

Efficient Approximation to Evacuation Planning

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1 Introduction

Evacuation planning problem is essential to the response to disasters like tsunamis. It can not only help evacuees to escape via efficient routes, but also help the government to identify how to execute emergency rescue.

2 Problem description

An evacuation planning problem is usually formulated as a dynamic multi-commodity flow problem. The problem takes as its input a graph from the map, which contains locations as nodes and roads as edges. To express the time-dependent flows, we discretize the time limit T into time steps and define a time-expanded graph G_T . The time-expanded graph copies the original graph for every time step. The evacuees' initial locations, transit areas, safe evacuation sites in a time-expanded graph are denoted as S_T, N_T, D_T , respectively. The set of roads is E . The solution guides the evacuees from different locations to arrive at one of the sinks. Assuming various types of evacuees exist, we use multi-commodity flow to formulate them. Let $f_{k,e_{ij}}$ be the flow variable for k th type of evacuees moving on road e_{ij} . The objective of the problem is to maximize the number of successful evacuees within a prescribed time limit T . The number of k th kind of evacuees in initial site i is $d_{i,k}$. The capacity for road e_{ij} is u_{ij} . E_{i-} represents the edges leaving node i , E_{i+} represents the edges entering node i . The linear programming formulation is as follows:

$$\max \sum_{k \in K} \sum_{i \in S_T} \sum_{e_{ij} \in E_{i+}} f_{k,e_{ij}} \quad (1)$$

$$\text{s.t.} \quad \sum_{k \in K} f_{k,e_{ij}} \leq u_{ij}, \quad \forall i, j \in N_T \quad (2)$$

$$0 \leq f_{k,e_{ij}}, \quad \forall i, j \in N_T, \forall k \in K \quad (3)$$

$$\sum_{e_{ij} \in E_{i-}} f_{k,e_{ij}} \leq d_{i,k}, \quad \forall i \in S_T \quad (4)$$

$$\sum_{e_{ij} \in E_{i-}} f_{k,e_{ij}} - \sum_{e_{ji} \in E_{i+}} f_{k,e_{ji}} = 0, \quad \forall i \in N_T, \forall k \in K \quad (5)$$

$$\sum_{i \in S_T} \sum_{e_{ij} \in E_{i+}} f_{k,e_{ij}} - \sum_{i \in D_T} \sum_{e_{ji} \in E_{i-}} f_{k,e_{ji}} = 0, \quad \forall k \in K \quad (6)$$

(1) maximizes the successful evacuees; (2) is the capacity constraint; (3) ensures the flow is non-negative; (4) ensures the sum of flows leaving sources is no larger than the initial evacuees; (5) makes no people stays in transit nodes; (6) makes the category of evacuees not to change on the way. For a large-scale problem, the linear programming solver usually fails to get efficient enough performance.

3 Proposed method

We adopt the proximal Jacobian ADMM[1] as the basis of our method. For a problem with the form:

$$\min_{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N} \sum_{i=1}^N f_i(\mathbf{x}_i) \quad \text{s.t.} \quad \sum_{i=1}^N A_i \mathbf{x}_i = c \quad (7)$$

The update formulas of the proximal Jacobian ADMM are:

$$\mathbf{x}_i^{k+1} = \arg \min_{\mathbf{x}_i} f_i(\mathbf{x}_i) + \langle \rho A_i^\top (A\mathbf{x}^k - c - \lambda^k/\rho), \mathbf{x}_i \rangle + \frac{\tau_1}{2} \|\mathbf{x}_i - \mathbf{x}_i^k\|^2, \quad (8)$$

$$\lambda^{k+1} = \lambda^k - \gamma\rho \left(\sum_{i=1}^N A_i \mathbf{x}_i^{k+1} - c \right), \quad (9)$$

Here we take the update of every single variable as a subproblem of ADMM. Since the separable objective function in our model is a linear function, each subproblem (8) becomes minimization of a quadratic function and has a closed-form solution. Thus, the algorithm can run with linear time complexity in every iteration.

4 Experiment

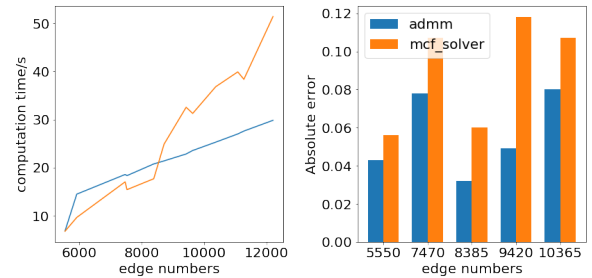


Figure 1: Left: computation time; Right: absolute error

The experiment compares the proposed method to the approximate algorithm proposed in [2] on the randomly generated graphs. We used the implementation of the latter called the mcf solver. The result is presented in Fig.1. The result shows the proposed method is more efficient for larger graphs, and it can achieve smaller errors to the optimal value derived from the LP solver.

5 Future work

For more realistic situations, we are going to divide various evacuees to specific lanes of the road and use an integer programming model to express it. Then the combination of the ADMM and other optimizing methods like Benders decomposition or McCormick relaxation is expected to lead to an efficient approximate algorithm.

References

- [1] Wei Deng et al. "Parallel Multi-Block ADMM with $o(1/k)$ Convergence". In: *Journal of Scientific Computing* 71.2 (2017), pp. 712–736.
- [2] George Karakostas. "Faster approximation schemes for fractional multicommodity flow problems". In: *ACM Transactions on Algorithms* 4.1 (2008), pp. 1–17.