Design and planning of reverse supply chain networks in a weighted Mean-Risk framework with risk-aversion and loss-aversion consideration

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Introduction

In a reverse supply chain network, used products (called return products) are collected from customers and reused.

Reverse Logistics is the process of

- planning,
- implementing,
- controlling

the efficient, effective inbound flow and storage of secondary goods and related information opposite to the traditional supply chain direction for the purpose of recovering value and proper disposal, Fleischmann (2001).

In this work we study the designing and planning of a reverse supply chain network in a two-stage stochastic structure with nondeterministic return and risk- and loss-averse consideratino.

Introduction

The problem addressed in this work has the objective of

- determining the supply chain structure
  along with
- planning decisions that maximize the total expected profit,
- by taking into account
  - the uncertainty in
    - (*i*) the demand and
    - (*ii*) prices of return products, and
  - the decision-maker's risk- and loss-aversion.

The general two-stage stochastic programming model

The general form of the two-stage stochastic programming (SP) model is the following:

$$\min_{\mathbf{x}\in\mathbb{R}^n} E\left(f\left(\mathbf{x},\omega\right)\right) \iff \min_{\mathbf{x}\in\mathbb{R}^n} \mathbf{c}^T\mathbf{x} + E\left(Q\left(\mathbf{x},\xi\left(\omega\right)\right)\right),$$

where $f\left(\mathbf{x},\omega\right)$ is the total cost function of the first-stage problem, and $Q\left(\mathbf{x},\xi\left(\omega\right)\right)$ is the optimal value of second-stage problem

$$\begin{cases} Q\left(\mathbf{x},\xi\left(\omega\right)\right) = \min\left\{q\left(\xi\right)^T\mathbf{y}\,\middle|\mathbf{y}\in\mathbb{R}^m\right\} \\ T\left(\xi\right)^T\mathbf{x}+W\left(\xi\right)^T\mathbf{y} = \mathbf{h}\left(\xi\right)^T \\ \mathbf{y}\geq\mathbf{0}. \end{cases}$$

We notice that this general form is risk-neutral.

The proposed two-stage SP model with loss aversion

In this paper we propose to incorporate the risk in the model such that decision-maker's (DM's) loss-aversion is also taken into consideration. In real life, DM has a certain risk profile and also a critical loss level - it is the case when the business conditions might change drastically if a certain critical loss level is reached. Therefore, a more realistic approach is to consider that DM is characterized by an increasing convex disutility function D with loss aversion which exhibits a kink at this critical loss level θ. When θ is reached, the perception of losses changes abruptly: the losses higher than this critical threshold are given disproportionate weight in accordance with a loss aversion parameter $\varepsilon > 0$. We consider the piecewise linear disutility function characterizing the DM:

$$D(z) = \begin{cases} z, \text{ for } z < \theta \\ (1+\varepsilon)z - \varepsilon\theta, \text{ for } z \geq \theta. \end{cases}$$

The proposed two-stage stochastic programming model

The proposed two-stage stochastic programming model is as follows:

$$\min_{\mathbf{x}\in\mathbb{R}^n} E\left(f\left(\mathbf{x},\omega\right)\right) + \lambda \cdot DCVaR_{\alpha}\left(f\left(\mathbf{x},\omega\right)\right),$$

where $DCVaR_{\alpha}$ of the monetary cost $f\left(\mathbf{x},\omega\right)$ is in fact $CVaR_{\alpha}$ of the modified/altered cost $D\left(f\left(\mathbf{x},\omega\right)\right)$, see Fulga, C. (2016).

For the case of finite probability space (a number of S scenarios available $\omega^1, ..., \omega^S$), we can equivalently reformulate the previous Mean-Risk problem as the following linear programming problem:

The proposed two-stage stochastic programming model

$$\min_{\mathbf{x},\mathbf{y}^s,\nu^s,\eta} \left\{ \mathbf{c}^T\mathbf{x} + \sum_{s=1}^{S} (q^s)^T \mathbf{y}^s p^s + \lambda \cdot \left(1 + \varepsilon \cdot \mathbf{1}_{\{\mathbf{c}^T\mathbf{x}+Q(\mathbf{x},\xi(\omega)) > \theta\}}\right) \cdot \right.$$

$$\left. \cdot \left(\mathbf{c}^T\mathbf{x} + \eta + \frac{1}{1-\alpha} \sum_{s=1}^{S} \nu^s p^s \right) - \varepsilon \cdot \theta \cdot \mathbf{1}_{\{\mathbf{c}^T\mathbf{x}+Q(\mathbf{x},\xi(\omega)) > \theta\}} \right\}$$

subject to :

$$\mathbf{x} \in \mathbb{R}^n, \eta \in \mathbb{R},$$

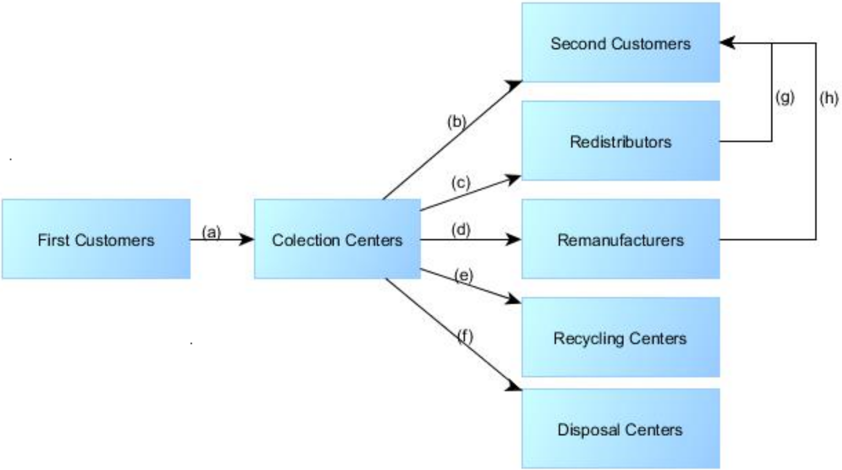
$$\mathbf{y}^s \geq 0,\ \forall s \in S = \{1, ..., S\},$$

$$\nu^s \geq \left[(q^s)^T \mathbf{y}^s - \eta \right]^+,\ \forall s \in S,$$

$$T^s\mathbf{x} + W^s\mathbf{y}^s = \mathbf{h}^s,\ \forall s \in S,$$

where $[z]^+ = \max\{0; z\}$.

The proposed network



Description of the network

First customer: Selling used products to collection centers - flow (a).

Collection centers: Purchasing used products from first customers and deciding about the next step based on the quality of used products. They segregate used products in four levels:

1. Some used products have acceptable quality, thus they can be repaired in the collection center and then sent
    1. to the second customers directly - flow (b),
    2. or through redistributors - flow (g).
2. Some fraction can be resold after undertaking some manufacturing processes so
    1. they are transferred to the manufacturers - flow (d),
    2. and then to the second customers - flow (h).
3. Some can be sold as recyclable products to the recycling centers in order to provide raw materials - flow (e).
4. The rest are sent to disposal centers - flow (f).

Description of the network

Recycling centers: They buy return products from collection centers and recycle them to provide raw materials.

Disposal centers: The return products of low-quality are sent to disposal centers to ensure proper green disposing.

Manufacturers: The return products are manufactured in order to increase their quality and make them ready to be sold to second customers.

Redistributors: Redistributors send repaired and/or remanufactured products to the second customers according to their demands.

Second customers: They are interested in repaired/remanufactured products, thus acquiring good quality products at a low price.

Notations

• The set of available scenarios $S = \{1, ..., S\}$. Depending on the context, it will be clear if S represents the set or the total number of scenarios. The scenarios are indexed by $s \in S$.

• The products are indexed by $u \in U = \{1, ..., U\}$

• The first customers $c \in C = \{1, ..., C\}$

• The collection centers $a \in A = \{1, ..., A\}$

• The second customers $k \in K = \{1, ..., K\}$

• The manufacturers $m \in M = \{1, ..., M\}$

• The redistributors $r \in R = \{1, ..., R\}$

• The recycling centers $l \in L = \{1, ..., L\}$

• The disposal centers $p \in P = \{1, ..., P\}$

• The nodes of the network $i \in \Omega$, where $\Omega = \{C, L, P, K, A, M, R\}$

• The destination nodes of the network are indexed by $j \in \Omega \setminus \{C\}$

Parameters of the model

- $\alpha \in (0, 1)$ The probability level in the definition of the risk measure DCVaR.
- $\lambda > 0$ The weight of the risk measure in the objective function.
- p_s Probability of scenario s,
- d_{kus} the amount of product u demanded by the second customer k on scenario s.
- $C^{(pur)}$ Purchase cost; $C^{(pur)}_{cus}$ Purchase cost of product u at the first customer c in scenario s.
- π Unit price
  - π_{kus} Unit price of product u at the second customer k on scenario s.
  - π_{lus} Unit price of product u at the recycling center l on scenario s.
- $C^{(fix)}_j$ Fixed cost for facility j, where $j \in \{A, M, R\}$.
- η_{iu} Capacity of facility i of product u, where $i \in \{A, L, P, M, R\}$.

Parameters of the model

- $C^{(rem)}_{fu}$ Remanufacturing cost of product u (unitary cost) for manufacturer f.
- $C^{(coll)}_{au}$ Collection cost of product u (unitary cost) for the collection center a.
- $C^{(rep)}_{au}$ Repairing cost of product u (unitary cost) for the collection center a'.
- $C^{(disp)}_{pu}$ Disposal cost of product u (unitary cost) of the disposal center p.
- $C^{(short)}_{ku}$ Shortage cost of product u (unitary cost) for second customer k.
- $C^{(hold)}_{ru}$ Holding and distributing cost of product u (unitary cost) for distributor r.
- $C^{(tr)}_{u}$ Transportation cost of product u (unitary cost) per kilometer.
- β_{iu} The batch size of product u when transported from location $i \in \{C, A, L, P, M, R\}$ to another location.
- D_{ij} Distance between location i and location j, where $(i, j) \in \{(C, A), (A, L), (A, M), (A, P), (A, R), (M, K), (M, R), (R, K), (A, K)\}$.

Parameters of the model

- $\rho^{(rec)}$ Recycling ratio, percentage of products that are sold to recycling centers.
- $\rho^{(rem)}$ Remanufacturing ratio, percentage of products that are sent to manufacturer for remanufacturing.
- $\rho^{(rep)}$ Repairing ratio, percentage of products that can be repaired and sold.
- $\rho^{(disp)}$ Disposal ratio, percentage of products sent to green disposal centers.

- $N^{(coll)}$ Maximum allowed number of collection centers
- $N^{(man)}$: Maximum allowed number of manufacturers.
- $N^{(red)}$: Maximum allowed number of redistributors.

Decision variables of the model

First-stage decision variables: location variables δ_j

$$\delta_j = \begin{cases} 1, & \text{if location } j \text{ is active} \\ 0, & \text{otherwise} \end{cases},$$

where $j \in \{A, M, R\}$.

Second-stage decision variables: flows φ_{ijus}

φ_{ijus}: Batch-flows of product u from location i to location j on scenario s, where $(i, j) \in \{(C, A), (A, L), (A, M), (A, P), (A, R), (M, K), (M, R), (R, K), (A, K)\}$.

Objective function of the model

The objective function to be maximized is the total expected profit, calculated as follows:

$$\begin{aligned} \textit{Total expected profit } (TEP) &= \textit{Total expected sales } (TES) - \\ &- \textit{Total expected costs } (TEC) - \\ &- \textit{Total DCVaR costs} \end{aligned}$$

Total expected sales

The total expected sales have two components:

1. Expected sales of return products to second customers that are supplied by
   1. collection centers, $\sum_{a \in A} \sum_{k \in K} \sum_{u \in U} \sum_{s \in S} \varphi_{akus} \beta_{au} \pi_{kus} p_s$, flow (b),
   2. manufacturers, $\sum_{m \in M} \sum_{k \in K} \sum_{u \in U} \sum_{s \in S} \varphi_{mkus} \beta_{mu} \pi_{kus} p_s$, flow (h),
   3. redistributors, $\sum_{r \in R} \sum_{k \in K} \sum_{u \in U} \sum_{s \in S} \varphi_{rkus} \beta_{ru} \pi_{kus} p_s$, flow (g), and
2. Expected sales of return products sent to recycling centers

$$\sum_{a \in A} \sum_{l \in L} \sum_{u \in U} \sum_{s \in S} \varphi_{alus} \beta_{au} P_{aus} p_s, \text{ flow } (e).$$

Total costs

The total cost (TC) is calculated as we previously discussed:

$$\begin{aligned} TC = {} & (1+\lambda) \cdot (\mathit{Fixed\ costs}) + \\ & + (\mathit{TEC}) + \\ & + \lambda \cdot (\mathit{Total\ DCVaR\ costs}) \end{aligned}$$

where, the three categories of costs are calculated as follows:

Components of the Total Cost (TC)

1. Fixed costs (FC)

$$FC = \sum_{j \in \{A,M,R\}} C_j^{(fix)} \cdot \delta_j$$

2. Total expected costs (TEC)

$$\begin{aligned} TEC = {} & \text{expected remanufacturing costs } (ERmC) \\ & + \text{expected shortage costs } (ESC) \\ & + \text{expected purchasing costs } (EPC) \\ & + \text{expected collection costs } (ECC) \\ & + \text{expected repairing costs } (ERC) \\ & + \text{expected disposal costs } (EDC) \\ & + \text{expected transportation costs } (ETC) \\ & + \text{expected inventory/holding costs } (EIHC). \end{aligned}$$

3. Total $DCVaR$ costs

Components of the Total expected cost (TEC)

Expected remanufacturing costs $(ERmC)$:

$$ERmC = \sum_{a \in A} \sum_{f \in F} \sum_{u \in U} \sum_{s \in S} \varphi_{afus} \beta_{au} C_{fu}^{(rem)} \cdot p_s$$

Components of the Total expected cost (TEC)

Expected shortage costs (*ESC*)

$$ESC = \sum_{k\in K}\sum_{u\in U}\sum_{s\in S}\Bigg(C_{ku}^{(short)}\cdot$$

$$\cdot\Bigg[d_{kus} - \Bigg(\sum_{a\in A}\sum_{k\in K}\sum_{u\in U}\sum_{s\in S}\varphi_{akus}\beta_{au}\dot{+}$$

$$+\sum_{r\in R}\sum_{k\in K}\sum_{u\in U}\sum_{s\in S}\varphi_{rkus}\beta_{ru}\dot{+}$$

$$\dot{+}\sum_{m\in M}\sum_{k\in K}\sum_{u\in U}\sum_{s\in S}\varphi_{mkus}\beta_{mu}\Bigg)\Bigg]^{+}\Bigg)$$

where $[z]^{+} = \max\{0; z\}$.

Components of the Total expected cost (TEC)

Expected purchasing costs (*EPC*)

$$EPC = \sum_{c \in C} \sum_{a \in A} \sum_{u \in U} \sum_{s \in S} \varphi_{caus} \beta_{cu} C_{cus}^{(pur)} \cdot p_s$$

Expected collection costs (*ECC*)

$$ECC = \sum_{c \in C} \sum_{a \in A} \sum_{u \in U} \sum_{s \in S} \varphi_{caus} \beta_{cu} C_{au}^{(coll)} \cdot p_s$$

Components of the Total expected cost (TEC)

Expected repairing costs (*ERC*)

$$ERC = \sum_{c \in C} \sum_{a \in A} \sum_{u \in U} \sum_{s \in S} \rho^{(rep)} \varphi_{caus} \beta_{cu} C_{au}^{(rep)} \cdot p_s$$

Expected disposal costs (*EDC*)

$$EDC = \sum_{a \in A} \sum_{p \in P} \sum_{u \in U} \sum_{s \in S} \varphi_{apus} \beta_{au} C_{pu}^{(disp)} \cdot p_s$$

Expected transportation costs (*ETC*)

$$ETC = \sum_{(i,j)\in\Gamma} \varphi_{ijus} D_{ij} \beta_{iu} C_u^{(tr)},$$

where

$$\Gamma = \{(C,A),(A,M),(A,R),(A,P),\\ (A,I),(A,K),(M,K),(M,R),(R,K)\}.$$

Components of the Total expected cost (TEC)

Expected inventory-holding costs (*EIHC*)

$$EIHC = \sum_{k\in K}\sum_{u\in U}\sum_{s\in S} C_{ru}^{(hold)} \cdot \left[\left(\sum_{a\in A}\sum_{k\in K}\sum_{u\in U}\sum_{s\in S}\varphi_{akus}\beta_{au} \dot{+} + \sum_{r\in R}\sum_{k\in K}\sum_{u\in U}\sum_{s\in S}\varphi_{rkus}\beta_{ru} \dot{+} + \sum_{m\in M}\sum_{k\in K}\sum_{u\in U}\sum_{s\in S}\varphi_{mkus}\beta_{mu}\right) - d_{kus}\right]^{+}.$$

Total DCVaR costs at probability level alfa (TDCVaRC))

$$TDCVaR_{\alpha}C =$$

$$= \eta + \frac{1}{1-\alpha} \cdot \{DCVaR_{\alpha}(RmC + SC + PC + CC + RC +$$

$$+ DC + RecC + TC + IHC\}$$

where

$RmC = remanufacturing\ costs$,

$SC = shortage\ costs$,

$PC = purchasing\ costs$,

$CC = collection\ costs$,

$RC = repairing\ cost$,

$DC = disposal\ cost$,

$RecC = recycling\ cost$,

$TC = transportation\ costs$,

$IHC = inventory\ holding\ costs$.

Constraints of the model

There are three groups of constraints:

(1) Balance constraints: Constraints that ensure the balance between input and output in each node of the network,

(2) Capacity constraints, and

(3) Limitations of facilities.

(1) Balance constraints

(1.1) Constraints related to collection centers. Input from first customers should be equal to total outputs to recycling centers, manufacturers, redistributors, disposal centers, and second customers.

$$\sum_{c\in C}\varphi_{caus}\beta_{cu} =$$
$$= \sum_{l\in L}\varphi_{alus}\beta_{au} + \sum_{m\in M}\varphi_{amus}\beta_{au} + \sum_{r\in R}\varphi_{arus}\beta_{au}+$$
$$+\sum_{p\in P}\varphi_{apus}\beta_{au} + \sum_{k\in K}\varphi_{akus}\beta_{au},$$

for all $s \in S$, $u \in U$, $a \in A$.

(1) Balance constraints

(1.2) Constraints related to manufacturers. Input from collection centers should be equal to total outputs to redistributors and second customers.

$$\sum_{m\in M}\varphi_{amus}\beta_{au} = \sum_{r\in R}\varphi_{arus}\beta_{au} + \sum_{k\in K}\varphi_{akus}\beta_{au},$$

for all $s \in S, u \in U, m \in M$.

(1) Balance constraints

(1.3) Constraints related to redistributors. Total inputs from collection centers and manufacturers should be equal to total outputs to second customers).

$$\sum_{a\in A}\varphi_{arus}\beta_{au} + \sum_{m\in M}\varphi_{mrus}\beta_{mu} = \sum_{k\in K}\varphi_{akus}\beta_{au},$$

for all $\forall s \in S, u \in U, r \in R$

(2) Capacity constraints

The capacity constraints control the maximum flows that can enter or issue from each node of the network.

$$\sum_{i \in \{C,A,M\}} \varphi_{ijus} \beta_{iu} \leq \eta_{ju} \delta_j,$$

for all $j \in \{A, L, P, R, M\}$, $u \in U$.

(3) Limitations of facilities

3.1 Limitations on the maximum number of collection centers:

$$\sum_{j \in A} \delta_j \leq N^{(coll)}.$$

3.2 Limitations on the maximum number of manufacturers:

$$\sum_{j \in M} \delta_j \leq N^{(man)}.$$

3.3 Limitations on the maximum number of redistributors:

$$\sum_{j \in R} \delta_j \leq N^{(red)}.$$

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Thank you for your attention

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