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Entropy in computer network model

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1. Introduction

The rapid development of computer network has provided great benefits, while it exhibits lots of complex phenomena as a complex system. These phenomena may have impact upon the state of network and have attracted many scientists in the fields of both computer networks and statistical physics. The present research work is mainly concentrated on detailed rules within networks, such as routing rules and transport protocols. Even then, more and more attention has been at present paid to network behaviors as a whole. It is done not only by computer scientists but also by mathematicians and statistical physicists [1, 2, 3, 4]. These complex behaviors include phase transitions, power laws in the fluctuation of data packets, fractal behaviors, and other nonlinear phenomena. In the popular point of view, computer network can be considered as a complex nonlinear system that consists of enormous nonlinear units. The interactions between these units lead to the complexities behaviors.

As we know, the term of entropy is used widely to describe the level of uncertainty about an event. According to the second law of thermodynamics, the entropy of an isolated system will increase monotonously as time goes by. A large value of entropy indicates a high level of uncertainty, and therefore corresponds to a disordered state. In some research fields of computer science entropy has been exploited. A statistically derived entropy maximization model is used in the study of the unintrusive targeted advertising on the web[5]. Entropy is also applied to the investigations of the cellular automata with rule of "game of life". However, computer network is a complex open system, and is affected by many environmental factors. In the next section, topologies used in network models are discussed. It is also pointed out in this section that the topologies, which can be viewed as the static factors of network, is not as important as the dynamic ones such as protocols, which are in charge of transmissions of data packets. In succedent section, the queue entropy of computer network is introduced for the first time. Based upon the concept of entropy, attempts are made to study of dynamic laws in the evolution of computer network. Finally, a short discussion is presented.

2. Network topological model

2.1. network topologies

The simulation of computer network is an important means to study the dynamic behaviors in them. However, the real computer network is very complex and can expand irregularly. The popular topologies in the simulations of computer network include two-dimension lattice, Cayley tree, Transit-Stub structure, and BRITE structure. The two-dimension topology gives the simplest description of the computer network in all these models, whatever, it is still in the study of the improvement of the performance of network while additional random links are introduced [7]. The Cayley Tree and Transit-Stub structures capture some properties of the topologies of real computer networks such as irregularity. They are used in the study of the complex behavior of computer networks [1, 8]. Furthermore, the BRITE structure can fit the power laws found in the topologies of real computer networks and give more accurate characterization of the network [9].

2.2. Key factors in complex behavior of networks

We investigate computer network models based on various of topologies and examine how the topologies influences the behaviors of computer networks [2]. These topologies represent the static rules of computer networks. However, the protocols, which are in charge of transmissions of data packets, can be viewed as main dynamic rules of networks. These dynamic rules have more important responsibilities for complexities phenomena in a dynamic system. Two commonly-used protocols, namely TCP and UDP, are applied to the above four popular topologies. The simulation results confirm the conjecture that network protocols play more crucial roles in complexities behaviors of networks instead of topologies. Phase transition can be observed in the plot of the average lifetime of packet v.s. the packet creation rate in the networks with UDP rules. In the TCP networks, however, the average lifetime of packets fails to reflect the congestion of networks. Another parameter, i.e. the sending ratio, is introduced to indicate the phase transition from non-congestion to congestion phase. Distinct diversities between UDP and TCP networks emerge in both the spectrum of the packet fluctuation and the queue length's one.

3. Queue entropy in computer network

3.1. Queue entropy in computer networks

The uncertainty property extensively exists in various systems. The uncertainty level of systems reflects its evolution in most time. Entropy is usually used to characterize such system uncertainties quantitatively. Given a data series x_i, $i = 1, 2, \ldots, n$, there are three commonly used definitions of entropy, and each of them can be described mathematically as follows [6]

1. Shannon's entropy

$$S_H(x) = -\sum_{i=1}^{n} x_i \ln x_i \tag{1}$$

2. Burg's entropy

$$S_B(x) = -\sum_{i=1}^{n} \ln x_i \tag{2}$$

3. Frieden's and Zoltani's entropy

$$S_{FZ}(x) = -\sum_{i=1}^{n} [x_i \ln x_i + (x_{imax} - x_i) \ln (x_{imax} - x_i)] \tag{3}$$

where, x_{imax} represents the max value of the data series x_i, $i = 1, 2, \ldots, n$.

The data packets traffic in a computer network can be thought as the origin of the various dynamic phenomena. In fact, states of network are affected by various external factors as well as internal ones. The distribution of traffic sources and the behaviors of users are regarded as the typical external factors, while the rules of packet transmission, and the routing policy et al can be obvious internal ones. To characterize the influence of these factors, the entropy of computer network is introduced, according to the above entropy definition.

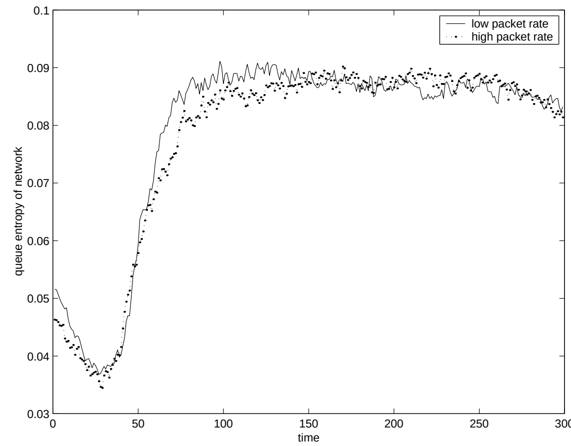

Figure 1: The queue entropy response of network

As we know, two of the important parameters to characterize the running status of network are the interval of arriving packets and the queue length of the nodes who are in charge of forwarding packets. Therefore, it is reasonable that the entropy of network is calculated based on these two parameters. In this paper, the queue entropy is used to explore the evolution of computer network model. It is derived from the time series of queue lengths of network nodes and defined in the following section.

3.2. Simulations result with queue entropy

We have built a simple network model having the two-dimension lattice topology. In this model, each network node forwards the arriving packets that the destination of them is not this node. At the same time, the data source on each network nodes generates the packet stream at predefined probability distribution. The initial status of the network is empty, that is, no packet exists at the initial point and the buffer of each forwarding node is empty. In this section, the response of the network is studied by means of the entropy introduced in last section, namely queue entropy. The queue entropy of network is defined as the average entropy of the all nodes in the network, just like:

$$S_q(x) = \frac{1}{N} \sum_{i=1}^{N} S_{qi} \tag{4}$$

where the $S_q i$ denotes the entropy of the ith node and N is the total number of the nodes in the network.

The simulations of the network models are carried out. The evolution of the queue entropy of the computer is calculated and is shown in Fig. 1. The two curves in this figure indicate the evolution process with two different

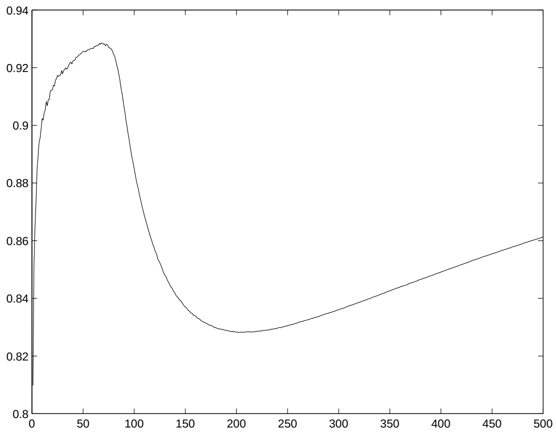

Figure 2: The queue entropy response of sudden burst of packet generation rate

packet generation rates. However, the response curves seems alike in spite of the difference of the packet generate rates. Weak oscillation are found in these process. However, the queue entropy will increase into a utmost point as the evolution is going. This implies that the network is running into a status that may be stable as a whole. In other words, network will not change visibly if the other parameters of the network remain unchanged, such as the packet generation rate, and bandwidth of links.

We also study how the changes of the sudden packet generation rate influence on the queue entropy of network. If sudden burst of packet generation rate occurs when the network is running into the stable state described above, violent variety of queue entropy can be observed. Fig. 2 shows another simulation process. This figure concentrates only on the process of the burst of packet generation rate. The x-axis indicates the time, while the y-axis represents the queue entropy. The network is assumed to be in the stable state with certain packet generation rates at the initial time. At this time point, additional burst of packets occurs, it can be seen from the figure that the queue entropy succeed to reach a peak. But it will arrive at the former value ultimately.

4. Conclusion

This paper attempts expression of entropy in computer network. Based upon the queue length of forwarding buffer, the queue entropy is introduced. The response process of computer network for the packet generation rate is explored with this entropy concept. However, these discussions are centralized on the experimental data of simulation process. And only qualitative analyses are carried out. We believe the concept of entropy will be important in the exploration of the behavior of computer network, especially with the theories on complex systems such as dissipative structure.

Acknowledgments

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