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Self-Emergence of Chaos in Identifying Irregular Periodic Behavior

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***Abstract* —** In order to exploit generalized chaotic synchronization phenomena for the solution of temporal pattern recognition problems, a chaotic dynamical system representing the class of signals that are to be recognized must be established. This system can be determined by means of identification techniques. Although the fulfillment of the chaotic condition could appear as a constraint, it is shown here that, for a very simple identification algorithm, chaos self-emerges when a model, fitting unprecise periodic signals, is identified.

1. Introduction

The recently discovered phenomenon of "qualitative resonance" of Shil'nikov-like chaotic attractors [1] and, in general, the phenomenon of synchronization of chaotic systems [2] have been proposed several times to be exploited for realizing associative memories and/or pattern recognizers [3]. In particular, as reported in [1, 3] and [4], the sharpness of the qualitative resonance phenomenon can be very easily exploited for the classification of approximately periodic patterns. Namely, approximately periodic patterns can be tested against a chaotic dynamical system that is able to reproduce the class of time-series that is to be recognized. This model has to be excited in a suitable way by an input signal such that qualitative resonance is realized, meaning that if the input signal belongs to the modeled class, the system approximately "locks" into it. If not, the trajectory of the system and the input signal remain unrelated.

To exploit qualitative resonance in concrete applications, it is necessary to build an algorithm which constructs the chaotic dynamical system representing the class of signals that are to be recognized. Such an algorithm would be a nonlinear identification procedure providing a chaotic dynamical system starting from observations of the concerned class. Constructing such a model could appear to be a very complex task in which the chaotic behavior has to be externally imposed as a constraint. This is only partially true. In fact, it is shown here that, for a very simple identification algorithm, the chaotic behavior self-emerges when a model, fitting unprecise periodic signals, is identified. Unfortunately, the self-emerging chaotic behavior is Feigenbaum-like, *i.e.* period doubling based, which is not suitable for qualitative resonance [4]. Nonetheless, on the basis of bifurcation theory [3, 5, 6], a Feigenbaum-like chaotic system can be modified, by means of continuation techniques, to satisfy Shil'nikov-like conditions. In this way, at least theoretically, a model suitable for qualitative resonance can be obtained.

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2. Identification Algorithm

The proposed nonlinear identification algorithm is very simple: the reference model used is a system of Lur'e type, *i.e.* a model where a linear dynamic and a nonlinear static part are separated and connected in a feedback loop (upper part of Fig. 1). The identification of the model parameters from the given data proceeds iteratively, adapting alternatively the linear and the nonlinear part. In this way, the difficult nonlinear dynamical system identification task is decomposed into the easier problems of linear dynamical and nonlinear static system identification (optimization). With regard to the linear identification part, since the measurements and the most common identification algorithms are in the discrete time domain [7], it follows that the most convenient modeling technique is in the discrete time domain. However, this does not correspond to the qualitative resonance phenomena which is a continuous time phenomena. This is not a real problem since a linear discrete time system has an equivalent in continuous time under the constraint that it has no eigenvalues with negative real part.

The working principle of the algorithm can be easily explained referring to Fig. 1. Given a Lur'e-like observed system described by an effective static nonlinearity $f(\cdot)$ and the transfer function $G(z)$, consider a parameterized nonlinear function $f_p(\cdot)$, where the index p highlights the dependence upon the parameter vector $p \in \mathbb{R}^m$. Applying an initial guess of the nonlinearity $f_p(\cdot)$ to the reference signal $y(t)$, the approximated input $\widehat{u}(t)$ of the linear dynamical system $G(z)$ is obtained. Accordingly, a parametric linear identification technique [7] on the pair $(\widehat{u}(t), y(t))$ can be applied obtaining an estimation $\widehat{G}(z)$ of the linear system. Besides the estimate of the transfer function $\widehat{G}(z)$, the linear identification techniques return also a measure of the quality of the identification result, the measure σ in Fig. 1. Namely, a measure saying how reliable the identified transfer function $\widehat{G}(z)$ is. Therefore, it is possible to alter the parameters describing the nonlinear function (p) optimizing the identification quality (which indeed would act as objective function in an optimization problem) with respect to the guessed nonlinearity. Then, the procedure can be iteratively repeated until, hopefully, the best possible pair $(p, \widehat{G}(z))$ is determined.

Summarizing, as shown in Fig. 1, the nonlinear identification problem is decomposed into a linear identification whose quality determines the value of a nonlinear objective function which must be optimized acting on the parameters describing the nonlinearity. Clearly, extra constraints must be added to avoid that the overall loop transfer function regresses, up to a very small delay (*i.e.* $1/z$), to the identity *input equals output*, which is obviously the best possible solution satisfying the data.

More detailed descriptions of the two main parts that compose this algorithm are reported in the next two paragraphs.

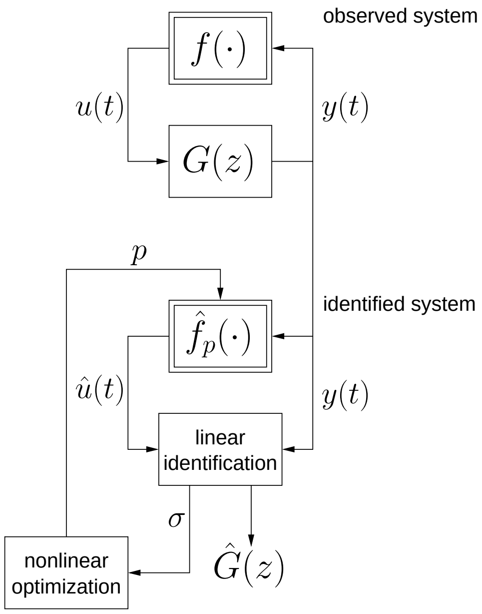

Figure 1: Operating scheme of the Lur'e model based nonlinear identification algorithm.

2.1. Linear Identification

The identification of linear dynamical systems is a common problem of control theory and a consistent literature is available on this topic. For what is relevant here, the linear identification is a subproblem; hence, the choice of a particular identification algorithm from those available in literature is subordinate to the accomplishment of the final goal, *i.e.* the Lur'e identification. Nonetheless, in a real application there would be several independent reference vectors $(y(t))$ which have to be taken into account for the identification, *e.g.* the entire database of signals representing the given class.

The easiest way in which several input–output relations can participate to the identification of a single dynamical system is to consider all of them together in a least square error problem. In fact, linear parametric identification algorithms are in general based on the solution of an overdimensioned linear system (with more equations than unknowns) which links the parameters of the linear system to the measurements. The solution of this linear system passes by the computation of the pseudo-inverse of a matrix which keeps the measurements in account. So the kind of particular pseudo-inverse chosen depends on the method of fitting (least square error, maximal error, etc.). In all the nonrecursive identification algorithms the pseudo-inverted matrix can be extended to take into account as many independent input–output relations as desired.

As simple example consider the least square error identification of an ARMA input–output relation

$$y(t) = -\sum_{i=1}^{n} a_i y(t-i) + \sum_{i=1}^{n} b_i u(t-i) \tag{1}$$

which corresponds to the transfer function

$$G(z) = \frac{\displaystyle\sum_{i=1}^{n} b_i z^{n-i}}{z^n + \displaystyle\sum_{i=1}^{n} a_i z^{n-i}}$$

Considered an ordered pair of observations $(u(k), y(k))\ k = 0, 1, \ldots, r$ with $r \gg n$. Then the complete ARMA relationship (1) can be rewritten as

$$\begin{bmatrix} y(n-1)\cdots & y(0) & u(n-1)\cdots & u(0) \\ y(n) & \ddots\ y(1) & u(n) & \ddots\ u(1) \\ y(n+1)\ddots & y(2) & u(n+1)\ddots & u(2) \\ \vdots\ \ \ddots & \vdots & \vdots\ \ \ddots & \vdots \\ y(r-1)\cdots & y(r-n) & u(r-1)\cdots & u(r-n) \end{bmatrix} \begin{bmatrix} -a_1 \\ -a_2 \\ \vdots \\ -a_n \\ b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} y(n) \\ y(n+1) \\ y(n+2) \\ \vdots \\ y(r) \end{bmatrix}$$

or

$$T\theta = Y$$

where T, θ, and Y are the corresponding Toeplitz matrix and the two vectors shown above, respectively. Hence, the least square error estimation of the parameter vector $\theta = [-a_i\ b_i]^T$ is given by

$$\theta = T^{-1\dagger} Y$$

where $T^{-1\dagger}$ is the Moore-Penrose pseudo-inverse of the Toeplitz matrix T. It should be clear that such an approach can easily take into account several pairs of independent observations $(u_i(k), y_i(k))$.

Finally, note that the critical point in the linear identification is the choice of the order [7] which, indeed, corresponds to the order of the entire identified dynamical system. Since the final aim is the equivalent of a continuous time chaotic system, at least a third order is necessary.

2.2. Nonlinear Optimization

Just as the identification, the optimization of nonlinear static functions is a common problem and is one of the main topics of *operations research* [8]. Here again, it is a subproblem subordinated to the attainment of the Lur'e model parameters and a suitable method must be chosen accordingly. Because of the possible bifurcations in the qualitative behavior of the Lur'e system upon changes of the parameter p, it follows that the objective function (quality of the linear identification) that is to be optimized is, in general, strongly wrinkled. Therefore, global searching methods have to be considered. The only optimization method considered in this work are *genetic algorithms*. Furthermore, for reasons described hereafter, the objective function has to be modified in order to account for some constraints necessary to avoid the degeneracy of the identification problem.

The critical point in the nonlinear optimization (identification of $f(\cdot)$) is the choice of the parameterization of $f(\cdot)$, *i.e.* the family $f_p(\cdot)$. For instance, to this aim, any finite generalized series expansion of functions represents a suitable parameterization of the nonlinearity, *e.g.* standard orthonormal polynomials series like *Laguerre, Legendre, Tchebyshev, etc*. An alternative to the series-based approximations of the nonlinearity are the geometrical-based ones as for instance the *cubic splines*, the *piecewise linear functions*, and the *piecewise polynomials*. Furthermore, the parameterization of the nonlinearity must compose, together with the linear transfer function, a unique parameterization of the Lur'e system. In particular, to uniquely fix the loop static gain, the slope of f_p, *i.e.* f'_p, must be assigned in a point, *e.g.* $f'_p(0) = 1$.

Finally, two extra constraints must be added to avoid that the overall loop transfer function regresses, up to a very small delay (*i.e.* $1/z$), to the identity *input equals output*, which is obviously the best but trivial possible solution satisfying the data. First, the regression of the nonlinearity to a linear function must be forbidden; namely, the combination of the parameters such that the nonlinearity $f_p(\cdot)$ regresses to a linear function $f_{\overline{p}}(x) = mx$ must be removed from the search space on which the optimization algorithm works. Next, it is necessary to forbid the degeneracy of the linear transfer function as well. This can be done, for instance, prohibiting cancellations between the poles and zeros of $\widehat{G}(z)$. These two constraints can be accounted for by altering the objective function, by means of penalty functions, such as to take into account the degree of compliance of the considered solution, *i.e.* the parameters vector p, to the constraints.

3. Tests & Discussion

The proposed identification algorithm has been tested on two sets of approximately periodic signals. The first set of signals is truly issued from a Feigenbaum-like chaotic attractor of a three dimensional system which, however, is not described by a Lur'e-like set of ODE. The second set of signals are concrete measures of ECG signals.

3.1. Test Framework

Twenty independent observations of 32 pseudo-periods (oscillations) each have been considered in every identification; the sampling of the observations is of 200 samples for each pseudo-period. The measurements have been scaled in amplitude and in time, over the whole set, such as to have the amplitude between ± 1 and to have a unitary nominal pseudo-period, subsequently the mean value has been removed. Concerning the reference model, a third order (exact matching) for the Feigenbaum-like signals and a fourth order for the ECG signals have been chosen, respectively. The nonlinearity has been modeled with a five segments smoothed piecewise linear function with the slope of the middle segment fixed at 1. This corresponds to have p 13-dimensional.

The objective function has been optimized with a genetic algorithm working for 500 evolution steps on 100 individuals of 13 chromosomes each (one for each parameter), which in turn are composed of 80 binary genes each, according to the IEEE representations of reals. The reproduction function is governed by a reproduction probability, given by the normalized fitness, and a standard Monte Carlo method, excepting the best three individuals which are surely reproduced at the next generation. The crossover operator is applied with probability $P_C = 0.7$ while the mutation operator is applied with a probability $P_M = 0.1$. The crossover works at chromosome level while the mutation works at gene level. The cut point, over the chromosomes for the crossover and the gene, over the genes for the mutation, are randomly drawn with an uniform distribution.

3.2. Rosenzweig–MacArthur Food Chain

The first time series set is generated by the 3-pulse Feigenbaum-like attractor observable, for suitable parameter values, in the Rosenzweig–MacArthur food chain model [9]. A sample of the data used for the identification is shown in Fig. 2; while in Fig. 3 a sample time series and the state space representation (reconstructor canonical form realization of the linear transfer function) of the identified model are reported.

3.3. ECG Signals

The second set of time series is composed of ECG from several healthy patients. This implicates that a single model is built for representing inter and intra patient diversity. A sample of the data used for the identification is shown in Fig. 4; while in Fig. 5 a sample time series and a three dimensional projection of the state space representation (reconstructor canonical form realization of the linear transfer function) of the identified model are reported.

3.4. Discussion

An accurate analysis of the results showed that, despite the simplicity of the identification algorithm, and without any external constraint requiring chaotic behavior, in both the cases

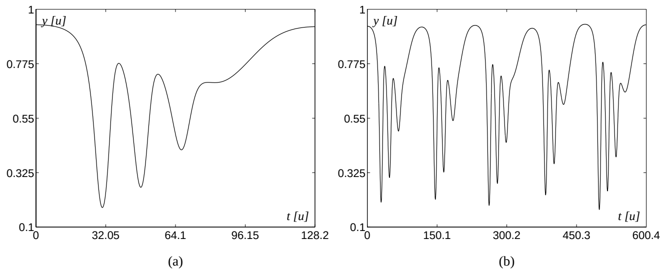

Figure 2: Example of data signal issued by the Rosenzweig–MacArthur food chain model which has been used for the identification: (a) – the pseudo-period prototype; (b) – a typical observation.

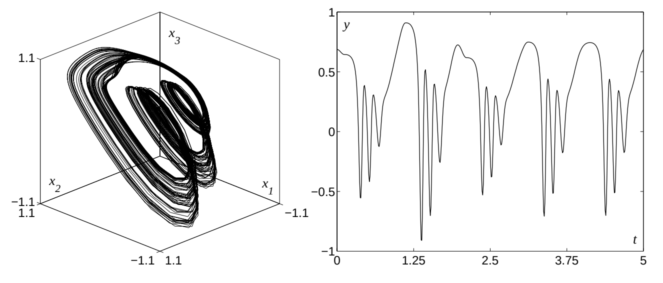

Figure 3: The result of the Rosenzweig–MacArthur food chain model identification: (a) – three-dimensional view of the resulting Feigenbaum-like strange attractor; (b) – a typical observation.

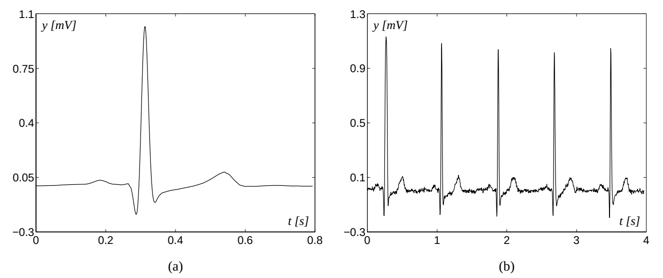

Figure 4: Example of healthy ECG data signal which has been used for the identification: (a) – the pseudo-period prototype; (b) – a typical observation.

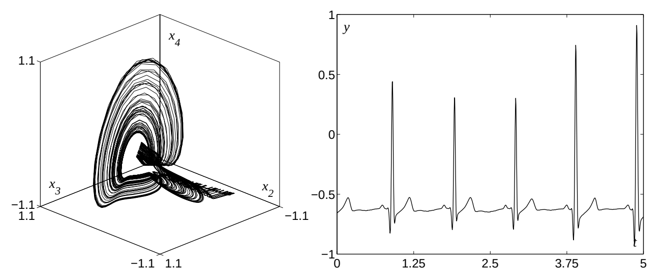

Figure 5: The result of the ECG identification: (a) – three-dimensional projection of the resulting Feigenbaum-like strange attractor; (b) – a typical observation.

the algorithm converges to a Feigenbaum-like chaotic system mimicking the intrinsic diversity of the signals used for the identification.

4. Conclusions

The chaotic behavior of Feigenbaum-like type self-emerges when identifying approximately periodic signals. These signals are rather common in nature, *e.g.* the physiological signals of living beings, such as the electrocardiograms (ECG), parts of speech signals, electroencephalograms (EEG), etc. Since there are strong arguments in favor of the chaotic nature of these signals, it could be thought that the self-emergence of chaos is a logical consequence of this fact. However, experiments conducted on signals for which the chaotic nature is an open and controversial question (ECG, EEG), allow to conjecture that the emergence of chaos should be quite general and due to the need of fitting the intrinsic diversity (randomness) of the fitted signals rather than being limited to signals produced by chaotic dynamics in nature.

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