sum_{l=1}^{M} y_i(l)y_j(l)\,. \quad (12)$$

: $i=1,2,,d \quad j=1,2,,d$

The Lyapunov spectrum is computed using estimated Jacobian $\{\mathbf{Jt}(n): n=1,2,,,N\}$[1,2].

Besides, this conventional method is required to find any data sets that exist near an orbital point of the system, so the much processing time is needed.

4. Generation of the points included in the small area

In this study, we try to intentionally generate the points $\mathbf{x}_n(1)$, $\mathbf{x}_n(2)$,, $\mathbf{x}_n(M)$ included in the small area for improving the computation time of the Lyapunov spectrum estimation based on the time series data. Figure 3 shows the methods that arrange the points in the small area. Originally, Type A in Figure 3 may be suitable for the arrangement of the points considering the conventional method of [1,2]. However, there is hardly a difference between Type A and Type C excluding the points in four corners, so the method of arrangement like the square is tested in this study.

4.1 Experiments based on the Henon map

In this chapter, the proposed method is applied to the Lyapunov spectrum estimation of Henon map.

$$x(n+1) = y(n) + 1 - ax(n)^2 \quad (13)$$

$$y(n+1) = bx(n) \quad (14)$$

The parameters a and b, the initial value of the internal state variables $x(0)$ and $y(0)$ are set to 4.0, 0.3, 0.1 and 0.0, respectively. Furthermore, 64-bit floating point computation is carried out.

Estimated results are shown in Table 1. As the table shows, there is no difference of the estimation result by the difference of the arrangement type A ~ D. The errors with the numerical value are minute.

Besides, comparison of the estimation time is shown in Table 2. In the table, the estimation time in each method is normalized based on the numerical computation. As the table shows, the estimation time of the proposed method is suppressed to several times the numerical computation, and can be executed from the method based on the time series data in a short time extremely. Therefore, if dynamics of the system is a clear, the proposed method is an effective method in the point of the estimation accuracy and the estimation time.

4.2 Lyapunov spectrum estimation of the private communication system.

The proposed method is applied to the Lyapunov spectrum

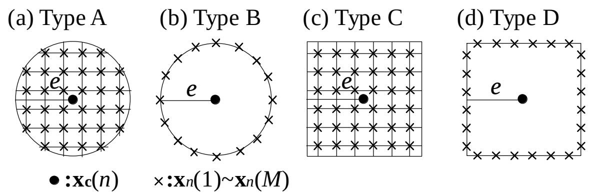

Figure 3: Arrangement of the points $\mathbf{x}_n(1)$, $\mathbf{x}_n(2)$,, $\mathbf{x}_n(M)$.

Table 1: Estimation results (N=10,000).

Numerical Results: $[\lambda_1,\lambda_2]$=[0.414306 -1.61828]

e	M	Type	λ_1	λ_2	e	M	Type	λ_1	λ_2
0.01	10	A	0.414356	-1.61833	0.1	10	A	0.414356	-1.61833
		B	0.414356	-1.61833			B	0.414356	-1.61833
		C	0.414356	-1.61833			C	0.414356	-1.61833
		D	0.414356	-1.61833			D	0.414356	-1.61833
	100	A	0.414356	-1.61833		100	A	0.414356	-1.61833
		B	0.413934	-1.61791			B	0.414568	-1.61854
		C	0.414356	-1.61833			C	0.414356	-1.61833
		D	0.414356	-1.61833			D	0.414356	-1.61833

Conventional Method based on the time series data[1,2]

e	M	λ_1	λ_2	e	M	λ_1	λ_2
0.01	10	0.414947	-1.583322	0.1	10	0.408866	-1.594060
	100	0.414421	-1.580030		100	0.414007	-1.622700

Table2: Comparison of the estimation time (N=10,000).

	e	M	Type	Time	
Numerical Computation	*	*	*	1.00	(Normalized)
Proposed Method	0.1 or 0.01	10	A	2.50	
			B	2.69	
			C	2.41	
			D	2.36	
		100	A	6.90	
			B	9.66	
			C	6.21	
			D	6.72	
Conventional Method based on the time series data	0.1	10		17.24	
		100		139.7	
	0.01	10		315.5	
		100		2500.	

estimation of the private communication system that includes the round-off function. As mentioned above, the derivative of the round-off function becomes zero, so the numerical computation of the Lyapunov spectrum cannot be obtained.

Figure 4 shows the estimation results of the Lyapunov spectrum using the proposed method. In this figure, the parameter *b* and γ that have high coefficient sensitivity[4] are variously set. In addition, Type A and B of the arrangement type are used for the intentionally point generation. Furthermore, Figure 4(c) shows the numerical result in case of the derivative of the round-off function is assumed to one. As the figure shows, each estimation result (a), (b) and (c) are mutually corresponding, so it is possible to think the derivative of the round-off function to be one.

5. Conclusions

In this study, we have proposed a Lyapunov spectrum estimation method for the system that the Jacobian cannot be identified though the dynamics is clear, and have shown that the computation time is extremely short compared with the conventional method based on the time series data. In the proposed method, the points near the orbit of the target system are intentionally generated and used for the Lyapunov spectrum estimation. However, intentional generation of the points does not influence the estimation result even if the generated points are not put on the orbit of the system.

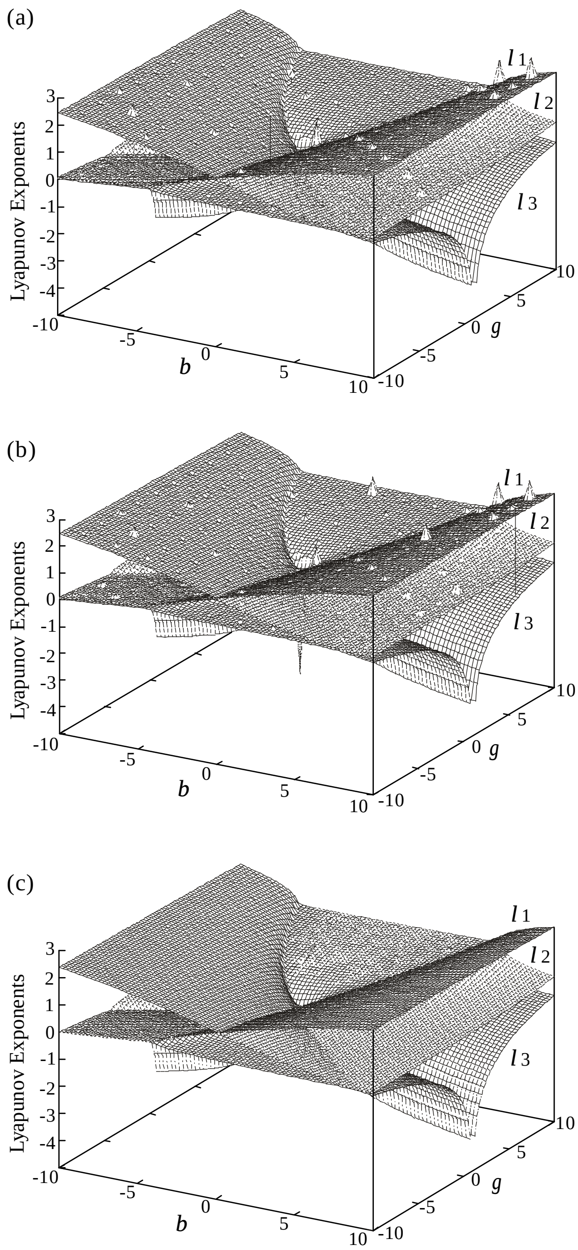

Figure 4: Lyapunov spectrum estimation of the private communication system Eq.(1)~(5). (a), (b) Estimation result using the proposed method. (c) Estimation example of the numerical Lyapunov spectrum in case of the derivative of the round-off function is assumed to be one. The arrangement type of generated points: (a) Type A in Fig. 3, (b) Type B in Fig. 3. Parameters of the private communication system: *q* = 0.138672, *a* = 1.0, *k* = 1.0, *s* = 10.0, *d* = 0.000977, *s*(*n*)=0.

Condition of the Lyapunov spectrum estimation: Number of data *N*=10,000, radius *e* =0.5, number of points near the orbit *M*=216.

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