High-Speed Periodicity Detection of Pseudo Chaos using Multirate Digital Processing and its Applications.

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Abstract - A pseudo chaotic signal generated by the finite bit-length computation has a much long period, so to detect the periodicity is required a lot of computation time. In this study, we propose a high-speed periodicity detection method for the pseudo chaos using multirate digital signal processing, and investigate the relation between the periodicity and the chaotic property based on the Logistic map.

Keywords: Pseudo chaos, Periodicity detection, Multirate digital signal processing, Logistic map.

1. Introduction

Originally, there must not be periodicity in the chaos signal. However, when the chaotic dynamical system is realized by finite bit-length computation, the periodicity arises in the generated signal. The chaotic signal with periodicity is called pseudo chaos; it is a problem which cannot avoid when the chaotic system is achieved[1].

In our study, the private communication system with chaos using fixed-point computation has been examined[2]-[4]. In this system, the generated signal becomes pseudo chaos. As an influence by pseudo chaos, we think that the coefficient sensitivity of parameter mismatching between transmitter and receiver sides is decreased, so the possibility of the information leakage to the third party rises. Therefore, to detect the periodicity of the pseudo chaos signal and to find the systems that can generate the signals similar to the true chaos are important problems.

Besides, the autocorrelation analysis is usually used to the periodicity detection of signal. The analysis method is a typical one of needing a lot of operation time and memories in computers. In addition, the periodicity of the pseudo chaos is extremely long, so the periodicity detection of pseudo chaos is not always easy if the personal computer is used for the analysis.

In this study, we propose a high-speed periodicity detection method using multirate digital signal processing[5], and try to investigate the relation between the parameters included in the Logistic map and the periodicity of the generated signal.

2. Multirate Digital Signal Processing

Multirate digital signal processing is a technique that can transform the sampling rate of signals. The technique of which the sampling rate of the signal is decreased is called decimation. In this study, the sampling rate of the pseudo chaotic signal that has a long periodicity is first extremely

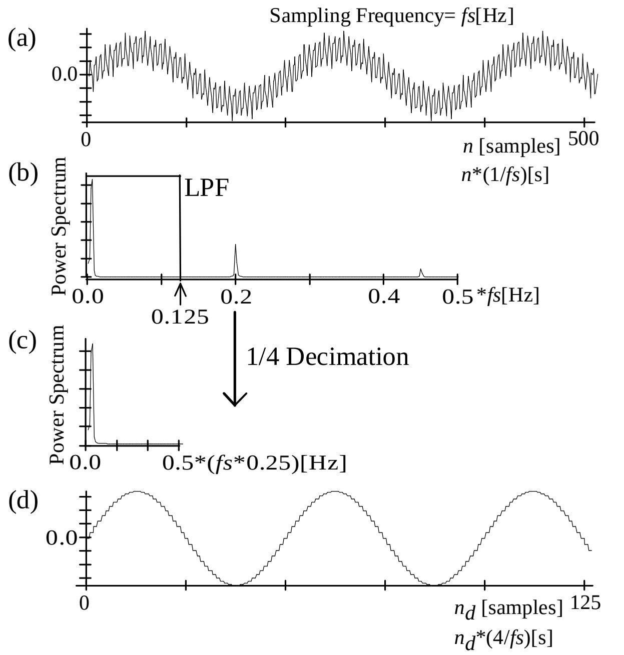

Figure 1: Example of 1/4 decimation. (a) Signal : *fs*[Hz], (b) Power spectrum of (a), (c) Band limited spectrum by LPF, (d) Decimated signal (*fs*/4[Hz])

reduced using the decimation technique. By this process, the characteristic of the pseudo chaos can be embedded to the short-time series data. Namely, if the reduction rate by the decimation is shown M, the number of decimated data becomes $1/M$ compared to the original signal.

$$x_d(n)=LPF(x(n*M)) : n=0,1,2,,, \quad (1)$$

where $x(n)$: n=0,1,2,,, are the pseudo chaotic signals, $x_d(n)$ is the decimated signal and $LPF()$ is the anti-aliasing filter.

When the decimation rate M is set to the value smaller than the length of the period P of the original pseudo chaos, the peak of autocorrelation that shows the periodicity must be able to detect based on the decimated time series data analysis.

Besides, when the decimation is carried out, the anti-aliasing filter $LPF()$ is required to the original chaotic signal. Namely, if the sampling rate of the original signal is fs[Hz], the cutoff frequency of the anti-aliasing filter becomes $fs/(2M)$[Hz]. This filter is needed to perform to the original long time series data[5].

Then, we propose a high-speed anti-aliasing filter that is suitable to the decimation method using Discrete Fourier Transformation (DFT), and realize an efficient decimation processing.

First, a pseudo chaotic signals $x(n)$: n=0,1,,,N-1 are transformed by DFT.

$$X(k)=\sum_{n=0}^{N-1} x(n)\exp\left(-j\frac{2p\,nk}{N}\right) : k=0,1,,N-1 \quad (2)$$

To realize the LPF, only $X(0)$, $X(1)$,,$X(L)$,$X(N\text{-}L)$,,$X(N\text{-}1)$ are used to the IDFT. Namely,

$$x_{LPF}(n)=\frac{1}{N}\left\{X(0)+\sum_{k=1}^{L}\left(X(k)+X(N-k)\right)\exp\left(j\frac{2p\,nk}{N}\right)\right\}$$

$$n=0,1,2,,,N-1. \quad (3)$$

In addition, $x_{LPF}(n)$ is decimated to

$$x_d(n)=x_{LPF}(n*M) : n=0,1,,,N/M\text{-}1. \quad (4)$$

Besides, the parameter $L\in Z$ must be decided to agree to the cutoff frequency of the anti-aliasing filter. The cutoff frequency is shown as $fs/(2M)$[Hz], so the parameter L becomes

$$\frac{fsL}{N}=\frac{fs}{2M}$$

$$L=\frac{N}{2M}. \quad (5)$$

In this study, the parameter L is set to 1 to reduce the amount of computation, so the data length N is changed following the decimation rate M.

$$L=1,\ N=2M \quad (6)$$

As the result, two decimated signals $x_d(0)=x_{LPF}(0)$ and $x_d(1)=x_{LPF}(M)$ are obtained by Eq. (4) in this process.

By the equations (2), (3), (4) and (6), $x_d(0)$ and $x_d(1)$ are shown as

$$x_d(0)=\frac{1}{M}\sum_{n=0}^{2M-1}x(n)+\frac{1}{M}\sum_{n=0}^{2M-1}x(n)\cos\left(\frac{2p\,n}{M}\right), \quad (7a)$$

$$x_d(1)=-\frac{1}{M}\sum_{n=0}^{2M-1}x(n)\cos\left(\frac{2p\,n}{M}\right). \quad (7b)$$

According to the process, the decimated data $x_d(n)$: n=0,1,2,, P-1 are obtained from a lot of data set $x(n)$.

$$\{x(2mM), x(2mM+1),,,x((2m+2)M-1)\}$$

$$\rightarrow x_d(2m), x_d(2m+1) : m=0,1,,,P/2-1 \quad (8)$$

The autocorrelation $R(n)$ for detecting the periodicity of $x_d(n)$ is calculated by

$$Pw(k)=\left|FFT[x_d(n)]\right|^2 : k=0,1,,,P-1 \quad (9a)$$

$$R(n)=IFFT\left[Pw(k)\right] : n=0,1,,,P-1 \quad (9b)$$

using Fast Fourier Transformation (FFT[ ]) and Inverse Fast Fourier Transformation (IFFT[ ]).

In addition, when the $R(n)$ is calculated in Eq. (9b), the power spectrum $Pw(0)$ and $Pw(P/2)$ are set to zero because these elements do not have the influence in the periodicity of the signal $x_d(n)$[6].

However, the length of period is obtained as a multiple of the decimation rate M in this method. For detecting the periodicity in precision, the autocorrelation analysis based on the original signal $x(n)$ is applied only near the rough period obtained from the decimated signal $x_d(n)$.

3. Experiments of the periodicity detection.

Figure2 shows the experimental example of the periodicity detection using the proposed method. The target signal is generated by the Logistic map.

$$x(n+1)=ax(n)(1-x(n)) \quad (10)$$

In this experiment, (1) initial 10^8[samples] of the generated signals are omitted to avoid the transition of the chaotic system, (b) the decimation rate M is set to 9,000, the order P of FFT and IFFT is 2,048(=2^{11}) and (d) the precise autocorre-

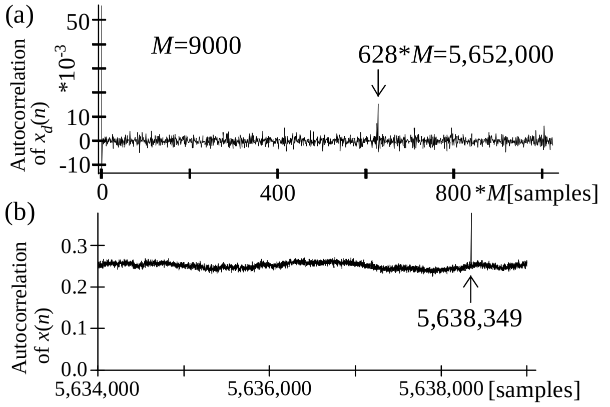

Figure 2: Periodicity detection of the Logistic map using the proposed method. (a) Autocorrelation function of the decimated signal $x_d(n)$. The decimation rate M is set to 9000. (b) Autocorrelation function of the original signal $x(n)$ by Eq. (11). Parameters: a=4.0, $x(0)$=0.1. 64-bit floating point computation is adopted.

lation analysis by (11) is carried out based on the rough period detected by the proposed method. Assuming the rough period is Pr, the precise analysis area by (11) is set to $[Pr\text{-}2M, Pr\text{+}2M]$.

$$R(k) = \sum_{n=0}^{L-1} x(n)x(n+k) \tag{11}$$

where L is used 1,000 in this study.

Figure 2(a) shows the autocorrelation of the decimated signal $x_d(n)$. As the figure shows, the peak based on the periodicity is estimated. By the estimated rough period (=628*M), the precise period is detected in Figure 2(b) by (11) in short computation time.

Furthermore, amount of computation is shown in Table. 1.

Table1: Number of times of the computation.

Autocorrelation computation by Eq.(11) only	Addition	Multiplication
	$(L\text{-}1)*Prd$	$L*Prd$
The proposed method	Addition	Multiplication
Anti-aliasing filter (Eq.(7))	$P/2*(2M\text{-}1)*2$	$P/2*2M$
Autocorrelation by FFT (Eq.(9))	$P*q*2+P$	$P/2*q*2+P$

Example(L=1,000, Prd=5,638,349, M=9,000, P=2,048(q=11))

	Addition	Multiplication
Autocorrelation only	5,632,710,651	5,638,349,000
The proposed method	36,909,056	18,456,576

where, L(in Eq.(11)) is the length of data on the autocorrelation computation, Prd is the length of period, M is the decimation rate and $P(=2^q)$ is the order of FFT.

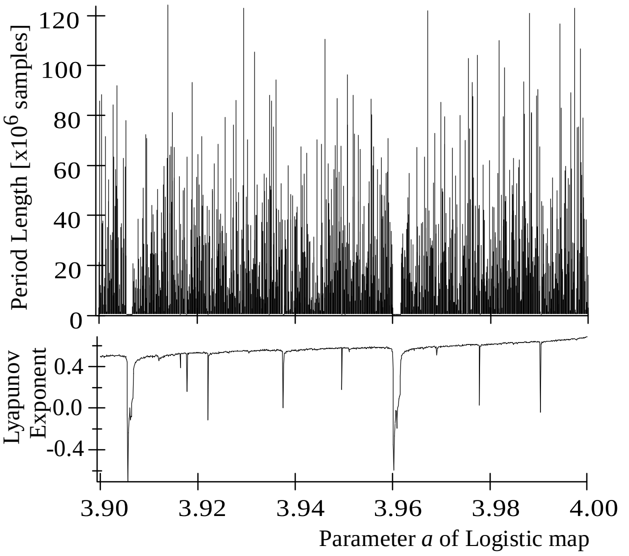

Figure 3: Detected period and Lyapunov exponent of the Logistic map(Initial value $x(0)$=0.1). Computation accuracy=64-bit floating point computation. The initial 10^8[samples] of data is omitted to avoid the transition.

As the table shows, the proposed method reduces amount of computation by 1/200 compared with the case that only the autocorrelation by (11) is used.

4. Periodicity of the Logistic map

As the results of Figure2 shows, when the parameter a is set to 4 that the Lyapunov exponent of Logistic map shows the maximum value, it is clear the the period length is not always long. Then, the period length when various values are set to the parameter a of Logistic map is estimated using the proposed method.

Figure 3 shows the detected period of Logistic map. In this figure, Lyapunov exponent on each parameter setting is also shown. Furthermore, the period length and Lyapunov exponent are analyzed at 0.0001 intervals between from 3.9 to 4.0. As the figure shows, (1) when the parameter that the system does not have the chaotic property is used, the periodicity becomes extremely short, (2) there is no relation between the length of the period and the size of value of Lyapunov exponent. The condition of the parameter because of the extension of the period length is not able to be clarified this time.

Besides, 64-bit floating point computation is adopted in the experiments shown in Figure 3, the periodicity of Logistic map often becomes more than 10 million samples. However, in case of that 32-bit floating point computation is used to the signal generation by Logistic map, we have confirmed that the periodicity becomes 100~1000 samples. In this case, because of the period length is smaller than the decimation rate, the proposed method is not needed.

On the other hand, the length of the transition of the Logistic map is assumed to be 10^8[samples] based on the detected period length in this experiments. We will examine the detection of accurate transition length with high speed operation at the next opportunity.

5. Conclusions

In this paper, the high-speed periodicity detection method using multirate digital signal processing has been proposed. The proposed method reduces amount of computation by 1/200 compared with the case that only the autocorrelation analysis of the time series data is used. We think that the property is suitable to the periodicity detection of pseudo chaotic signal which has much long period.

Furthermore, the periodicity of Logistic map in various parameter setting has been estimated by the proposed method in this study. As the results, we have confirmed that there is no causal relationship between the Lyapunov exponent of the chaotic map and the periodicity of the generated pseudo chaotic signal.

Besides, the relation of the parameter setting and the period length was not able to be clarified by the research at this time. It seems that the factor that the period length is extended is in the form of the chaotic map, the parameters, and the computation precision. To clarify the relation, the periodicity of the pseudo chaos which is generated by Henon map and other map[4] is detected by the proposed method, and examined in the future.

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