# Inferring Phase Synchronization from Nonsynchronized Chaotic Data

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Abstract

A novel modeling approach is presented for reconstruction of global behavior of coupled chaotic systems from bivariate time series. We analyze two coupled chaotic oscillators, which are able to phase synchronize due to coupling. It is shown that our technique enables the recovery of the synchronization diagram from only three data sets. This allows the estimate of the relative strength of the coupling and the parameter mismatch of both subsystems. We also apply this approach to experimental data of a paced plasma tube.

## 1. Introduction

Synchronization is a basic phenomenon of coupled nonlinear oscillations [1]. In the studies of coupled chaotic systems, four basic types of synchronization have been found recently: complete [2, 3], generalized [4], phase [5], and lag synchronization [6]. Among them, we focus on phase synchronization as a novel nonlinear phenomenon which has many important applications (see ref. [7] for review), including laboratory experiments, such as circuits [8], lasers [9], and plasmas [10], as well as natural systems, such as cardiorespiratory interaction [11, 12], brain activity of Parkinsonian patients [13], paddlefish electrosensitive cells [14], Canadian lynx-hare populations [15], and solar activity [16]. To analyze such experimental data, some special techniques of synchronization analysis have been proposed and it has been shown that they are very efficient even for noisy and instationary data [7, 11, 13, 17]. However, the important problem, how to reconstruct models from such synchronized data, remains open. It is of special interest to estimate the system parameters such as the coupling strength of interaction and the frequency difference between the two oscillatory systems. Such parameters often cannot be measured, in particular in natural systems. To retrieve the synchronization regime of the underlying dynamical systems, reconstruction of a parametrized model family from sets of bivariate recording data is required as a first basic step.

There are several approaches to recover such a family from time series [18, 19]. By using radial basis functions, the bifurcation diagram of a discrete-time dynamical system has been successfully reconstructed, however, under the restriction that the value of the bifurcation parameters that underlie each of the given time series are known a priori [20]. Also, a nonlinear regression technique based on maximal correlation enables, in a nonparametric way, to retrieve the time delay in experimental laser data [21].

In this study, we focus on a special technique that enables to estimate the underlying bifurcation parameters of dynamical systems [22]. By using nonlinear predictors, the algorithm reconstructs a bifurcation parameter family from only a set of time series associated with different parameter values. This approach has an important practical advantage that no prior knowledge of the parametrized family of the dynamics, in particular the underlying bifurcation parameter values, is necessary. Efficiency of this technique has been shown for prototypical model systems of chaos, such as the Hénon, the coupled delayed-logistic, and the Rössler systems [22] and even for instationary data [23].

The aim of the present study is to demonstrate this modeling approach applied to coupled chaotic oscillators using bivariate time series. The global behavior of the underlying coupled system is learned from measurements made with different parameter values of the system and the model is then used to reconstruct the synchronization diagram.

## 2. Modelling Technique

Suppose we have two coupled nonlinear oscillators whose dynamical states are simultaneously recorded into bivariate time series $\{\xi_1(t), \xi_2(t)\}$. Among several dynamical components, only a single variable $\xi_{1,2}(t)$ is observed from each oscillator. With a change in the system parameters $p$ such as the coupling strength parameter, the coupled system exhibits a variety of dynamical patterns, e.g., regimes of phase synchronization, non-phase synchronization patterns, borderlines between both, etc. From $N$ sets of bivariate time series $\{\xi_1(t,p_i),\xi_2(t,p_i)\}_{i=1,\ldots,N}$ recorded with different system parameter values $\{p_i\}_{i=1,\ldots,N}$, we want to estimate the system parameter values hidden behind the time series data. This is equivalent to a practical experimental situation, since in real experimental measurements, it often happens to have a variety of data acquired under different recording conditions. Assuming that we have no prior knowledge about the coupled systems, it is very difficult to estimate the exact parameter values from only the measured time series.

The main idea of our approach is, therefore, to estimate system parameters $\Gamma$ which are qualitatively similar to the original parameters $p$. For a one-to-one correspondence between $\Gamma$ and $p$, $\Gamma$ gives rise to a parametrized family of nonlinear dynamics $F(\Gamma,\cdot)$ which exhibits qualitatively similar bifurcation phenomena inherent to the original system.

Our algorithm is mainly composed of two steps:

(A) Embed the bivariate time series $\{\xi_1(t),\xi_2(t)\}$ into delay coordinates [24]:

$$\begin{aligned} X_1 &= \{\xi_1(t),\xi_1(t-\tau),\ldots,\xi_1(t-(d-1)\tau)\}, \quad (1)\\ X_2 &= \{\xi_2(t),\xi_2(t-\tau),\ldots,\xi_2(t-(d-1)\tau)\}. \end{aligned}$$

Construct, within the same parametrized family, two coupled nonlinear predictors $F_{1,2}$ that model the $N$ sets of time series as

$$\begin{aligned} \frac{dX_1}{dt} &= F_1(W,X_1,X_2), \quad (2)\\ \frac{dX_2}{dt} &= F_2(W,X_2,X_1). \end{aligned}$$

This means that we seek the $N$ sets of nonlinear prediction parameters $\{W(p_i)\}_{i=1,\ldots,N}$ that correspond to each time series $\{\xi_1(t,p_i),\xi_2(t,p_i)\}_{i=1,\ldots,N}$ (learning algorithm of the neural networks is described in detail in ref. [22]). Note that the dimension of the prediction parameters $W$ is larger than that of the original parameters $p$. For the nonlinear predictors $F_{1,2}$, any global functional model, such as polynomial functions [19] or radial basis functions [20] may be used. In this study, we exploit 3-layer feed forward neural network [25] having $2d$ neurons in the input layer and $d$ neurons in the output layer for $F_{1,2}$. The generalization capability of the neural network [26] works efficiently for the learning of a dynamical parametrized family.

(B) Use the singular value decomposition to extract the principal components $\Gamma$ from the nonlinear prediction parameters $\{W(p_i)\}_{i=1,\ldots,N}$. The number of the principal components $\Gamma$ is determined in such a way that the sum of the normalized eigenvalues that measure significances of the principal components is larger than 0.95. On this principal component parameter space $\Gamma$, we define a parametrized family of coupled nonlinear predictors $F_{1,2}$. By computing the mean frequency difference of the nonlinear predictors $F_{1,2}$, which depends upon the principal component parameter values $\Gamma$, a synchronization diagram qualitatively similar to the original one is recovered.

The present algorithm is based on the following mechanism. Supposed that a coupled nonlinear system $f_{1,2}(p)$ whose bifurcation parameters are fixed as $p=p^*$ is modelled by $F_{1,2}(W^*)$. When the original parameters $p^*$ are slightly perturbed as $p^*+\Delta p$, we can expect from the smoothness of nonlinear dynamics that the corresponding model is given by $F_{1,2}(W^*+\Delta W)$, which is also a slight perturbation of the model parameters $W^*$. In this way, in parameter space around $p^*$ and $W^*$, there may exist a one-to-one correspondence between the original and the model parameters. The original parameter space $p$ which is projected into the model parameter space $W$ is extracted via the singular value decomposition.

## 3. Applications

### 3.1. Coupled Rössler Oscillators

Let us apply our technique to two bidirectionally coupled Rössler oscillators:

$$\begin{aligned} \frac{dx_{1,2}}{dt} &= -\omega_{1,2}y_{1,2}-z_{1,2}+C(x_{2,1}-x_{1,2}),\\ \frac{dy_{1,2}}{dt} &= \omega_{1,2}x_{1,2}+0.15y_{1,2}, \quad (3)\\ \frac{dz_{1,2}}{dt} &= 0.2+z_{1,2}(x_{1,2}-10). \end{aligned}$$

Note that both subsystems are not identical, but have a mismatch $\Delta\omega$ in the parameters $\omega_{1,2}=1\pm\Delta\omega$. In the absence of coupling (C=0) both oscillators are chaotic. It has been recently shown that such two non-identical Rössler systems are able to phase synchronize due to a weak coupling strength $C$ [5]. To learn the global behavior of this coupled system [Eqs. (3)] in dependence on both parameters, we simulated 3 sets of bivariate time series $\{\xi_1(t),\xi_2(t)\}_{i=1,2,3}$ associated with 3 different parameter values of $(C,\Delta\omega)=(0.06,0.045)$, $(0.05,0.04)$, $(0.06,0.035)$. For the time series variables $\xi_{1,2}$, we picked the $y_{1,2}$-coordinate from each oscillator.

At the first step of the algorithm, the time lag and the embedding dimension were set as $(\tau,d)=(0.2,3)$. Next, we modeled each of the 3 records using the coupled nonlinear predictors Eqs. (2). A 2-dimensional principal component parameters $(\Gamma_1,\Gamma_2)$ were extracted from the 3 sets of nonlinear prediction parameters that correspond to the 3 records. Then we performed a linear rescaling of this space, in order to bring these parameters into close correspondence with the system's parameters $C$ and $\Delta\omega$. The reconstructed synchronization diagram shows a surprising similarity to the original diagram (see Fig. 1). The critical lines of both diagrams, i.e., the borderlines between the synchronized and non-synchronized regimes, are in good agreement.

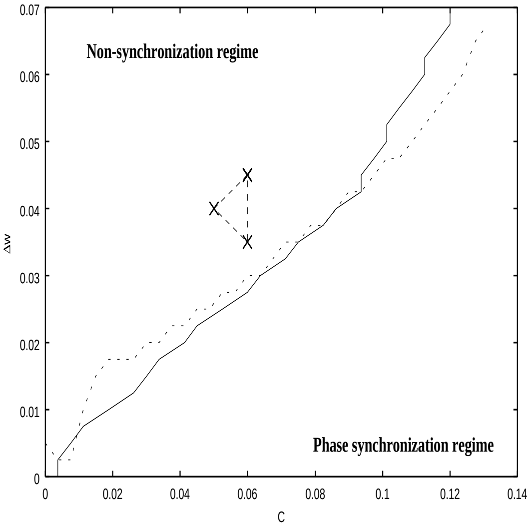


Figure 1: Borderlines between regimes of phase synchronization and non-synchronization for the coupled Rössler systems (solid line) and the and the nonlinear model (dotted line). The triangle indicates the locations of the 3 sets of parameter values used for the learning.

## 3.2. Paced Plasma Data

We now test whether our technique works for experimental data obtained from a chaotic plasma discharge tube subject to the action of a periodic wave generator [10]. This tube (Geissler tube) is filled with spectroscopically pure helium gas and the anode and cathode are connected to a high dc voltage (850 V) through a current limiting resistor $R = 30k\Omega$. In parallel with the resistor, a capacitor $C = 3.5pF$ cuts out the high voltage and a transformer adds a very low amplitude sine wave forcing (range: 0 - 0.4 V) to the system. It has been reported that such a weak forcing is sufficient to phase synchronize with the whole system in a rather large range of forcing amplitude and frequency (see Fig. 2) [10].

As an experimental measurement $\xi(t)$, the current through the plasma was used, where one record was non-paced and the other was paced with a pacing frequency of 6960 Hz and an amplitude of 0.4 V. The paced data was in a phase synchronized regime and the sampling rate was set as 200 kHz. For the modeling of this driven system, an input-output model $\frac{d}{dt}X_1 = F_1(\Omega, X_1, X_2)$ was used instead of the coupled nonlinear model (2). For the two sets of bivariate time series $\{\xi(t), \cos\Omega t\}$ with $\Omega = 2\pi * 6960$, corresponding parameter values $\Omega$ for the input-output model were computed and then the principal component parameter $\Gamma_1$ was extracted. By changing the pacing frequency $\Omega$ and the principal component parameter $\Gamma_1$, which was scaled to match the real pacing amplitude, the regimes of phase synchronization and non-phase synchronization were identified in Fig. 2. This is an Arnold tongue-like shape similar to the original synchronization diagram obtained directly from experimental data [10]. In this case the borderlines did not show the same level of agreement as before because (1) experimental results were subject to all sorts of distortion and noise; (2) one set of data used was from the synchronization regime. Nevertheless, taking into account these difficulties, it seems fair to say that the outcome of our reconstruction technique is good.

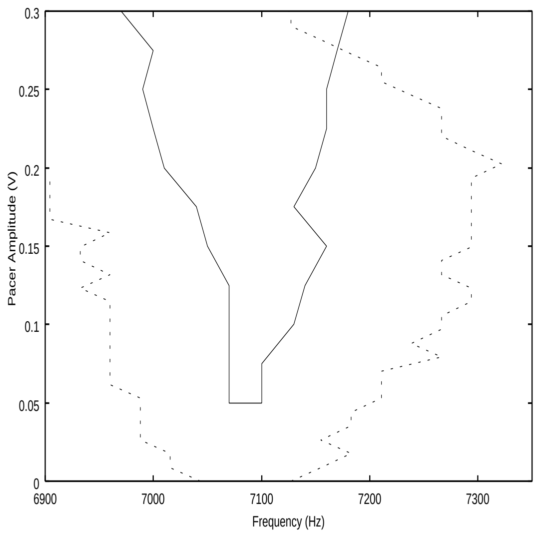


Figure 2: Borderlines between regimes of phase synchronization and non-synchronization for a paced chaotic plasma discharge (solid line) and the input-output model (dotted line). The plasma data is recorded from the real experiment of ref. [10].

## 4. Conclusions

In conclusion, the modeling approach presented in this study enables to reconstruct the synchronization diagram of two coupled chaotic oscillators and paced plasma system from only few records of bivariate time series. Our technique allows to estimate the relative strength of coupling and to evaluate how close a system is to the borderline of phase synchronization as well as non-phase synchronization, even when no further information about the underlying dynamics is available. This should be of importance especially for natural coupled system, e.g., neural systems, parasystole, circadian rhythms, insulin dynamics, vocal fold vibrations, locomotion, and ventilation versus respiration. In a future study, our technique will be extended to coupled chains and lattices of nonlinear oscillators with real applications, e.g., to laser, convection, and electrochemical data,

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