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A New Approach for Description of Complexity in Economic Time Series

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1. Introduction

The financial market which is continuing a revolutionary change has been activated by improvement in the communication speed of information. Since the financial theory which was believed in Japan has been broken by the bankruptcy of financial company, a new approach for recovery is considered. For example, the fixed deposit in Japan had been in fashion as the application of asset so far, but the stock's investing public is increasing rapidly by deregulation of finance. They make the investment mainly depending on economic analyst's information, which is evaluated by statistics and the economic state of a company, etc. The vast quantity of the economic data per seconds can be got easily by development of the computer. Recently, the analysis using these data is made by scientists, who study the structure of economic time series itself, and propose mainly the price change model.

Since there are too much exogenous and endogenous factors in economic time series, the complex system can not be defined clearly now. The analysis of a complexity is made by many researchers. Especially the price change phenomena including the fluctuation on a complex system are one of the important research works.

In this paper we propose a new representation to extract characteristics of the complicated time series, which is called the ascended time series representation (ATS representation). In section 2 we illustrate our representation method using several time series. We study numerically the price change mechanism using the actual economic data in section 3. In last section we give the conclusion and conjectures.

2. The Ascended Time Series (ATS)

In this paper we introduce a new representation method, and propose the price change mechanism for economic time series.

2.1. ATS representation

We rearrange a given data in ascending order. By using this method the rearranged data shows of course monotonically increasing behavior. We investigate fractal nature in time series by using this method.

We illustrate the ascended time series representation (ATS repesentation) in this section. Let us consider a certain time series,

$$x_n : \{x_1, x_2, x_3, \cdots, x_N\}, \tag{1}$$

where x_n denotes the value at step n. Then we rearrange the time series in asecding order.

$$x_{i_n} : \{x_{i_1}, x_{i_2}, x_{i_3}, \cdots, x_{i_N}\}, \tag{2}$$
$$(x_{i_1} < x_{i_2} < x_{i_3} < \cdots < x_{i_N}),$$

where x_{i_1} is the lowest one and x_{i_N} the highest one in $\{x_n\}$. This is called "the ascended time series". We focus our attention on the order of time i_n in the ascended time series. We plot (i_n, n) plot and the return map for the pairs of time, $\{(i_{n+1}, i_n)\}$ $(n = 1, 2, 3, \cdots, N-1)$. To demonstrate ATS representation, two examples are given as follows.

2.2. Periodic Time Series

First we demonstrate ATS representation by using the periodic time series $x_n = sin((\pi/1000)n)$, $(n = 0, 1, 2, \cdots, 10000)$ as a simple example. Figure 1 (a) shows the original time series (x_n, n). In figure 1 (b) the ascended time series is ploted: (x_{i_n}, n). Figure 1 (c) shows (i_n, n) plot. Figure 1 (d) to (g) show (i_{n+1}, i_n) plot in different sample regions: (d) $n = 0 \sim 500$, (e) $n = 0 \sim 1\times10^3$, (f) $n = 0 \sim 5\times10^3$, (g) $n = 0 \sim 1\times10^4$. In figure 1 (d) (i_{n+1}, i_n) plot is shown in the monotonically increasing region ($n = 0 \sim 500$), in which the line with slope 45° appears. In figure 1 (e) ($n = 0 \sim 1\times10^3$) two lines intersects perpendicularly with each other. In figure 1 (g) we can see many parallel lines with slopes 45° and 135°.

2.3. Nonperiodic Time Series

Figures 2 (a1) and 2 (b1) show that chaotic and random time series are distinguished by ATS representation. As a time series, which exhibits chaotic behavior, we employ the logistic map,

$$x_{n+1} = 4x_n(1 - x_n), (n = 0, 1, 2, \cdots, 3000). \quad (3)$$

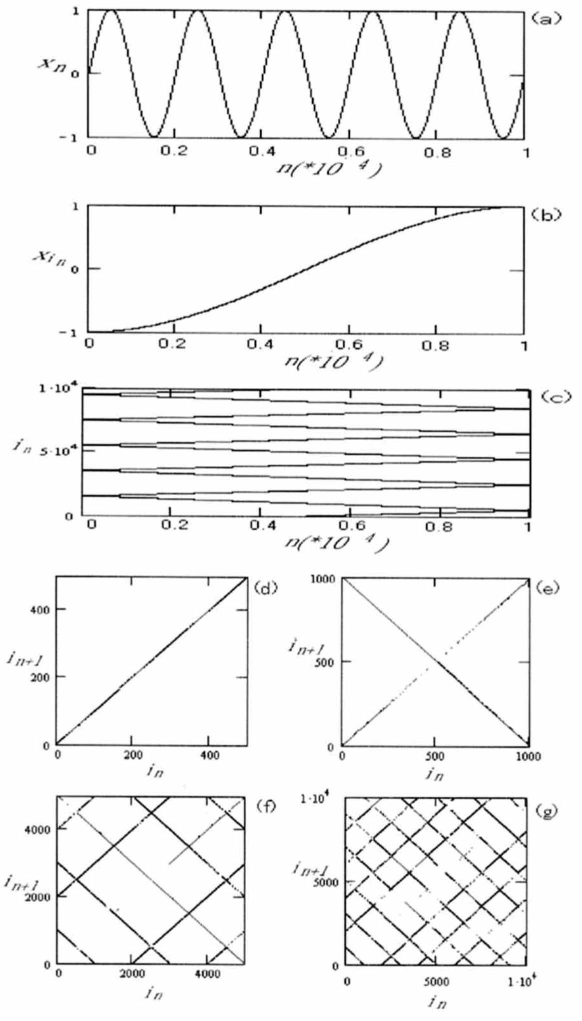


Figure 1: (a) the periodic time series, (b) the ascended time series, (c) (i_n, n) plot, and (d) to (g) (i_{n+1}, i_n) plot. ((d) $n = 0 \sim 500$, (e) $n = 0 \sim 1{\times}10^3$, (f) $n = 0 \sim 5{\times}10^3$, (g) $n = 0 \sim 1 \times 10^4$.)

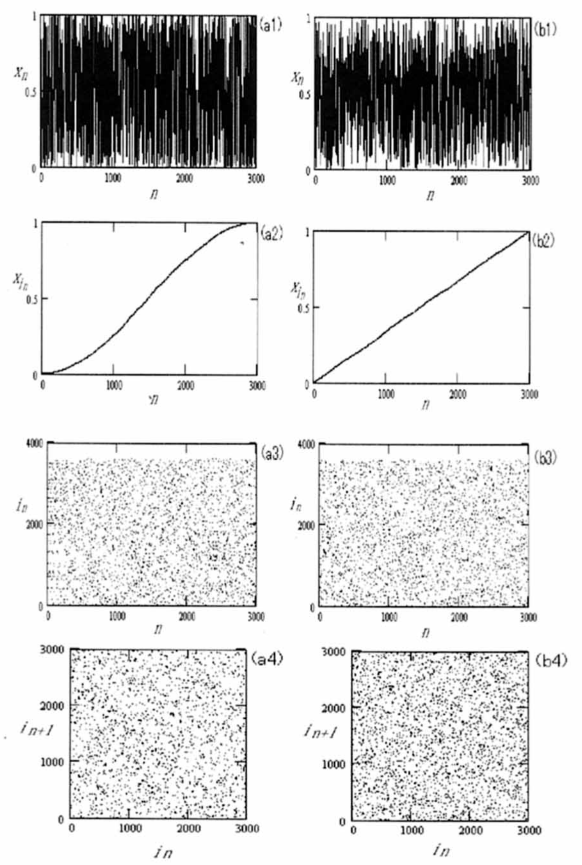


Figure 2: (a1) original time sereis, (a2) the ascended time series, (a3) (i_n, n) plot (a4) (i_{n+1}, i_n) plot for chaotic time series. (b1), (b2), (b3), and (b4) show the corresponding ones for uniform random time series.

The ascended time series are shown in figure 2(a2) (chaos) and 2 (b2) (random). Since the invariant density of the logistic map is not uniform, but it has sharp peaks at x=0 and x=1, the ascended time series exhibits a small deviation from the straight line, while the straight line appears in uniform random noise. In (i_n, n) plot and (i_{n+1}, i_n) plot it is very difficult to find any difference. In the next section we try to extract useful information from the actual market data by using ATS representation.

3. Application

The analysis of economic time series has been tried by many researchers. Especially they are very interested in a study of a stock price time series. First we employ Nikkei Stock Average data as a standard. Figure 3 (a) shows the normalized price, from -1 to 1. In this time series the 700th day's neighborhood is called Japanese bubble era. In figure 3 (b) the line has steep slope in the term of 0 to 200 steps and the term of 3100 to 3600steps, which mean that price change is large near low price and high price. The term of 500 to 3000 steps has gentle slope, which means that the change is almost constant. In figure 3 (c) (i_n, n) is shown compared figure 2 (a3) or 2 (b3) with figure 3 (c), we can understand usually that the economic time series is quite different from chaotic or random time series.

We analyze the stock price time series in local range. As a standard term we employ the term from the beginning to the end of Japanese bubble (about 820days). We decompose the whole time series into 4 terms by using this standard term: (a) $n = 0 \sim 821$(days), (b) $n = 822 \sim 1643$(days), (c) $n = 1644 \sim 2465$(days), and (d) $n = 2466 \sim 3487$(days). In figure 5 (i_n, n) plot is shown in each term.

Next we apply ATS representation to the stock price time series of the individual company. Figure 6 (a) shows the stock price time series of one Japanese electric company for the same term as that in figure 3 (a). Therefore it should be noted that the highest value appears not just at the bubble era, but after the bubble era. In figure 6 (c) to 6 (f) (i_n, n) plot is shown. In figure 6 (c) we can find that i_n takes the value of -0.043 to 0.290 between 0 and 250steps, which is quite different from the behavior of Nikkei Stock Average in figure 5 (a), in which i_n takes the value of -0.043 to 1.00. This may originate in the speed of business communication between one Japanese electric company and Nikkei Stock Average.

4. Conclusion and Conjecture

In this paper we proposed the ascended time series representation (ATS representation) to extract useful information, which is not easy to be found in the original time series.

To demonstrate characteristics of ATS representation, we have discussed ATS representation to the actual economic time series: Nikkei Stock Average and stock price time series of one Japanese electric company. We have found that there exists a large difference in the time evolution between Nikkei Stock Average figure 5 (a) and that of one Japanese electric company figure 6 (c).

This paper is the first step to analysis by using ATS representation. Developing ATS representation further,

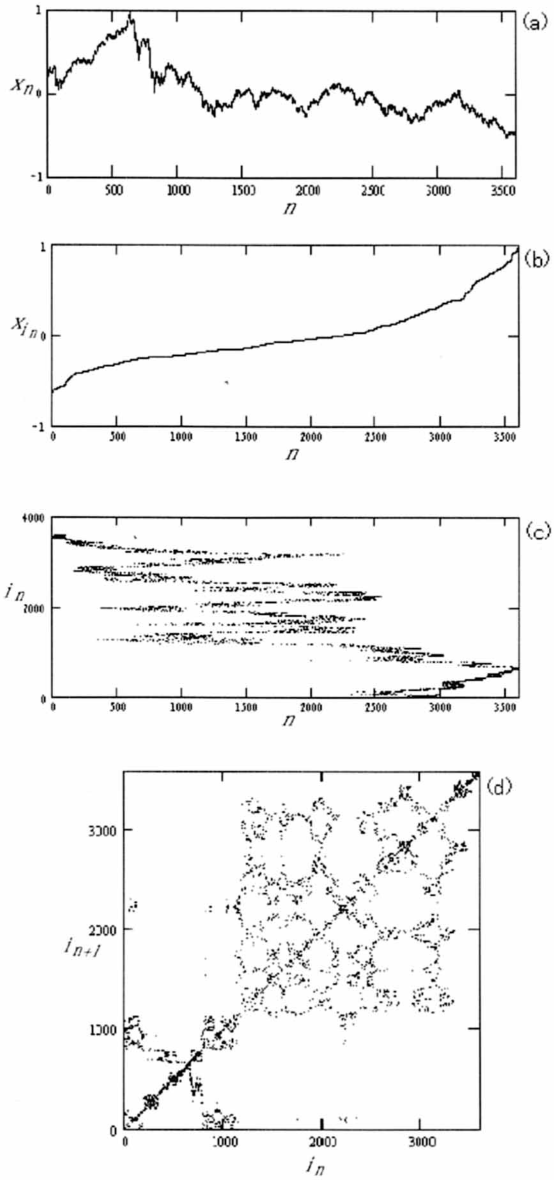


Figure 3: (a) shows price time series of Nikkei Stock Average, (b) the ascended time series, (c) (i_n, n) plot, and (d) the return map of (i_{n+1}, i_n) of (c).

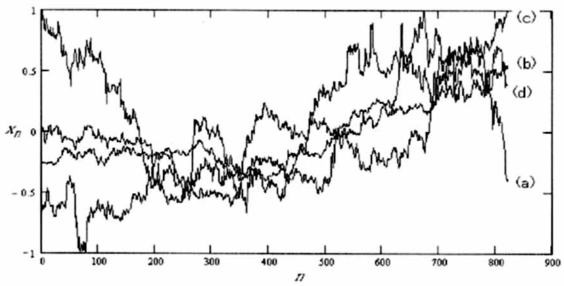


Figure 4: the decomosed Nikkei Stock Average price time series

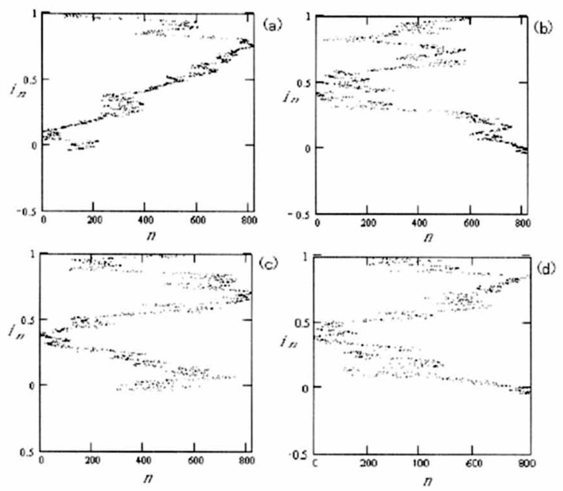
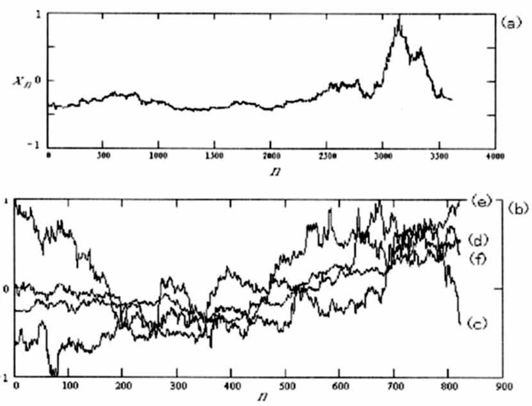


Figure 5: (a) to (d) show (i_n, n) plot of Nikkei Stock Average. ((a) $n = 0 \sim 821$(days), (b) $n = 822 \sim 1643$(days), (c) $n = 1644 \sim 2465$(days), and (d) $n = 2466 \sim 3487$(days).)



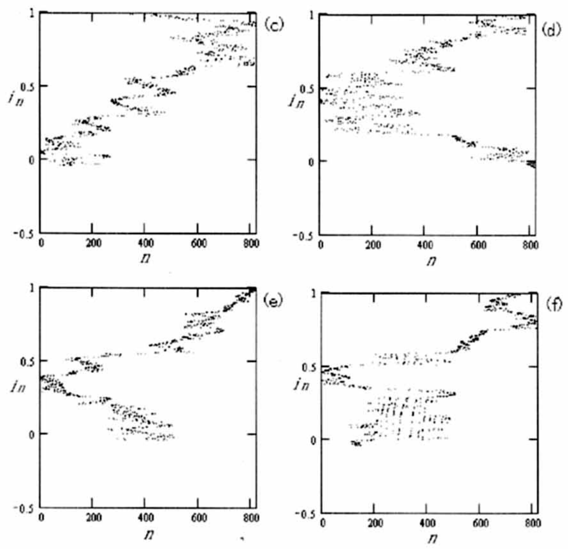


Figure 6: (a) shows the time series of one electric company, (b) the local term in the original time series. (c) to (f) show (i_n, n) plot of one Japanese electric company. ((c) $n = 0 \sim 821$(days), (d) $n = 822 \sim 1643$(days), (e) $n = 1644 \sim 2465$(days), and (f) $n = 2466 \sim 3487$(days).)

we can get more useful information from the original time series.

References

[1] R. N. Mantegna and H. E. Stanley: *An Introduction to Econophysics*, Cambridge University Press, 2000.