ANALYSIS ON THE COLLAPSE OF COMPLEX SYSTEM BASED ON THE BRITTLE CHARACTERISTIC

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Abstract: The research on open complex system has been carried out in this paper, in which brittleness as one of the basic characteristics is presented on the basis of the traits of its collapse. Based on game theory, brittleness is demonstrated by the way of the non-cooperative game relationship between the subsystems. Moreover, brittle potential function has been induced applying the distribution function of random depending on catastrophe theory after the collectivity probability density has been got. Finally, brittleness link function is defined, which can serve the further quantitative research on optimizing open complex system.

Keywords: complex system, brittleness, collapse, catastrophe, game theory

1.Introduction

Complex systems are characterized by complexity, opening, self-organization, nonlinear[1]. However, many kinds of disturbs from inside and outside the systems are induced to systems themselves. Some of them are large, certain and predictable, while the others are small, uncertain and unpredictable. Furthermore, owing to the non-cooperative game relation between the subsystems , which a small damage can results in the collapse of the whole system suddenly. Consequently, new characteristic should be defined to describe the performance of complex systems.

In this paper, brittleness is defined to serve this purpose as a basic characteristic of the complex system. The concept of brittleness and its related theory are presented to study the invalid degree of performances under the influence of different kinds of damage factors originated from inside and outside the complex system so that the excellent designed performance can be kept in operation.

The theory of catastrophe is based on topology and used to study the state of critical point under the equilibrium.

Critical points are classified by means of potential function. Furthermore, non-continuous characteristics are studied nearby the critical points. However, it is very difficult to confirm the potential function of special system. In this paper, the potential function is confirmed by applying the relationship between distributing function of random events and it.

When one of subsystem of complex system becomes collapse, it is the brittle link function between different subsystems that determines whether the other subsystems will be influenced to have the same result. Based on the brittle link function, a series of concepts are defined such as unanimous degree, opposite degree, fluctuating degree, brittleness varying ratio, which analyzes the brittle link between the subsystems.

2. Brittleness Of Complex System

As far as complex system is concerned, it has more subsystems belonging to different hiberarchy than the simple system. At the same time, the relationship between different subsystems becomes more complex. After many years study on it, it comes to the result that complex system whose basic characteristics are complexity, opening, self-organization, nonlinear, is composed of many subsystems belonging to different hiberarchy. Any two subsystems are not only related between each other but also independent. This paper aims to present that brittleness in also one of the basic characteristics of the complex system.

Because the evolution and development of system is from simple to complexity, from small scale to large scale, the control of system will become more and more difficult. As a component, a subsystem exchanges the material, energy and information with the other ones and the surrounding environment. It is open so that it cannot avoid being disturbed by some certain and uncertain factors directly. Some of them may bring fatal effect. The subsystem will be collapse accordingly. The other undisturbed subsystems will be influenced by the collapse subsystem. The influence is spread over the whole complex system. In the end, the complex system will lose some important functions. When all the functions are invalid, the complex system is collapse because of its brittleness. Brittleness is dormant before it is triggered. Once it is active, we should find some ways to control it. It is necessary to carry out enough study on it.

Brittleness can be defined by the use of probabilistic way[2] in order to measure the damage degree of systems under the condition of being disturbed. It is related to two helpful concepts: loading and state. To define brittleness, let $P(r,s)$ denote the probability of collapse of the system in a state r for loading S. Denote a pristine state of system by r_0 and a special damaged state r_d. Then the brittleness B of the system in state r_d for loading S is the ratio

$$B = B(r_d, S) = P(r_d, S) / P(r_0, S) \qquad (1)$$

The brittleness is unity if the probability of collapse is the same in the damaged and undamaged states. If variety from r_0 to r_d increases the probability of collapse by a factor B, then the brittleness is equal to B.

To some extent, brittleness is also a kind of information. Uncertainty of collapse of the system is indicated by how much brittle information that the system contains. From the definition of entropy, the brittle entropy of system can be defined like the following

$$H_b = -\frac{P(r_d, S)}{P(r_0, S)} \ln \frac{P(r_d, S)}{P(r_0, S)} \qquad (2)$$

Let H_b denote the brittle entropy when state is r_d and the load is S.

Under the different disturbs, the system comes to different states, which the probability of collapse is different. The total brittle entropy definition is

$$H = -\sum_i \frac{P(r_i, S)}{P(r_0, S)} \ln \frac{P(r_i, S)}{P(r_0, S)} \qquad (3)$$

Generally speaking, it is hoped that the system can receive negative entropy from the environment in order to keep the balance of its entropy. When some subsystems are under the attack, energy is transmitted from the outside to inside the system so that energy of parts of the system increases sharply. The subsystems with increased entropy need obtaining negative entropy from surrounding in order to reduce their value of entropy and keep their stability, which increases the entropy value of the other subsystems.

According to the game theory, the relationship between different subsystems is non-cooperative. Every subsystem after being attacked wants to maximize its payoff (i.e. negative entropy) from the environment by means of its strategy and action so that it can resume as soon as possible. It is certain that the combination of different strategies determines the whole interest of complex system. However, the complex system keeps in operation under the condition of incomplete information so that each subsystem does not know the strategies of other subsystems. It is unavoidable that some wrong actions will be made, which can incur the unreasonable allocation of negative entropy. The subsystem that has got negative entropy can survive the catastrophe for the time being. On the other hand, the other subsystems that cannot get the negative will be collapse accordingly. Because the negative entropy of the environment is limited, all the negative entropy will be exhausted. If the complex system cannot be supplied with negative entropy, disorder of the system increases until the system is collapse. For an example, the prisoner’s dilemma can show the brittleness of the system. Just because the two prisoners want to maximize their outcome, they confess to the policeman, which makes the system including the two prisoners collapse.

3. Brittle Potential Function Of System

Based on theory of information diffusion, the collapse probability of the system can be got in different states. $P(r,s)$, the probability of collapse of the system in a state r for loading S, can be computed accordingly. Furthermore, the brittleness degree can be got by formula (1).

Theory of classification is one of the important parts as far as the theory of catastrophe[3] is concerned. Its main idea is that all the formats of catastrophe can be classified according to the dimension of control space and state space. Because the space-time people live is four-dimensioned, it is important to study four-dimensioned space-time. It is proved by Thom that there are seven formats of catastrophe at most when the number of controlling variables is not more than four. They are fold, cusp, swallowtail, elliptic umbilic, hyperbolic umbilic, butterfly, parabolic umbilic.

The collectivity density $f(y)$ can be estimated by theory of information diffusion and the brittle distribution function can be got

$$P(x,c)=\int_{-\infty}^{x} f(y)dy \tag{4}$$

As for random system, distribution function $P(x,c)$ is related to brittle potential function $V(x,c)$ as follow

$$P(x,c)=Ne^{-V(x,c)/D} \tag{5}$$

Then

$$V(x,c)=-D\ln\frac{P(x,c)}{N} \tag{6}$$

Let x denote the state variables, c denote control variable, D denote dissipation coefficient, N denote fluctuating level. $V(x,c)$ can be described using polynomial according to splitting theorem. Then compared with the basic formats of catastrophe, the closest format can be found.

4. Brittle Link Function Between Subsystems

Assuming that two subsystems X,Y can be describe with the following state vector sets respectively : $X=\{x_1,x_2,\cdots x_i\cdots x_n\}, Y=\{y_1,y_2,\cdots y_j\cdots y_n\}$.If there are n states that can be observed considering two subsystems, their change under the disturbs will play important roles in the operation of the system.

If a subsystem X becomes collapse under the disturbs and there is at least one vector $y_j(1<j<n)$ in state vector sets that turns to be worse and worse, say that $y_j(1<j<n)$ has the unanimous brittleness with X, While the other vectors that have never been changed are thought of as brittle opposite to X. Those vectors that sometimes incline to be unanimous, sometimes incline to be opposite, say them fluctuant.

Define brittle weight coefficient vectors $\omega=\{\omega_1,\omega_2,\cdots\omega_j,\cdots\omega_n\}$ according to different states of the subsystem Y, describing the influencing degree on the whole subsystem when X is collapse and performance of different states of Y worsens.

Let $\omega_j=1$ when y_j is the same brittleness with X, let $\omega_j=0$ when y_j is brittle opposite to X, let $0<\omega_j<1$ when y_j is brittle variety with X. Let $a=\sum_{j=1}^{k}\omega_j(t)$ denote the unanimous brittleness degree of Y, let $b=\sum_{j=1}^{h}\omega_j(t)$ denote the opposite brittleness degree of Y, let $c=\sum_{j=1}^{n-k-h}\omega_j(t)$ denote the fluctuant brittleness degree of Y, where

$1 \le k \le n, 1 \le h \le n$,when time is t .

Brittleness is transmitted and amplified from one subsystem to another related to the brittle link function

$$F = f(a,b,c) \tag{7}$$

Let $\dot{F}$ denote the differential of F ,i.e. brittle variety ratio

$$\dot{F} = \frac{\partial F}{\partial a}\frac{da}{dt} + \frac{\partial F}{\partial b}\frac{db}{dt} + \frac{\partial F}{\partial c}\frac{dc}{dt} \tag{8}$$

When $\dot{F} > 0$, the brittle relation becomes tighter and tighter. When $\dot{F} < 0$, the systems are losing brittle relation gradually. When $\dot{F} = 0$, the brittle relation stands unchanged. If two subsystems have the tight brittleness relation, when one is collapse, another will be inevitable collapse in the end. It is hoped that brittle relation should be reduced lest the whole complex system would be collapse under the condition that one subsystem is disturbed.

5. Brittle Game Dynamics Of Complex System

Assume that there are three subsystems, let X, Y, Z denote them, described by state vectors respectively $X = \{x_1, x_2, \cdots x_i \cdots x_n\}$, $Y = \{y_1, y_2, \cdots y_j \cdots y_n\}$, $Z = \{z_1, z_2, \cdots z_j, \cdots z_n\}$, $1 \le i \le n, 1 \le j \le n$

When X comes to end at first because of some disturbance, let $x_i = 1$. Applying the discrete expressing of the dynamic system, the system changes according to the time t , i.e. $X(t) \to X(t+1)$; $Y(t) \to Y(t+1)$; $Z(t) \to Z(t+1)$. Denote E the environment, so E changes according to the time describing by $E(t) \to E(t+1)$. As in the real world, the system we consider changes autonomously, even without subsystems' actions. We call this property the natural law of the system. A decision made by a subsystem may also affect the game environment the other subsystems, and itself. Denote T the map function of the system dynamics that include three parts. g , the map the order of nature. r , the effect of disturbs. u , the effect from the decision making of subsystems. Then the game dynamics can be expressed as follow:

$$g : (Y(t), Z(t), E(t)) \to (Y(t)', Z(t)', E(t)')$$
$$r : (Y(t)', Z(t)', E(t)') \to (Y(t)'', Z(t)'', E(t)'')$$
$$u : (Y(t)'', Z(t)'', E(t)'') \to (Y(t+1), Z(t+1), E(t+1))$$
$$T = g \circ r \circ u \tag{9}$$

Once the brittleness of complex system is triggered, nearly all the subsystems need to get negative entropy from the surrounding environment. Whether the environment can give enough negative entropy will be determined by the game dynamics of the complex system.

6.CONCLUSION

Brittleness as the basic characteristic of system is presented for the first time and probabilistic way was applied to define the brittle degree. From the view of brittle entropy and game theory, the reason that brittleness is the main factor of the collapse of system is explained. Based on the theory of catastrophe, brittle potential function has been got in order to evaluate and predict the collapse of one subsystem. Brittle link function and brittle game dynamics of complex system have been also built for further quantificational and qualitative study on complex system.

References

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