Analysis of 2-Port VCCS Chaotic Oscillators

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Abstract– This paper presents a 4-D chaotic oscillator using linear and hysteresis 2-port VCCSs. The oscillator has various chaotic and periodic attractors. The circuit dynamics is described by a Jordan form equation. Using the equation, various periodic attractors are demonstrated.

1. Introduction

Synthesis and analysis of chaotic oscillators are important topics. The oscillators are useful tools to investigate interesting nonlinear phenomena and the circuits configuration relates deeply to physical mechanism to generate the phenomena. There exist a number of synthesis methods of chaotic oscillators: designs based on Chua's circuit [1, 2], designs based on hysteresis elements [3, 4, 5], designs based on cellular neural networks [7] and designs based on classical or basic sinusoidal oscillators [8, 9, 10, 11, 12]. Note that rigorous analysis of chaotic attractors is possible for piecewise linear oscillators [1, 3, 5, 10].

This paper considers a 4-D chaotic oscillators using 2-port voltage-controlled current sources (ab. 2PVCCSs) and capacitors. The design procedure is based on concepts of linear/nonlinear two-port and KCL. It is convenient for simple and systematic synthesis and analysis. As a simple example, we present a 4-D chaotic oscillator consisting of one linear 2PVCCS, one hysteresis 2PVCCS and two capacitors. The hysteresis 2PVCCS has two jumps which correspond to the third and forth states apart from the two capacitor voltages. The oscillator has various chaotic and periodic attractors. In [6], we have presented various chaotic attractors and their co-existence phenomena. In this paper, various periodic attractors are demonstrated. The circuit dynamics is described by a Jordan form equation that is convenient for theoretical analysis. We should note that [5] presents the first 2PVCCS-based 4-D chaotic oscillator but discusses neither the Jordan form equation nor co-existence phenomena of two symmetric periodic attractors.

2. A 4-D chaotic oscillator

We construct a 4-D chaotic oscillator using one linear 2PVCCS, one hysteresis 2PVCCS and two capacitors. The linear 2PVCCS is characterized by

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -g_1 & -g_2 \\ g_2 & -g_1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \tag{1}$$

The hysteresis 2PVCCS is characterized by

$$\begin{cases} i'_1 = H_1(v_2) \\ i'_2 = H_2(-v_1) \end{cases} \quad H_i(v) = \begin{cases} -I_i & \text{for } v \le E_i \\ I_i & \text{for } v \ge -E_i \end{cases} \tag{2}$$

where $i = 1, 2$. As shown in Fig. 1, the output is

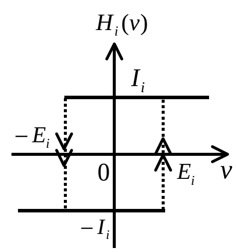

Figure 1: Hysteresis characteristics.

switched from $-I_i$ to I_i (respectively, from I_i to $-I_i$) if v reaches the right threshold E_i (respectively, the left threshold $-E_i$). Fig. 2 shows a 4-D chaotic oscillator. The oscillator is described by Equation (3).

$$\frac{d}{dt}\begin{bmatrix} Cv_1 \\ Cv_2 \end{bmatrix} = \begin{bmatrix} g_1 & g_2 \\ -g_2 & g_1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \begin{bmatrix} H_1(v_2) \\ H_2(-v_1) \end{bmatrix}. \tag{3}$$

Using the dimensionless variables and parameters:

$$\begin{gathered} \tau = \frac{g_2}{C}t, \quad E = E_1 = E_2, \; x = \frac{v_1}{E}, \quad y = \frac{v_2}{E}, \\ \delta = \frac{g_1}{g_2}, \quad p = \frac{g_2}{g_1^2 + g_2^2} \cdot \frac{I_2}{E}, \quad q = \frac{g_2}{g_1^2 + g_2^2} \cdot \frac{I_1}{E}, \\ h(x) = \frac{1}{I_2}H_2(Ex), \; h(y) = \frac{1}{I_1}H_1(Ey), \end{gathered} \tag{4}$$

Equation (3) is transformed into the Jordan from

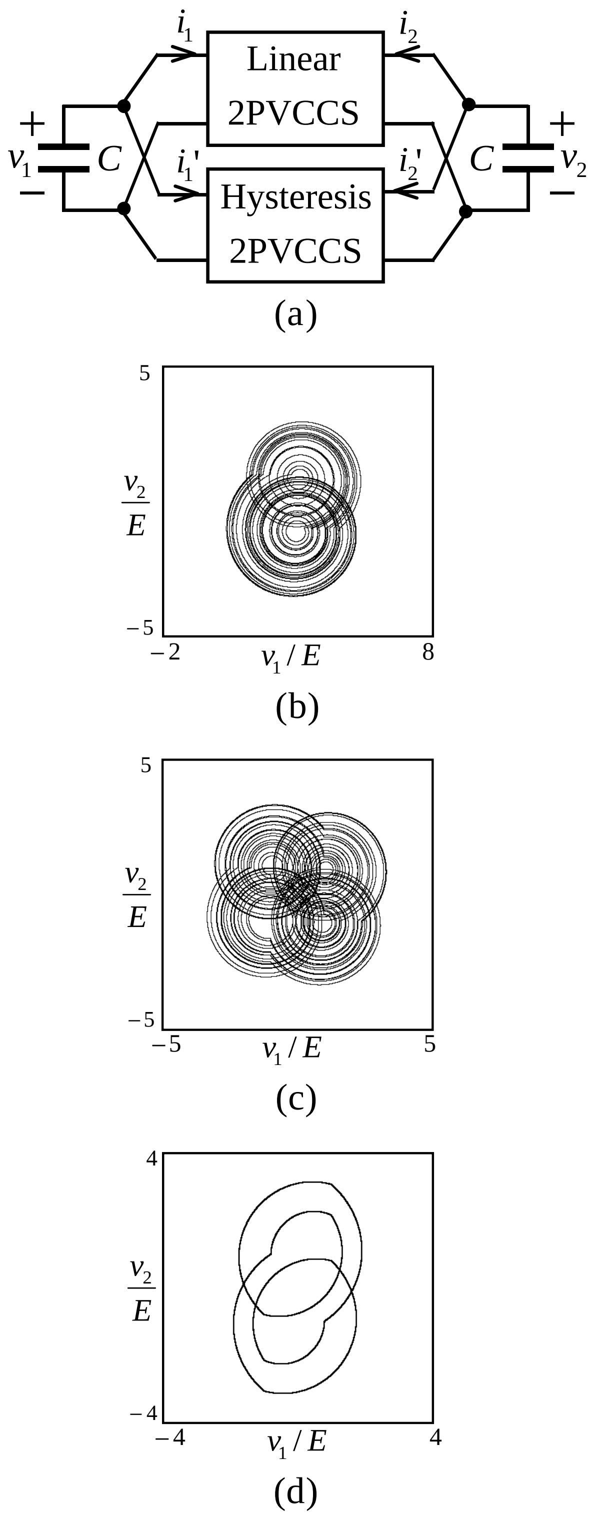

Figure 2: 4-D oscillator and attractors ($E = E_1 = E_2$, (b):double-screw chaotic attractor ($\frac{g_1}{g_2} = 0.05, \frac{g_2}{g_1^2+g_2^2} \cdot \frac{I_2}{E} = 3, \frac{g_2}{g_1^2+g_2^2} \cdot \frac{I_1}{E} = 1$), (c):quad-screw chaotic attractor ($\frac{g_1}{g_2} = 0.05, \frac{g_2}{g_1^2+g_2^2} \cdot \frac{I_2}{E} = 1, \frac{g_2}{g_1^2+g_2^2} \cdot \frac{I_1}{E} = 1$), (d):periodic attractor ($\frac{g_1}{g_2} = -0.05, \frac{g_2}{g_1^2+g_2^2} \cdot \frac{I_2}{E} = -0.6, \frac{g_2}{g_1^2+g_2^2} \cdot \frac{I_1}{E} = 1$)).

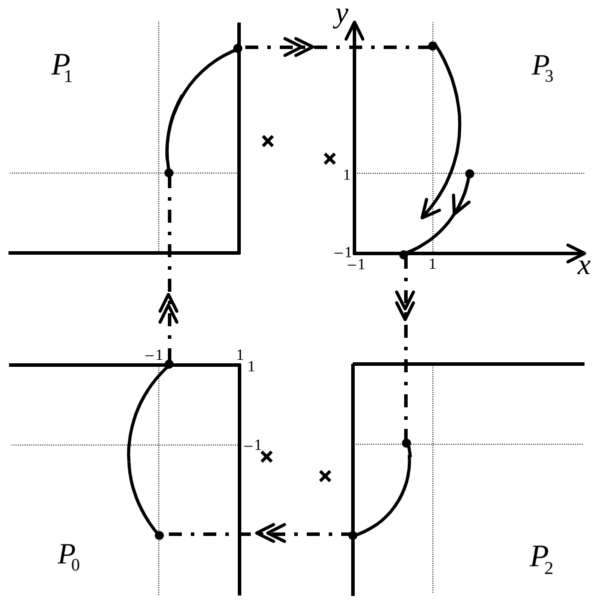

Figure 3: Trajectory on phase space ($\times$: equilibrium point).

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \delta & 1 \\ -1 & \delta \end{bmatrix} \left[\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 1 & \delta \\ -\delta & 1 \end{bmatrix} \begin{bmatrix} ph(x) \\ qh(y) \end{bmatrix} \right], \tag{5}$$

where $\frac{dx}{d\tau} \equiv \dot{x}$. This oscillator can exhibit various chaotic and periodic attractors (see Fig. 2). In [6], we have clarified chaotic attractors (Fig. 2(b), (c)) and their coexistence phenomena. In this paper, we consider periodic attractors only. In order to explain the system dynamics, we define four half-planes (see Fig. 3):

$$\begin{aligned} P_3 &\equiv \{\boldsymbol{x} | -1 \le x,\ -1 \le y,\ h(x) = 1,\ h(y) = 1\}, \\ P_2 &\equiv \{\boldsymbol{x} | -1 \le x,\ y \le 1,\ h(x) = 1,\ h(y) = -1\}, \\ P_1 &\equiv \{\boldsymbol{x} |\ x \le 1,\ -1 \le y,\ h(x) = -1,\ h(y) = 1\}, \\ P_0 &\equiv \{\boldsymbol{x} |\ x \le 1,\ y \le 1,\ h(x) = -1,\ h(y) = -1\}, \end{aligned} \tag{6}$$

where $\boldsymbol{x} \equiv (x, y, h(x), h(y))$. Equation (5) has two complex characteristic roots, $\delta \pm j$. The exact piecewise solution is given by

$$\begin{bmatrix} X \\ Y \end{bmatrix} = e^{\delta\tau} \begin{bmatrix} \cos\tau & \sin\tau \\ -\sin\tau & \cos\tau \end{bmatrix} \begin{bmatrix} X(0) \\ Y(0) \end{bmatrix},$$

where

$$\begin{aligned} \begin{bmatrix} X \\ Y \end{bmatrix} &= \begin{bmatrix} x - (p + \delta q) \\ y - (-\delta p + q) \end{bmatrix} \quad \text{for } \boldsymbol{x} \in P_3, \\ \begin{bmatrix} X \\ Y \end{bmatrix} &= \begin{bmatrix} x - (p - \delta q) \\ y + (\delta p + q) \end{bmatrix} \quad \text{for } \boldsymbol{x} \in P_2, \\ \begin{bmatrix} X \\ Y \end{bmatrix} &= \begin{bmatrix} x - (-p + \delta q) \\ y - (\delta p + q) \end{bmatrix} \quad \text{for } \boldsymbol{x} \in P_1, \\ \begin{bmatrix} X \\ Y \end{bmatrix} &= \begin{bmatrix} x + (p + \delta q) \\ y + (-\delta p + q) \end{bmatrix} \quad \text{for } \boldsymbol{x} \in P_0. \end{aligned} \tag{7}$$

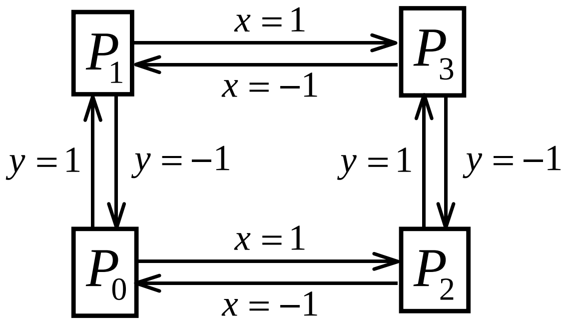

Figure 4: Switching rules.

In this paper, we focus on the following parameter range

$$\delta < 0, \quad p \cdot q < 0. \tag{8}$$

In this case, a trajectory rotates convergently around the equilibrium point on each half-plane. Let us consider a trajectory on P_3. It rotates convergently around the equilibrium point $(p+\delta q,\ -\delta p+q,\ 1,\ 1)$ as shown in Fig. 3. It hits either $x=-1$ or $y=-1$. If it hits $x=-1$ (respectively, $y=-1$), it jumps onto P_1 (respectively, P_2) holding $(x,\ y) =$ const. In a likewise manner, we can define switching rules from other half-planes, and the rules are summarized in Fig. 4.

3. Periodic attractors

In the case of Equation (8), this oscillator exhibits various periodic attractors. Also, this oscillator exhibits co-existence phenomena of periodic attractors(Fig. 5(a), (b), (g), (h), (k)). Fig. 5 shows typical attractors from this oscillator.

4. Conclusions

We have considered a 4-D chaotic oscillator consisting of 2PVCCSs and two capacitors. This oscillator exhibits various periodic attractors and co-existence phenomena of periodic attractors. Future problems include to classification of periodic attractors and analyze the bifurcation phenomena.

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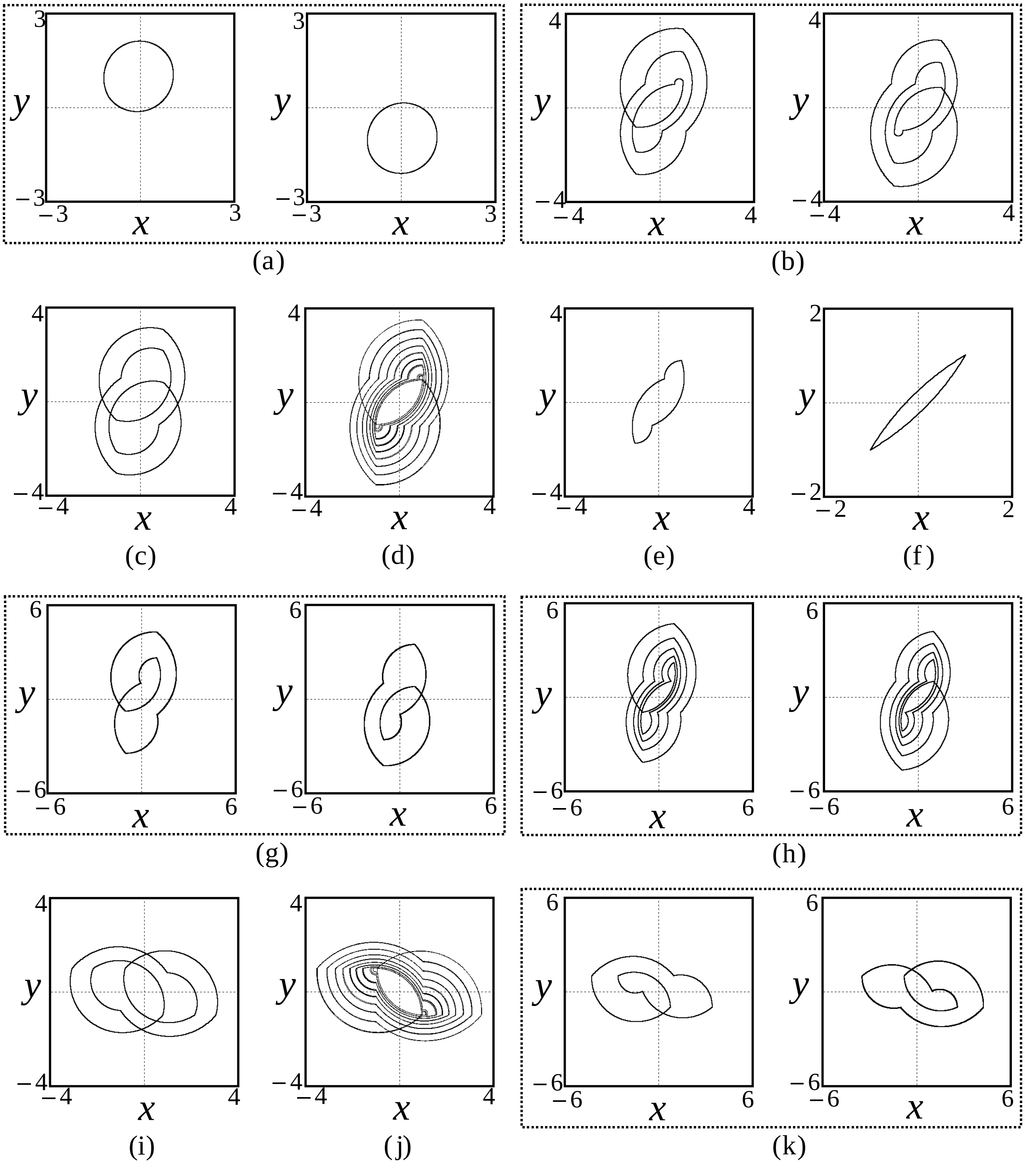

Figure 5: Typical periodic attractors ($\delta = -0.05$, (a)-(e):$q = 1$, (a):$p = -0.1$, (b):$p = -0.9$, (c):$p = -0.6$, (d):$p = -1$, (e):$p = -1.1$, (f):$p = -4, q = 4$, (g), (h):$q = 1.5$, (g):$p = -1.1$, (h):$p = -1.5$, (i):$p = 1, q = -0.6$, (j):$p = 1, q = -1$, (k):$p = 1.5, q = -1.1$.).