International Symposium on Nonlinear Theory and its Applications
Xi'an, PRC, October 7–11, 2002

Influences of the Driving Sequence to Synchronized Spatiotemporal Chaos

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Abstract

In the driving synchronization of two one-way coupled map lattices (OCML) spatiotemporally chaotic systems, the driving sequence plays a very important role. In this paper, we investigate some of its influences to the chaotic system. First, we study the OCML system driven by a periodic sequence. The resulting lattice sequences are also periodic, with the same period as the driving sequence is. secondly, we study the synchronization of two OCML systems in the case that an error occurs in the driving sequences. The result proves the robustness of the driving synchronization method.

1. Introduction

Chaos synchronization has attracted a great deal of research interests during the past decade. Practically, it has potential applications to communications, control, and system identification.

Recently, the synchronization of spatiotemporal chaos has become an active research area. Spatiotemporal chaos [1] is a kind of chaos with high dimensions. Compared with low-dimensional one, more complicated phenomena could be observed in a spatiotemporally chaotic system. So the synchronization of this high-dimensional chaos has more potentials.

In the driving synchronization of spatiotemporal chaos, the driving sequence plays a very important role. Several papers have investigated some aspects of the influences of the driving sequence to the spatiotemporal chaos. If the driving sequence is taken from an one-way couple ring map lattice (OCRML) system [2], then the spatially periodic and temporally chaotic states may be observed. In contrast, if a compound driving sequence is adopted [3], the spatial periodicity disappears, and the sequences from different lattice sites are uncorrelated.

However, the influences of driving sequence to the synchronized spatiotemporal chaos are not thoroughly studied. In this paper, we will investigate some significant influences.

The first is about the period of chaotic sequences. In theory, it should be infinite. However, because in practice the digital equipments are finite-word-length, what we actually get are sequences with a finite period, which depends on the precision of the computation. The driving sequence of spatiotemporal chaos is from another chaotic system, so it must be periodic. How does spatiotemporal chaos perform when it is driven by a periodic sequence?

The second is about the effect of an error. In a chaotic system, if an error occurs at time T, all the values after this time will be changed. In a spatiotemporal chaos system, lattice sites couple with each other. If an error occurs in the driving sequence, it will influence other lattice sequences. Chaos synchronization must be robust enough so that a minor error could not terminate the synchronization. Does the driving synchronization method satisfy this condition?

2. Spatiotemporally Chaotic System and Driving Synchronization

Spatiotemporal chaos is often modelled by one-way coupled map lattices (OCML) [4]

$$x_{n+1}^i = (1-\varepsilon)f(x_n^i) + \varepsilon f(x_n^{i-1}), \tag{1}$$

where n is the discrete time, i is the index of the lattice site, and ε is the coupling coefficient. $f(.)$ is the chaotic logistic map

$$f(x) = 1 - 2x^2. \tag{2}$$

If we limit the coupling coefficient ε in a proper interval and inject a one-dimensional chaotic sequence $\{x_n, n = 0, 1, 2, \cdots\}$ into system (1) with $x_n = x_n^0$, each lattice will perform more complex chaotic dynamics than the driving sequence $\{x_n, n = 0, 1, 2, \cdots\}$. In this way, a large number of time series can be acquired parallelly in one spatiotemporally chaotic system. They are all chaotic.

Two spatiotemporally chaotic systems could be synchronized using the driving synchronization method. To show this method, we assume that there is another OCML system:

$$y_{n+1}^i = (1-\varepsilon)f(y_n^i) + \varepsilon f(y_n^{i-1}). \tag{3}$$

It is similar to system (1) with only one difference of changing variables from x to y.

If these two chaotic systems (1) and (3) are linked to the same driving sequence (*i.e.*, $x_n^0 = y_n^0 = k_n, n = 0, 1, 2, \cdots$), and the coupled coefficient ε is large enough, then systems (1) and (3) will be synchronized [3]. In other words, the corresponding lattice sequences from systems (1) and (3) will be uniform eventually.

3. Periodicity

In theory, the period of a chaotic sequence is infinite. However, due to the finite word-length effect, what we actually get is not a infinite-period sequence but a periodic one with an indeterminate period. In a spatiotemporally chaotic system, there are many chaotic sequences. Are there any relationships in the periods of all these chaotic sequences? In this section, we will answer this question.

The driving sequence $\{k_n, n = 0, 1, 2, \cdots\}$ is from another chaotic system, so it must also be periodic. Assume its period is N. Now we study the period of the rest lattice sequences.

$$x_n^0 = k_n = k_{n+N} = x_{n+N}^0, \quad n = 0, 1, 2, \cdots$$

$$x_{n+1}^1 = (1-\varepsilon)f(x_n^1) + \varepsilon f(x_n^0)$$

$$x_{n+N+1}^1 = (1-\varepsilon)f(x_{n+N}^1) + \varepsilon f(x_{n+N}^0)$$

$$\begin{aligned} |x_{n+N+1}^1 - x_{n+1}^1| &= (1-\varepsilon)|f(x_{n+N}^1) - f(x_n^1)| \\ &= 2(1-\varepsilon)|x_{n+N}^1 + x_n^1||x_{n+N}^1 - x_n^1| \end{aligned}$$

Because

$$|x_n^1| \le 1 \quad \text{and} \quad |x_{n+N}^1| \le 1,$$

we have

$$|x_{n+N+1}^1 - x_{n+1}^1| \le 4(1-\varepsilon)|x_{n+N}^1 - x_n^1|.$$

If ε is confided to the interval $(0.75, 1)$, then the above equation can be rewritten as

$$|x_{n+N+1}^1 - x_{n+1}^1| \le \lambda |x_{n+N}^1 - x_n^1|$$

where

$$0 < \lambda = 4(1-\varepsilon) < 1$$

So we have

$$\lim_{n\to\infty} |x_{n+N+1}^1 - x_{n+1}^1| = 0. \tag{4}$$

Assume the period of the sequence at lattice 1 is N_1. According to equation (4), the following equation must hold:

$$N = lN_1, \qquad l = 1, 2, 3, \cdots \tag{5}$$

If $l > 1$, then N_1 is less than N. So we have

$$x_{n+N_1}^1 = x_n^1$$

and

$$x_{n+N_1}^0 \ne x_n^0.$$

So

$$|x_{n+N_1+1}^1 - x_{n+1}^1| = \varepsilon |f(x_{n+N_1}^0) - f(x_n^0)| \ne 0. \tag{6}$$

Equation (6) implies that N_1 is not the period of the sequence $\{x_n^1, n = 0, 1, 2, \cdots\}$. It is inconsistent with the definition of N_1. Hence l must be equal to 1, and N_1 must be equal to N.

The above procedure shows that after some time steps, the sequence at lattice 1 is with the same period as the driving sequence is. Repeating the same procedure, it can be proved that the periods of the rest lattice sequences are also equal to that of the driving sequence.

$\varepsilon > 0.75$ is a sufficient condition but not necessary. In fact, ε can be less than 0.75. Figure 1 shows the OCML system under the condition $\varepsilon = 0.5$. In the figure, because the sequence $\{x_n^0, n = 0, 1, 2, \cdots\}$ is periodic with period N, $x_n^0 - x_{n+N}^0$ is always equal to 0. Concerning other lattice sequences, because of the random initial values, they are not periodic at the beginning. However, after some time steps, they are all with the same period.

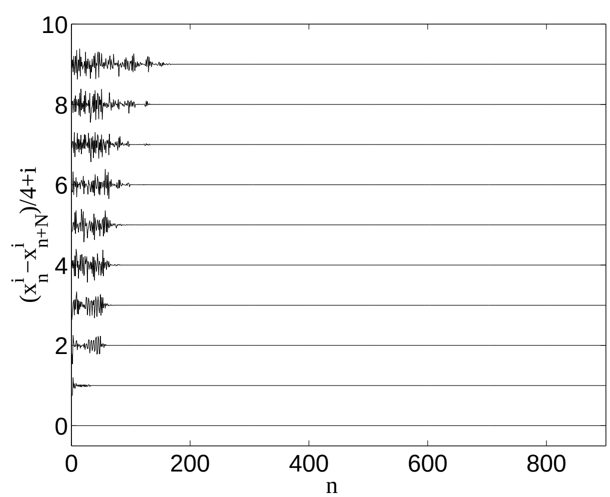

Figure 1: The periodicity of OCML sequences with the coupling coefficient $\varepsilon = 0.5$.

The period of a chaotic sequence is indeterminate. It depends on the precision of the computation. The traditional method for getting chaotic sequences with desired period is cutting and reduplicating the sequences. In an OCML system, there are a large number of chaotic sequences. Cutting and reduplicating all of them is a heavy work. Using our conclusion, we can easily get the sequences with the desired period by only controlling the period of the driving sequence.

4. Effect of An Error

If an error occurs in an OCML system and causes a difference between systems (1) and (3), will these two chaotic systems be still synchronized? This is a significant question. In the practical system, errors are almost inevitable. Furthermore, in a spatiotemporal chaos system, lattice sites couple with each other. If an error occurs, it will influence not only this lattice sequence but also other lattice sequences. If the chaotic systems are so sensitive that a minor error will make them out of synchronization, they cannot be used in practice.

In our case, due to the importance of driving sequence, we will firstly investigate the influence of an error occurring in the driving sequence to chaos synchronization. Figure 2 gives an example. In the figure, the driving sequences of systems (1) and (3) have a difference at the $100th$ value (*i.e.*, $x^0_{100} \neq y^0_{100}$). This difference causes further differences in the posterior lattice sites. Fortunately the influences are not fatal. After few time steps, the synchronization will be recovered again.

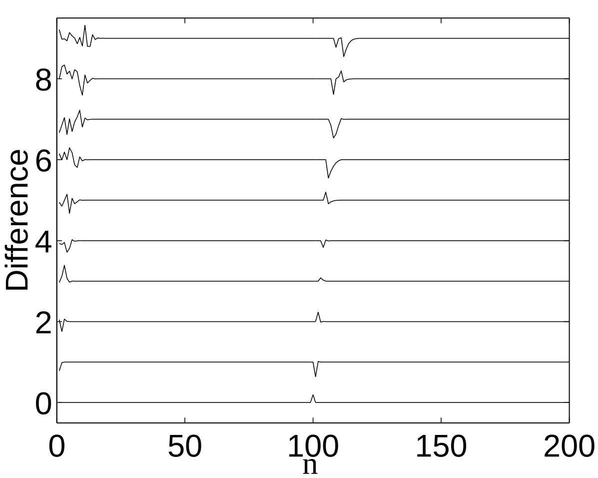

Figure 2: The influence of an error occurring in the driving sequence to the synchronization. The Y-coordinate is defined as $(y^i_n - x^i_n)/4 + i$.

This fact could be proved as same as the proof of driving synchronization (see [3]). In fact, if the difference happens at time step T, then we can consider the time T+1 as the initial time. Just similar to the initial synchronization, two OCML systems will be synchronized again after several time steps.

If an error occurs in another lattice site, it will influence the sites behind it. And the synchronization can also be recovered after some time steps.

5. Conclusions

This paper investigates two kinds of influences of driving sequence to the OCML spatiotemporal chaos. The first is about the period of chaotic sequences. Though in theory the period of these sequences is infinite, what we get in practice are periodic ones. we find that in the synchronized OCML systems, all of the lattice sequences are periodic with the period as same as that of the driving sequence. Using this conclusion, we can easily control the period of OCML sequences by only controlling the driving sequence. The second influence is about an error occurring in an OCML sequence. The investigation shows that if this occurs in the driving sequence, then it will influence the posterior lattice sequences and cause the transitory non-synchronization between two OCML systems. However, the influences are not fatal. After few time steps, the synchronization will be recovered again. Similarly, if an error occurs in another lattice sequence, then it will only influence the lattice sites behind it. And the influences are also transitory.

Acknowledgments

This work was supported by National Natural Science Foundation of China under grants No. 60172023.

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