Chaos in Mutually Coupled Oscillators with Hard Nonlinear Conductance with the Increase of Nonlinearity

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Abstract— Chaos occurring in two inductively-coupled oscillators with hard nonlinear conductance, has investigated. This system has the same-phase oscillation, the reverse-phase oscillation and the double-mode oscillation for weak nonlinearity. The double-mode oscillation becomes a periodic oscillation via synchronization when the nonlinearity becomes strong. Before and after the periodic oscillation region, it is confirmed that chaotic oscillation occurs in this system.

1. Introduction

It is derived from averaging method that weakly-nonlinear two inductively-coupled oscillators with hard nonlinear conductance can possess three stable steady states for small ε; namely, the same-phase oscillation, the reverse-phase oscillation and the double mode oscillation [1]. We focus our attention to how these stable steady states change or bifurcate as nonlinearity increases. As reported previously, the same-phase oscillation becomes unstable via pitchfork bifurcation for some value of $0 < \varepsilon < 1$. Here ε presents the degree of nonlinearity. The reverse- phase oscillation survive for $0 < \varepsilon < 1$ and seems stable even for $\varepsilon > 1$. On the other hand, the double-mode oscillation presents variety of interesting phenomena as ε tends to large. They are periodic, quasi-periodic and chaotic oscillations. In this paper, we will investigate chaotic oscillation bifurcated from the double-mode oscillation occurring before and after the periodic oscillation regions for comparatively large ε.

2. Results of averaging method

The two inductively-coupled oscillators with hard nonlinearity can be described as :

$$\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = -\varepsilon(1-\beta x_1^2 + x_1^4)x_2 - x_1 + \alpha x_3 \\ \dot{x_3} = x_4 \\ \dot{x_4} = -\varepsilon(1-\beta x_3^2 + x_3^4)x_4 - x_3 + \alpha x_1 \end{cases} \tag{1}$$

where x_1 denotes the normalized output voltage of one oscillator, x_2 denotes its derivative, and where x_3 denotes the normalized output voltage of the other oscillator, and x_4 denotes its derivative. The parameter α denotes a coupling factor ($\alpha = 0$ means no coupling and $\alpha = 1$ means maximum coupling). The parameter β determines the amplitude of oscillation. For small ε, one can apply the averaging method to (1), and the following solutions are proved stable, where $\omega_1 = \sqrt{1-\alpha}$ denotes the same-phase angular frequency and $\omega_2 = \sqrt{1+\alpha}$ denotes the reverse-phase angular frequency, and θ_1 and θ_2 are arbitrary constants.

- The same-phase attractor:
  (The stable region: $\beta > 2\sqrt{2}$)

$$x_1 = x_2 = \sqrt{\beta + \sqrt{\beta^2 - 8}}\cos(\omega_1 t + \theta_1) \tag{2a}$$

- The reverse-phase attractor;
  (The stable region: $\beta > 2\sqrt{2}$)

$$x_1 = -x_2 = \sqrt{\beta + \sqrt{\beta^2 - 8}}\cos(\omega_2 t + \theta_2) \tag{2b}$$

- The double-mode attractor:
  (The stable region: $4\sqrt{5}/3 < \beta < 4$)

$$\begin{cases} x_1 = \sqrt{0.3}\sqrt{\beta + \sqrt{\beta^2 - \frac{80}{9}}}\cos(\omega_1 t + \theta_1) \\ \qquad + \sqrt{0.3}\sqrt{\beta + \sqrt{\beta^2 - \frac{80}{9}}}\cos(\omega_2 t + \theta_2) \\ x_2 = \sqrt{0.3}\sqrt{\beta + \sqrt{\beta^2 - \frac{80}{9}}}\cos(\omega_1 t + \theta_1) \\ \qquad - \sqrt{0.3}\sqrt{\beta + \sqrt{\beta^2 - \frac{80}{9}}}\cos(\omega_2 t + \theta_2) \end{cases} \tag{2c}$$

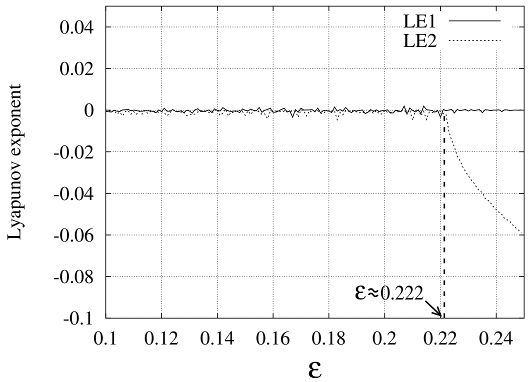
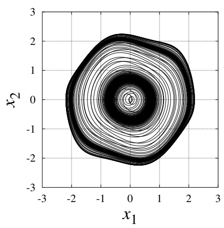
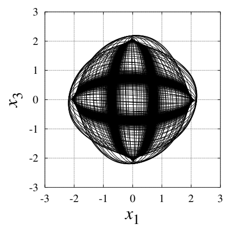


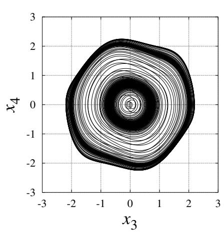
Figure 1: Lyapunov exponents for $\alpha = 0.05$ and $\beta = 3.1$.



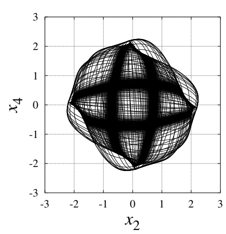
(a) x_1–x_2 plane.



(b) x_1–x_3 plane.



(c) x_3–x_4 plane.



(d) x_2–x_4 plane.

Figure 2: The intermittent almost-periodic attractor for $\alpha = 0.05$, $\beta = 3.1$ and $\varepsilon = 0.221$.

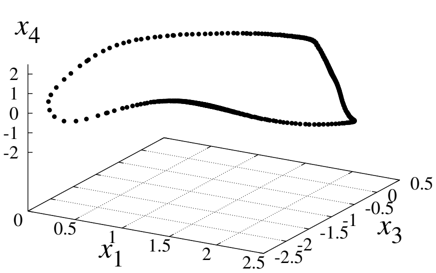
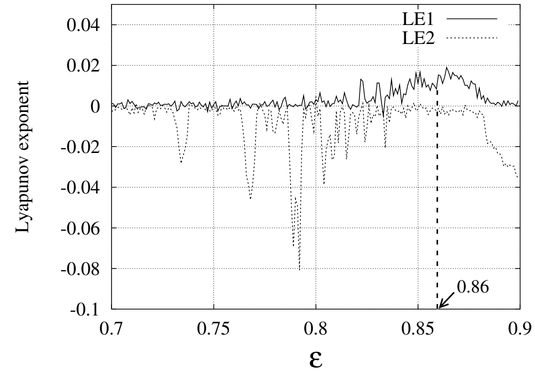
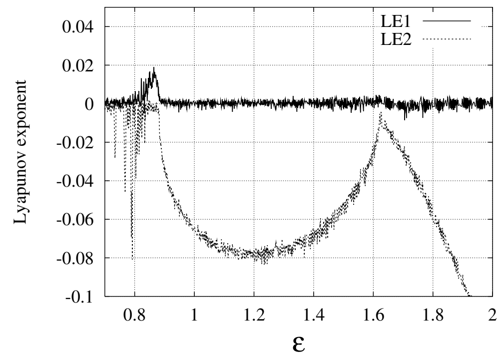


Figure 3: Poincare map for $\alpha = 0.05$, $\beta = 3.1$ and $\varepsilon = 0.221$.



(a)



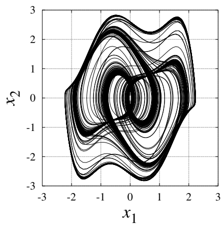
(b)

Figure 4: Lyapunov exponents for $\alpha = 0.5$ and $\beta = 3.2$.

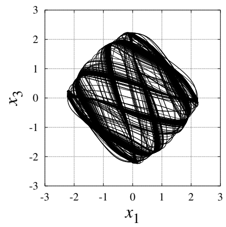
3. Chaos bifurcated from the double-mode oscillation

In general, the double-mode attractor in (2c) is a quasi-periodic solution. As ε tends to large, this quasi-periodic attractor presents various bifurcation depending on the values of α and β. Fig.1 presents the variation of the largest and second largest Lyapunov exponents in terms of ε for $\alpha = 0.05$ and $\beta = 3.1$. In this case the quasi-periodic oscillation can be observed for almost all $0 < \varepsilon < 0.222$, and at $\varepsilon \approx 0.222$ it becomes a periodic solution which exists for $0.222 < \varepsilon < 1$. The process in which the periodic solution becomes a quasi-periodic solution at $\varepsilon \approx 0.222$ when the value of ε is decreased, is a saddle-node bifurcation [2]. In the last part of the quasi-periodic oscillation for $\varepsilon = 0.221$ one can observe an intermittent quasi-periodic oscillation as seen in Fig.2, whose Poincare map($x_2 = 0$ is the Poincare section.) is shown in Fig.3. It is noticed from Fig.3 that this is certainly a quasi-periodic oscillation.

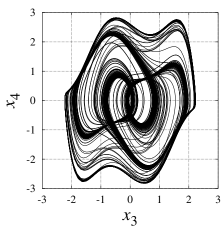
Fig.4(a)(b) shows the largest and the second largest Lyapunov exponents in terms of ε for $\alpha = 0.5$ and $\beta = 3.2$. Small periodic regions are sandwiched between quasi-periodic regions for $0.7 < \varepsilon < 0.82$, and chaos will



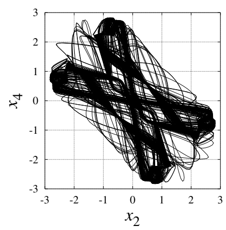
(a) x_1–x_2 plane.



(b) x_1–x_3 plane.



(c) x_3–x_4 plane.



(d) x_2–x_4 plane.

Figure 5: The chaos attractor for $\alpha = 0.5$, $\beta = 3.2$ and $\varepsilon = 0.86$.

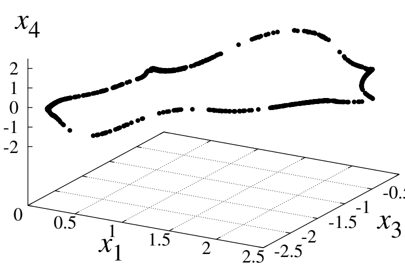
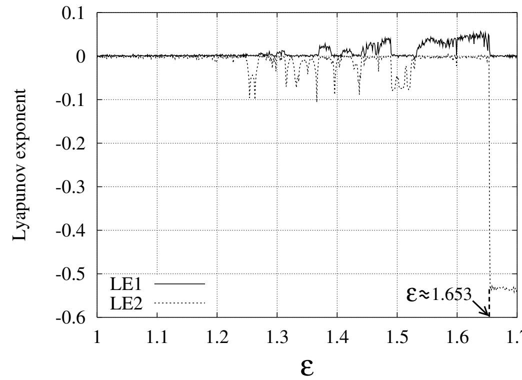
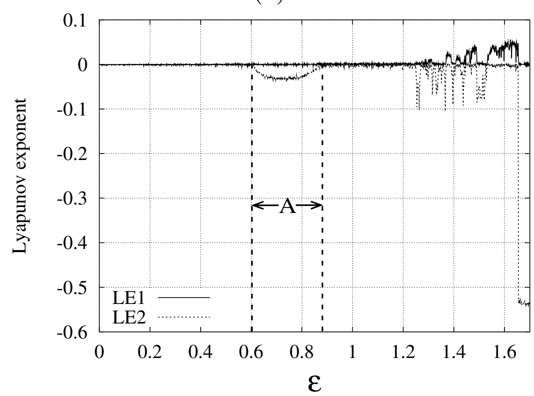


Figure 6: Poincare map for $\alpha = 0.5$, $\beta = 3.2$ and $\varepsilon = 0.86$.



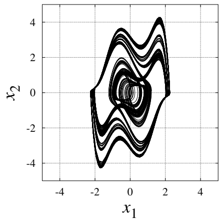
(a)



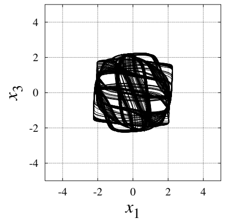
(b)

Figure 7: Lyapunov exponents for $\alpha = 0.4$ and $\beta = 3.2$.

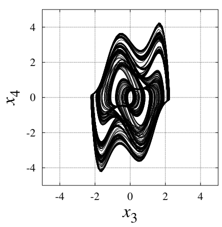
be observed in $0.82 < \varepsilon < 0.88$. For $\varepsilon > 0.88$, the 1-to-2 periodic solution will appear. The time waveform just before the 1-to-2 synchronization shows intermittency as seen in Fig.5. Fig.6 shows the Poincare map (taken at $x_2 = 0$) for Fig.5. Fig. The invariant curve has many thorns which are one of the characteristics of chaos. Moreover, the largest Lyapunov exponent is positive for $\varepsilon = 0.86$ as in Fig.4. Fig.7(a)(b) presents the largest and the second largest Lyapunov exponents in terms of ε for $\alpha = 0.4$ and $\beta = 3.2$. It also shows quasi-periodic, periodic and chaotic regions, and at last at $\varepsilon \approx 1.653$ this solution jumps to the reverse phase solution. In particular, a large chaotic region can be seen for ε just before jumping to the reverse-phase solution. Fig.8 demonstrates an example of chaos in this parameter region, and Fig.9 shows Poincare maps of various periodic and chaotic attractors. At last, Fig.10 shows the saddle-node bifurcation in region A of Fig.7(b). It is noticed that two sets of period-3 solution exist in this region, and when the saddle-node bifurcation disappears, switching between two sets of solutions will be observed as seen in Fig.2.



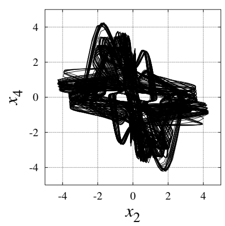
(a) x_1–x_2 plane.



(b) x_1–x_3 plane.

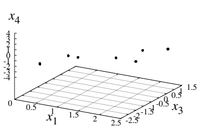


(c) x_3–x_4 plane.

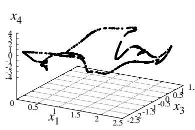


(d) x_2–x_4 plane.

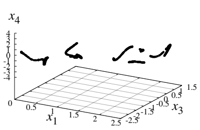
Figure 8: The chaos attractor for $\alpha = 0.4$, $\beta = 3.2$ and $\varepsilon = 1.6$.



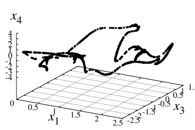
(a) $\varepsilon = 1.52$



(b) $\varepsilon = 1.56$



(a) $\varepsilon = 1.6$



(b) $\varepsilon = 1.64$

Figure 9: Poincare map for $\alpha = 0.4$, $\beta = 3.2$.

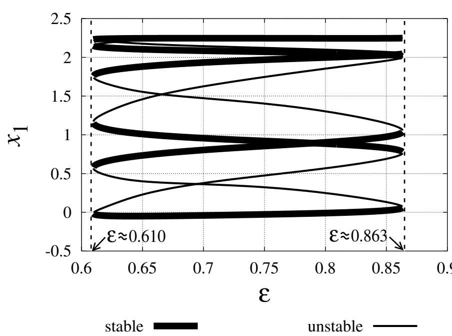


Figure 10: Saddle-node bifurcation in region A for $\alpha = 0.4$, $\beta = 3.2$

4. Conclusions

We investigate chaos bifurcated from the double-mode oscillation in two inductively-coupled oscillators with hard nonlinearity. Namely, as ε tends to large, the quasi-periodic oscillation becomes periodic oscillation or chaos which alternately appear.

References

[1] T. Endo and T. Ohta, "Multimode oscillators in a coupled oscillator system with fifth-power nonlinear characteristics," *IEEE Trans. Circuits and Systems*, Vol. CAS-27, No. 4, pp. 277–283, 1980.

[2] Y. Aruga and T. Endo, "Instability of the Same-phase Solution in a System of Two Coupled Oscillators," *Proc. of NOLTA2001*, 2001.