An Improved Resonant Parametric Perturbation for Controlling and Anti-Controlling Chaos in DC/DC Converters

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Abstract

This paper presents an improved resonant parametric perturbation method for controlling chaos. The control target can be any periodic orbit. Anti-control of chaos or chaotization is also possible using this method. Application of the method is illustrated for a common current-programmed dc/dc converter which can easily become chaotic for a wide parameter range.

Keywords— Chaos control, resonant parametric perturbation, dc/dc converter.

1. Introduction

Chaos is known to exist in many physical and engineering systems, be it a beneficial or harmful phenomenon. When chaos is found undesirable for a particular system, as would be the case for most engineering applications, controlling it becomes necessary to avoid harmful consequences [1]. However, there are also applications where chaos has been found to be beneficial [2]. For these applications, chaos has to be purposely generated. The generation of chaos in an otherwise non-chaotic system has been known in the literature as *anti-control of chaos* or *chaotization* [3].

Various methods for the control of chaos have been proposed [4]–[6]. Our attention in this paper is focused on the non-feedback type of control for controlling chaos as well as anti-controlling it. We consider the resonant parametric perturbation method [7]–[10] and derive an improved strategy based on modulation of a nonlinear map. It is shown that this technique is highly suitable for controlling chaos in periodically driven systems. In this paper, we consider a common current-programmed dc/dc boost converter, and our objective is to stabilize it to any periodic state (period-1, period-2, period-3, etc.). Also, chaotization is demonstrated for the same current-programmed dc/dc converter.

2. Review of Resonant Parametric Perturbation

The usual procedure is to choose a parameter that strongly affects the system and can be easily varied. Suppose this parameter is c. This parameter is then perturbed with the function $(1+\alpha\sin 2\pi f t)$, where $\alpha \ll 1$ and f is the perturbation frequency to be chosen. Effectively, we are replacing c by $c(1+\alpha\sin 2\pi f t)$ such that

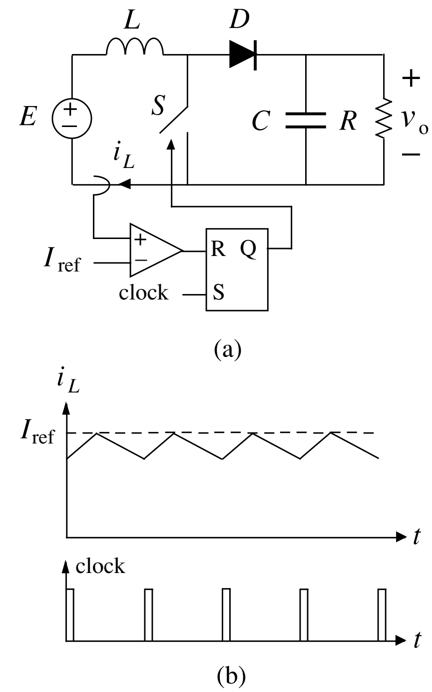

Fig. 1: Current-programmed boost converter. (a) Schematic diagram; (b) inductor waveform.

the largest Lyapunov exponent is reduced to below zero. This approach has been used by Lima and Pettini for stabilizing a chaotic Duffing-Holmes system [7]. In particular, it has been shown that when the perturbation frequency f resonates with the periodic driving frequency, say f_s, the largest Lyapunov exponent will approach zero from positive, and eventually chaos subsides and the periodic state emerges as the Lyapunov exponent falls further below 0. This phenomenon has been demonstrated analytically with Melnikov's method, and verified by simulations and experiments [7]–[9].

3. Application to Current-Programmed DC/DC Converters

3.1. Overview of Circuit Operation

The current-programmed boost converter is considered here [11, 12]. It consists of an inductor, a switch, a diode, a capacitor and a re-

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sistor load, as shown in Fig. 1 (a). When the switch is turned on, the inductor current rises up linearly, and when the switch is turned off, the inductor current ramps down and discharges through the diode to the load. To determine the turn-off instant, the inductor current is compared with a reference current, $I_{\rm ref}$. When the inductor current reaches $I_{\rm ref}$, the switch is turned off. Moreover, turn-on is done periodically. Fig. 1 (b) shows some typical waveforms. The state equation can be written as

$$\dot{x} = \begin{cases} A_{\rm on}x + B_{\rm on}E & \text{switch on} \\ A_{\rm off}x + B_{\rm off}E & \text{switch off} \end{cases} \tag{1}$$

where x denotes the state variable, i.e., $x = [i_L, v_o]^T$, the A's and B's are the system matrices given by

$$A_{\rm on} = \begin{bmatrix} 0 & 0 \\ 0 & -1/RC \end{bmatrix}, \quad B_{\rm on} = \begin{bmatrix} 1/L \\ 0 \end{bmatrix}, \tag{2}$$

$$A_{\rm off} = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix}, \quad B_{\rm off} = \begin{bmatrix} 1/L \\ 0 \end{bmatrix}. \tag{3}$$

To complete the model, we need to include the control equation, which can be expressed as $i_L(dT) - I_{\rm ref} = 0$, where d is the duty ratio defined as the fractional period in which the switch is turned on, and T is the switching period. "Exact" cycle-by-cycle simulation can thus be performed using the above equations.

The afore-described boost converter has been shown previously to exhibit period-doubling bifurcation, border-collision bifurcation and chaos [11],[12].

3.2. Control of Chaos by Resonant Parametric Perturbation

Consider the current-programmed dc/dc converter, with L = 1 mH, C = 12 μF, R = 20 Ω, T = 1/10000 s, E = 10 V and $I_{\rm ref}$ = 4 A. For this set of parameters, the circuit operates chaotically with a largest Lyapunov exponent of 3782.91. Our objective now is to stabilize this chaotic converter to some periodic orbit, e.g., period-1, period-2, etc. The application of the resonant parametric perturbation method involves, first of all, selecting an appropriate parameter to be perturbed. Here we choose the reference peak inductor current $I_{\rm ref}$ as the perturbing parameter. Essentially we replace $I_{\rm ref}$ by $I_{\rm ref}(1+\alpha \sin 2\pi f t)$, where α is the perturbation amplitude, f is the perturbation frequency, and thus the term $\alpha \sin 2\pi f t$ is the resonant perturbation applied to $I_{\rm ref}$.

The perturbation frequency f is set to the driving frequency and its subharmonics, i.e., 10 kHz, 5 kHz, 2.5 kHz, 1.25 kHz, etc. We observe the following from the simulation results (details omitted to save space):

1. The converter stabilizes to period-1, period-2 and period-4 operation when the perturbation frequency f is set, respectively, at 10 kHz, 5 kHz and 2.5 kHz. In these cases, the perturbation amplitude α has been varied in the range 0.0025 to 0.25.
2. The converter cannot be stabilized to a period-8 (1.25 kHz) orbit when the perturbation frequency f is set at 1.25 kHz. No effective perturbation amplitude can be found in this case.

Thus, we have demonstrated the control of chaos in this converter using the resonant parametric perturbation method. However, the method fails to stabilize orbits of long periods.

Period	μ used	Measured Lyapunov exponent
1	0.50	−3763.2
2	1.00	−5172.7
3	1.75	−967.75
4	1.30	−4531.7
8	1.37	−167.53
16	1.395	−521.4135
32	1.40	−137.9463

Table 1: Choice of μ for stabilization of periodic orbits

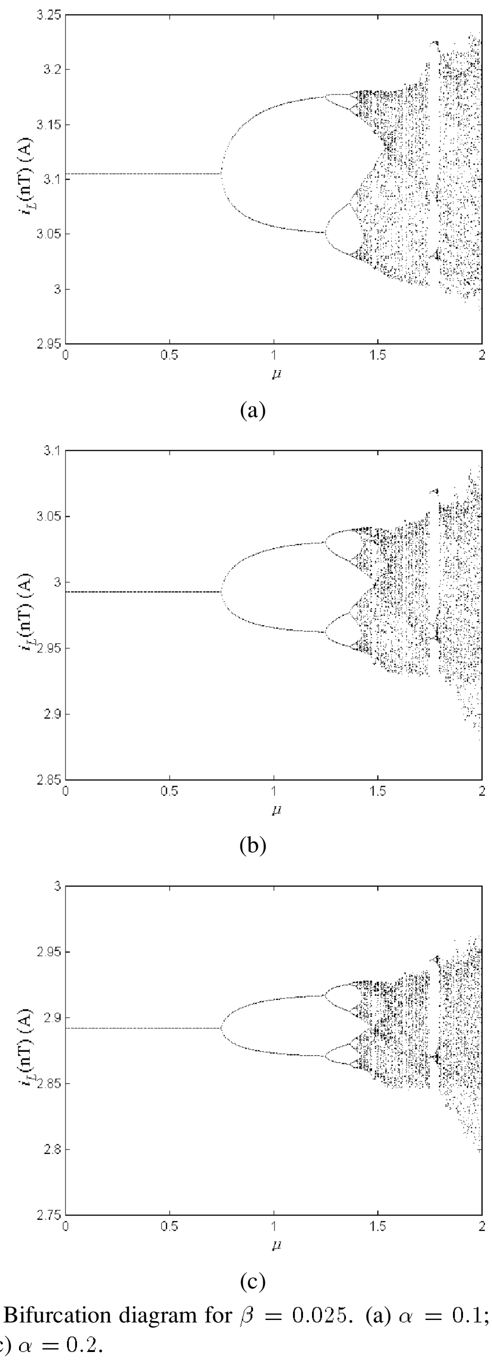
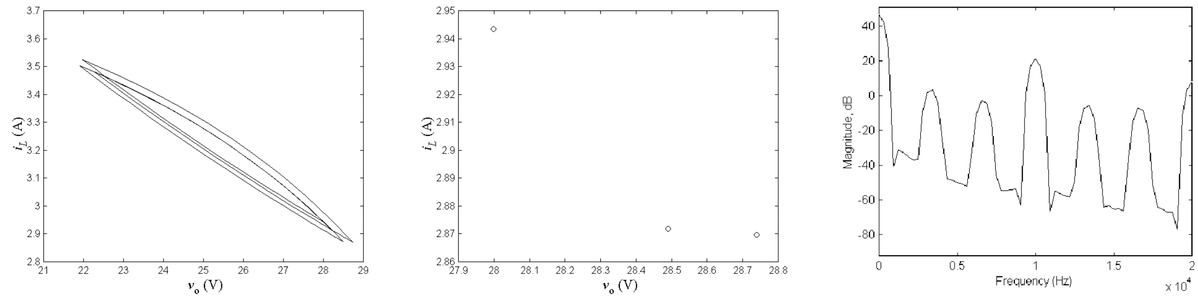
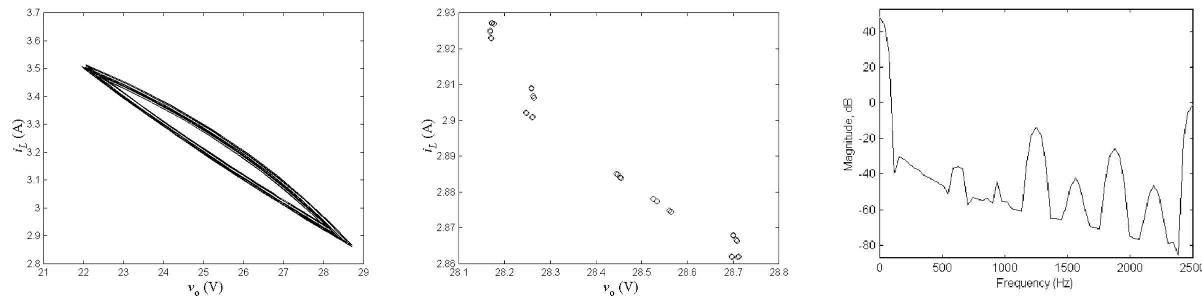


Fig. 2: Bifurcation diagram for $\beta = 0.025$. (a) $\alpha = 0.1$; (b) $\alpha = 0.15$; (c) $\alpha = 0.2$.



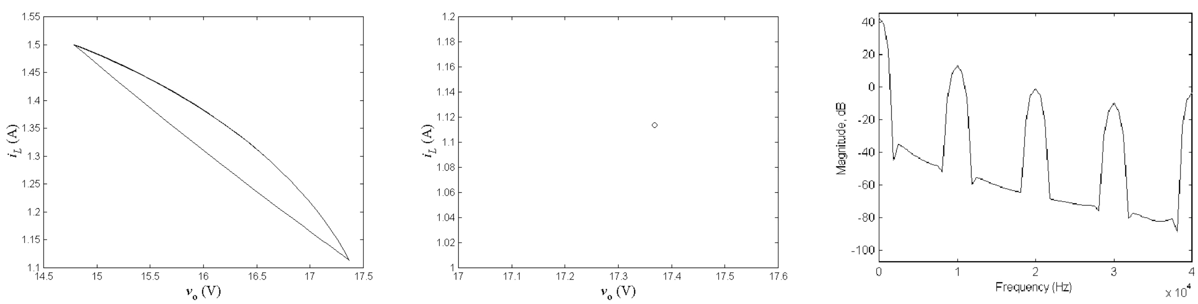
(a) (b) (c)

Fig. 3: Stabilized period-3 orbit, with $\mu = 1.75$. (a) Phase portrait; (b) Poincaré section; (c) power spectrum.

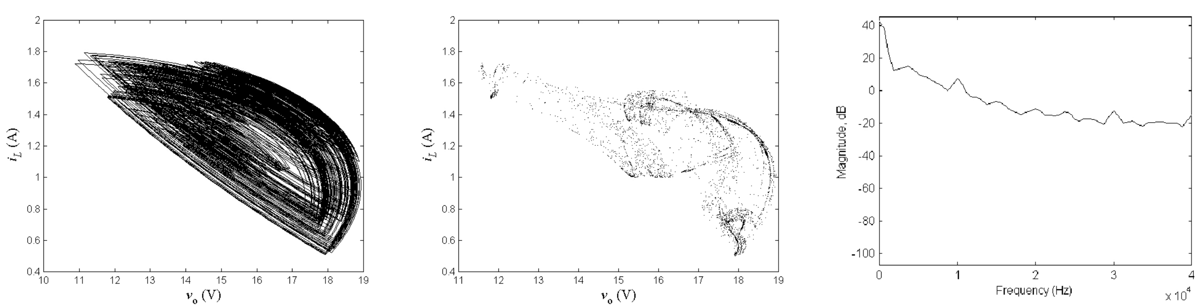


(a) (b) (c)

Fig. 4: Stabilized period-32 orbit, with $\mu = 1.40$. (a) Phase portrait; (b) Poincaré section; (c) power spectrum.



(a) (b) (c)



(d) (e) (f)

Fig. 5: (a) Phase portrait, (b) Poincar section, and (c) power spectrum of stable period-1 operation. (d) Phase portrait, (e) Poincaré section, and (f) power spectrum of the “chaotized” system ($\mu = 1.9$).

4. An Improved Resonant Parametric Perturbation Based on Modulation of Nonlinear Maps

The basic idea of the proposed technique is to inject stronger periodicity to the parametric perturbation. Suppose the parametric perturbation is applied to c. Essentially the perturbed parameter, $\tilde{c}$, is

$$\tilde{c} = c\,(1 + \alpha \sin 2\pi f t + \beta L(t)) \tag{4}$$

where f is set at the driving frequency of the converter, $L(t)$ is the output from a nonlinear iterative map, α and β are the perturbation amplitudes. A convenient choice for $L(t)$ is the logistic map because it can generate sequences of any period. Here, we define $L(t)$ as

$$L(t) = x_n - \bar{x} \quad \text{for } \frac{n}{f} \le t < \frac{n+1}{f} \tag{5}$$

where x_n is generated by the recursive formula

$$x_n = 1 - \mu x_{n-1}^2 \tag{6}$$

for $n = 1, 2, 3, \cdots$, and $\bar{x}$ is the average value of the sequence $\{x_n\}$ in the steady state. Thus, $L(t)$ always has a zero average. Now, by choosing μ, we can inject the desired periodicity to the parametric perturbation. For example, $\mu = 0.5$ corresponds to period-1, $\mu = 1$ corresponds to period-2, etc. Thus, by choosing α, β and μ, we expect to stabilize the system to an orbit whose period is determined by the chosen $L(t)$.

5. Application of Improved Resonant Parametric Perturbation

We consider again the current-programmed boost converter which operates chaotically for some given ranges of parameters. Here we make an attempt to stabilize the system to a periodic orbit of any desired period using the improved resonant parametric perturbation method described earlier. The parameter to be perturbed is again $I_{\rm ref}$, and the perturbed parameter is as given in (4), i.e.,

$$\tilde{I}_{\rm ref} = I_{\rm ref}\,(1 + \alpha \sin 2\pi f t + \beta L(t)) \tag{7}$$

The choice of $L(t)$ is as given in (5) and (6), i.e., the logistic map with zero mean. In the following we demonstrate the stabilization of periodic orbits by computer simulations.

The system is set to operate in a chaotic regime, with L = 1 mH, C = 12 μF, R = 20 Ω, T = 1/10000 s, E = 10 V and $I_{\rm ref}$ = 4 A. We then introduce the logistic map modulation by assigning a non-zero value for β, say 0.025. The effect of this additional perturbation can be illustrated by plotting the bifurcation diagrams, as shown in Fig. 2 for the cases where α = 0.1, 0.15 and 0.2. Clearly, in all these cases, we can stabilize the system to a periodic orbit of any period by choosing an appropriate value for μ. For instance, choosing $\mu = 1.75$ and 1.40 should give a period-3 and period-32 orbit, respectively, as illustrated in Figs. 3 and 4. Table 1 summarizes the values of μ used in each case and the measured largest Lyapunov exponents.

6. "Chaotizing" Current-Programmed Boost Converters

It should be apparent that the afore-described method of resonant parametric perturbation with nonlinear map modulation can also be used to "chaotize" a periodic system. Basically, if we choose μ = 1.9, for instance, a periodic system can be driven to operate chaotically. We consider the same current-programmed boost converter. This time, however, we set the parameters such that the system operates in a stable period-1 orbit. An appropriate choice would be $I_{\rm ref} = 1.5$ A, with all other circuit parameters remained the same as before. Figs. 5 (a), (b) and (c) show the stable period-1 orbit, a Poincaré section and the power spectrum corresponding to this set of parameters. To "chaotize" this system, we apply the parametric perturbation as given in (7), with $\mu = 1.9$, $\alpha = 0.1$ and $\beta = 0.2$. The corresponding phase portrait, Poincaré section and the power spectrum are shown in Figs. 5 (d), (e) and (f), which verify the successful chaotization of the system. In this case, the computed largest Lyapunov exponent is 6646.1.

7. Conclusion

In this paper a resonant parametric perturbation method, based on modulation of a nonlinear map, has been proposed for suppressing chaos. The method is applied to a current-programmed dc/dc boost converter. Moreover, there are applications where chaos may be desirable for power converters [13]. In this paper it has also been shown that the same parametric perturbation method can be used for chaotization of periodic systems.

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