

Coding Theorems

Based on Constrained-Random-Number Generators

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Acknowledgments

- This research includes a joint work with Prof. Shigeki Miyake, who is currently affiliated with International Professional University of Technology in Tokyo, Japan.
- This research was done when we were working at NTT Laboratories, Japan.
- I am grateful for their support.

Why do we need a new code construction?

- Shannon's coding theorems guarantee the existence of optimal codes
 - ▶ Rate-distortion function for lossy source coding
 - ▶ Channel capacity for channel coding
- Classical constructions (random codebook generation) are impractical
 - ▶ Exponentially large lookup table
 - ▶ Impractical encoding/decoding
- Practical codes (LDPC, Polar codes) have limitations
 - ▶ Optimal only for specific sources/channels
 - e.g., single-input-single-output, stationary memoryless, symmetric channels
 - ▶ Additional techniques needed for general sources/channels
- **Goal:** A unified, tractable code construction achieving fundamental limits for arbitrary sources and channels including multi-terminal settings

About this talk

■ This talk is based on the following YouTube lecture:

▶ “How to achieve Shannon limit: Introduction of CoCoNuTS”

■ The YouTube lecture covers:

▶ Constrained-random-number generator (CRNG)

▶ Lossy source code

▶ Channel code

■ New in this talk

← not covered in the YouTube lecture

▶ Extensions to multi-terminal problems

- Multi-terminal source coding

- Multi-terminal channel coding

■ For details of the proofs, please refer to:

▶ [M.,2025] [arxiv:2410.07939](#)

▶ [M.,2023] [arxiv:2206.00792](#)



Notations

- A random variable is denoted by capital letter (e.g. X).
- A realization of random variable is denoted by corresponding small letter (e.g. x).
- An n -dimensional column vector is denoted with superscript (e.g. $x^n \equiv {}^t(x_1, x_2, \dots, x_n)$).
- A matrix is denoted by capital letter (only A , B , and F).
- Small letter r and capital letter R denote rates of vectors.

Outline of this talk

■ Our contributions

- ▶ Constrained-random-number generator (CRNG)

■ Lossy source code

← Review of YouTube lecture

- ▶ Lossless source coding

- ▶ Independent uniform random number generation

■ Channel code

← Review of YouTube lecture

- ▶ Lossless source coding with side information at decoder

- ▶ Uniform random number generation

■ Extensions to multi-terminal problems

← New

■ Concluding remarks

Our contributions

- Lossy source code and channel code are constructed by using constrained-random-number generators generating a random sequence x^n subject to the distribution

$$\nu_{\tilde{X}_F^n|C^l}(x^n|c^l) \equiv \begin{cases} \frac{\mu_{X^n}(x^n)}{\mu_{X^n}(\{x^n: Fx^n=c^l\})} & \text{if } Fx^n = c^l, \\ 0 & \text{otherwise} \end{cases}$$

for given probability distribution μ_{X^n} , matrix F , and vector c^l .

- When a given probability distribution is memoryless, there is a tractable algorithm for a constrained-random-number generator by using the Belief Propagation algorithm or the Markov Chain Monte Carlo method.
- We call this type of codes **CoCoNuTS** (Codes based on Constrained Numbers Theoretically-achieving Shannon limits).

Algorithm of constrained-random-number generator [M., 2014]

■ Assume that X^n is memoryless.

1: **for** $k \in \{1, \dots, n\}$ **do**

2: Calculate the following conditional probability distribution with the Belief Propagation algorithm:

$$\nu_{\tilde{X}_k | \tilde{X}_1^{k-1} C^l}(x_k | x_1^{k-1}, c^l) \equiv \frac{\sum_{x_{k+1}^n} \prod_{j=k}^n \mu_{X_j}(x_j) \prod_{i=1}^l \chi(\mathbf{f}_{\mathcal{S}_i} \cdot x_{\mathcal{S}_i}^n = c_i)}{\sum_{x_k^n} \prod_{j=k}^n \mu_{X_j}(x_j) \prod_{i=1}^l \chi(\mathbf{f}_{\mathcal{S}_i} \cdot x_{\mathcal{S}_i}^n = c_i)},$$

where $\chi(\mathbf{f}_{\mathcal{S}_i} \cdot x_{\mathcal{S}_i}^n = c_i)$ is a parity check function.

3: Generate x_k at random subject to $\nu_{\tilde{X}_k | \tilde{X}_1^{k-1} C^l}(\cdot | x_1^{k-1}, c^l)$.

4: Output $x^n \equiv x_1^n$.

■ Time complexity is $O(tw^2n^2)$, where t is the iteration number of Belief Propagation and $w = \Theta(\log n)$ is the maximum row weight of the matrix.

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- ▶ Lossless source coding
- ▶ Independent uniform random number generation

■ Channel code

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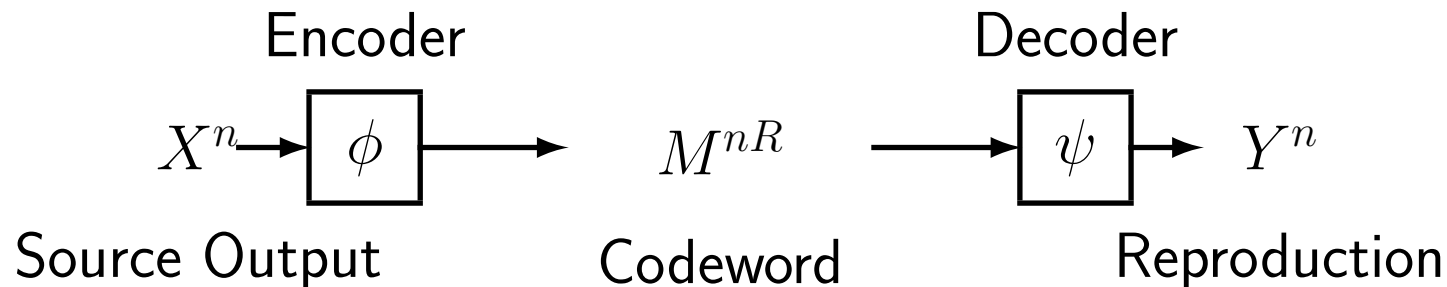
- ▶ Lossless source coding with side information at decoder
- ▶ Uniform random number generation

■ Extensions to multi-terminal problems

← New

■ Concluding remarks

Lossy source code



Distortion criterion

$$\text{Prob} (d_n(X^n, \psi(\phi(X^n))) \leq D) \rightarrow 1 \text{ as } n \rightarrow \infty$$

Rate-distortion function [Shannon, 1959]

$$R_X(D) = \max_{\mu_{Y|X}: E[d(X,Y)] \leq D} [H(Y) - H(Y|X)] = \min_{\mu_{Y|X}: E[d(X,Y)] \leq D} I(X; Y)$$

- Optimal conditional distribution $\mu_{Y|X}$ can be obtained by using Blahut-Arimoto algorithm.

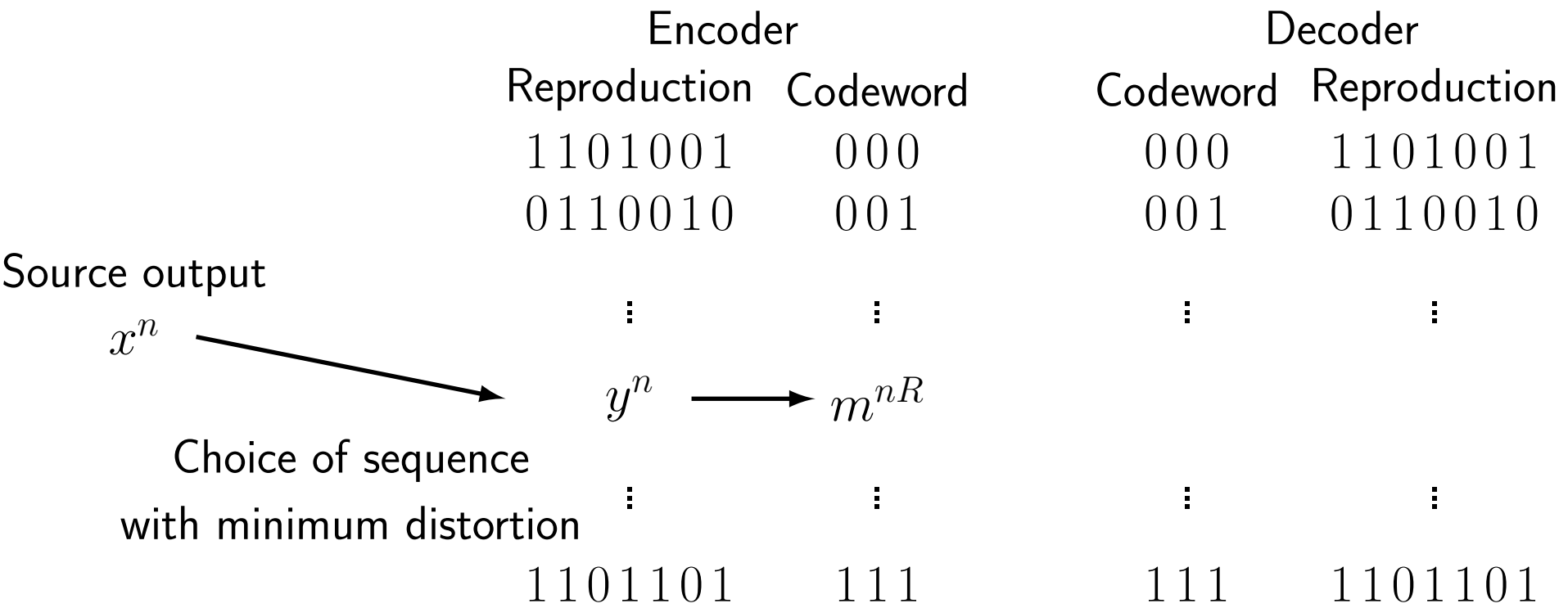
Random codebook generation [Shannon, 1959]

Encoder		Decoder	
Reproduction	Codeword	Codeword	Reproduction
1101001	000	000	1101001
0110010	001	001	0110010
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
1101101	111	111	1101101

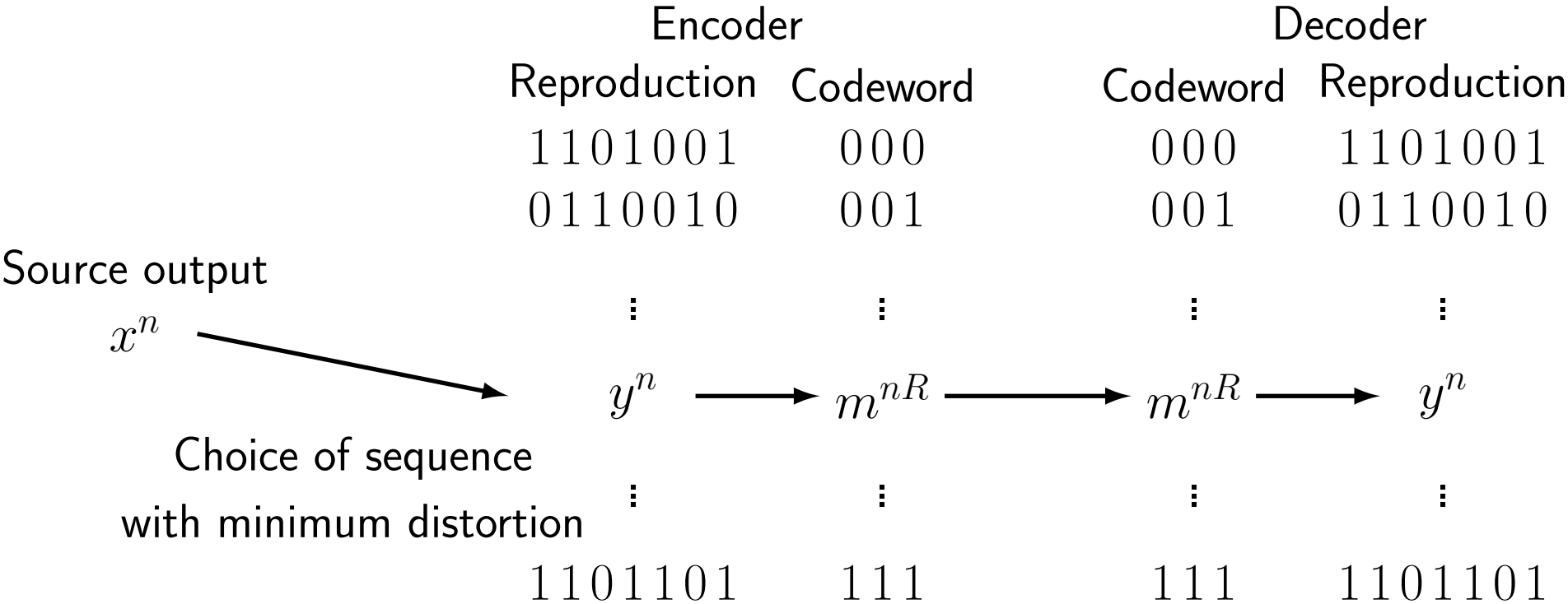
Generate
 2^{nR} reproductions
subject to μ_Y

$$\mu_Y(y) \equiv \sum_x \mu_{Y|X}(y|x) \mu_X(x)$$

Random codebook generation [Shannon, 1959]



Random codebook generation [Shannon, 1959]



- Assume that $\mu_{XY} (d_n(X^n, Y^n) > D) \rightarrow 0$ as $n \rightarrow \infty$.
- The error probability $\rightarrow 0$ by generating more than $2^{nI(X;Y)}$ reproductions.

Shortcomings of random codebook generation

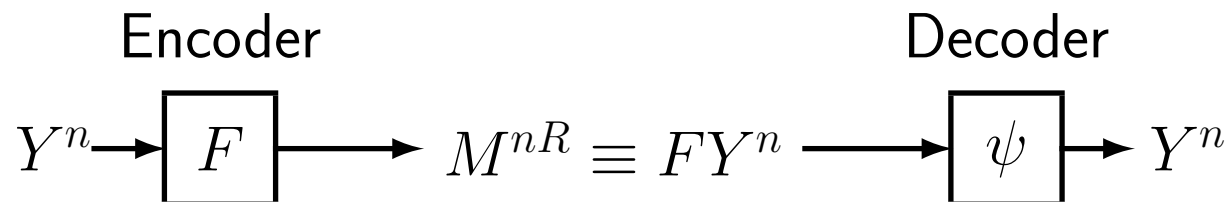
- Quantization points are generated at random.
 - ▶ A large lookup table for (codeword, quantization point) is necessary.
 - ▶ Encoding (looking for nearest quantization point) is impractical.

LDGM codes [Matsunaga-Yamamoto, 2003]

[Gupta-Verdú, 2009][Wainwright-Martinian, 2009]

- Encoding is practical.
- These codes achieve the rate-distortion function of a **uniform** source and Hamming distortion.
- An additional technique is necessary to achieve the rate distortion function of a general **(non-uniform)** source.

Lossless source coding



- $nR \times n$ matrix F is operated to Y^n .

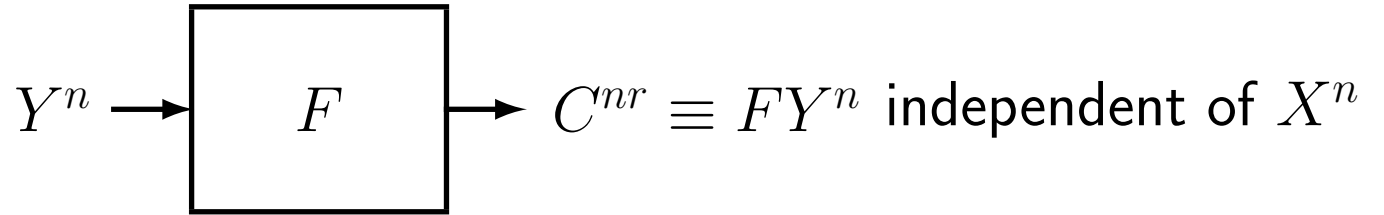
Theorem [Shannon, 1948]

- The infimum achievable rate is $H(Y)$. $\Rightarrow$ Code exists when $R > H(Y)$.

Theorem

- For any Y , the rate $H(Y)$ is achievable by using
 - ▶ random binning [Cover, 1975]
 - ▶ linear codes [Csiszár, 1984]
 - ▶ LDPC codes [M.-Uyematsu-Wadayama, 2005].

Independent uniform random number generation

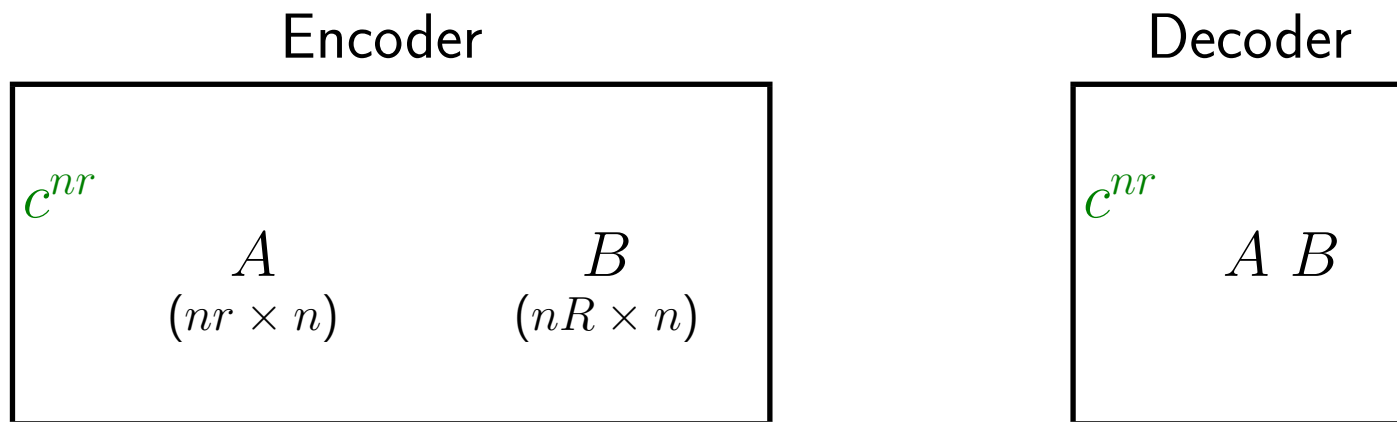


- $nr \times n$ matrix F is operated to a non-uniform random vector Y^n .
- C^{nr} is expected to be a uniform random vector which is independent of X^n .

Theorem [Maurer, 1993][Ahlsvede-Csiszár, 1993]

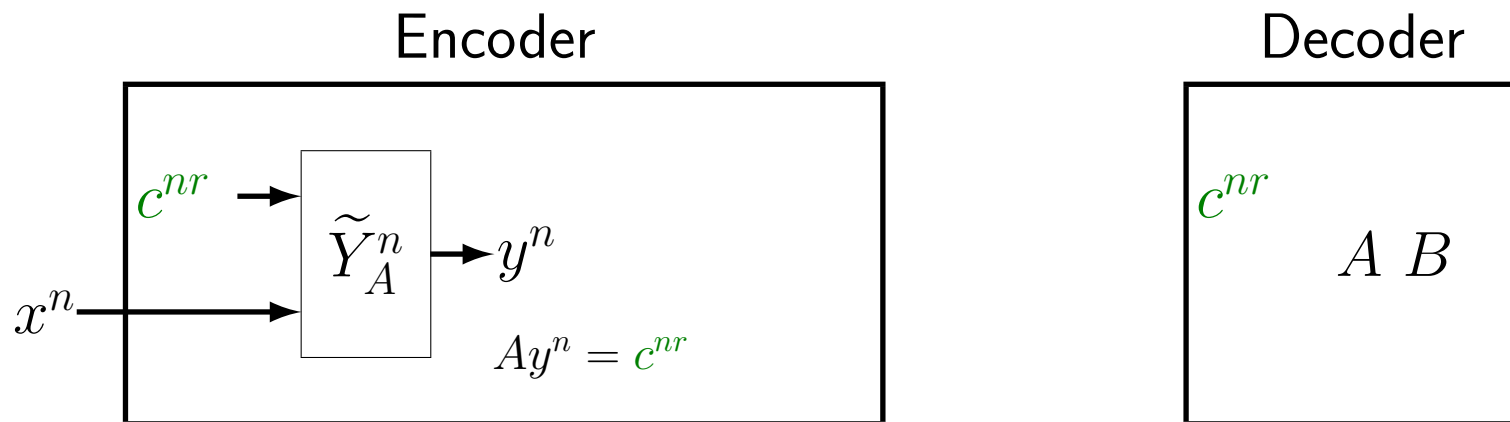
- The supremum achievable rate is $H(Y|X)$. $\Rightarrow$ Code exists when $r < H(Y|X)$.

Lossy source code (CoCo lossy) [M., 2014]



- An encoder and a decoder share (sparse) matrices A , B and a vector $c^{nr} \in \text{Im}A$.

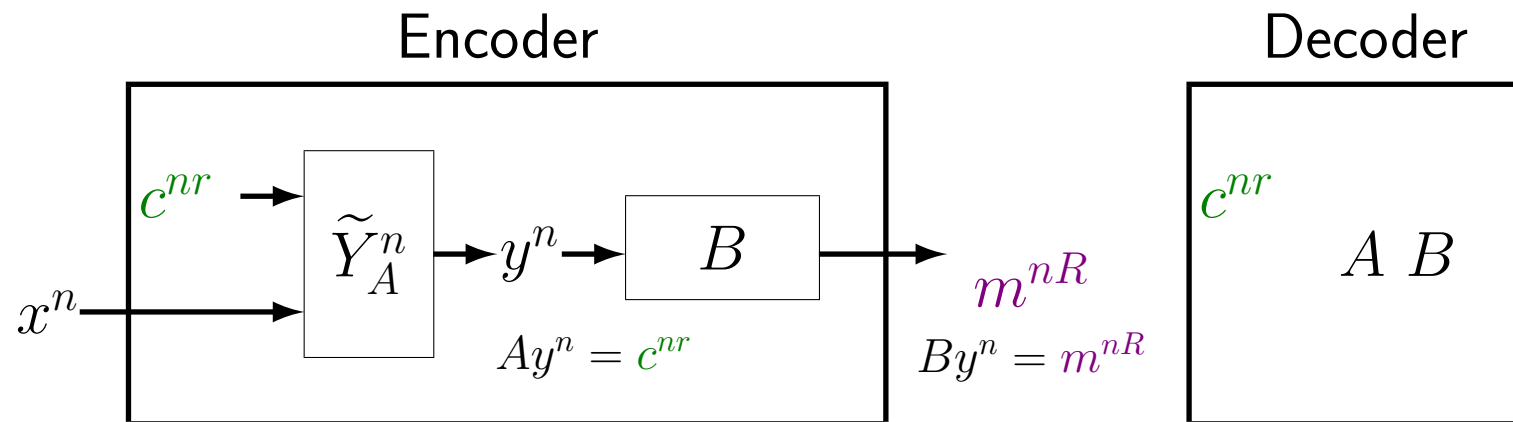
Lossy source code (CoCo lossy) [M., 2014]



$$\nu_{\tilde{Y}_A^n | X^n C^{nr}}(y^n | x^n, c^{nr}) \equiv \frac{\mu_{Y^n | X^n}(y^n | x^n) \chi(Ay^n = c^{nr})}{\sum_{y^n} \mu_{Y^n | X^n}(y^n | x^n) \chi(Ay^n = c^{nr})} \quad \text{CRNG}$$

- CRNG $\tilde{Y}_A^n$ generates y^n satisfying $Ay^n = c^{nr}$.
 - The probability is proportional to $\mu_{Y^n | X^n}(y^n | x^n)$.

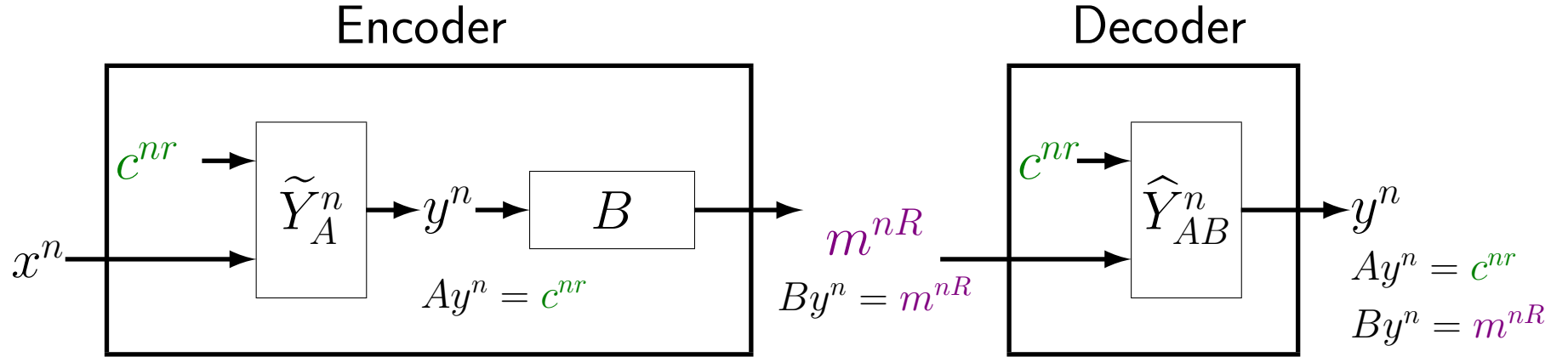
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- Encoder generates codeword $m^{nR} = By^n$.

Lossy source code (CoCo lossy) [M., 2014]



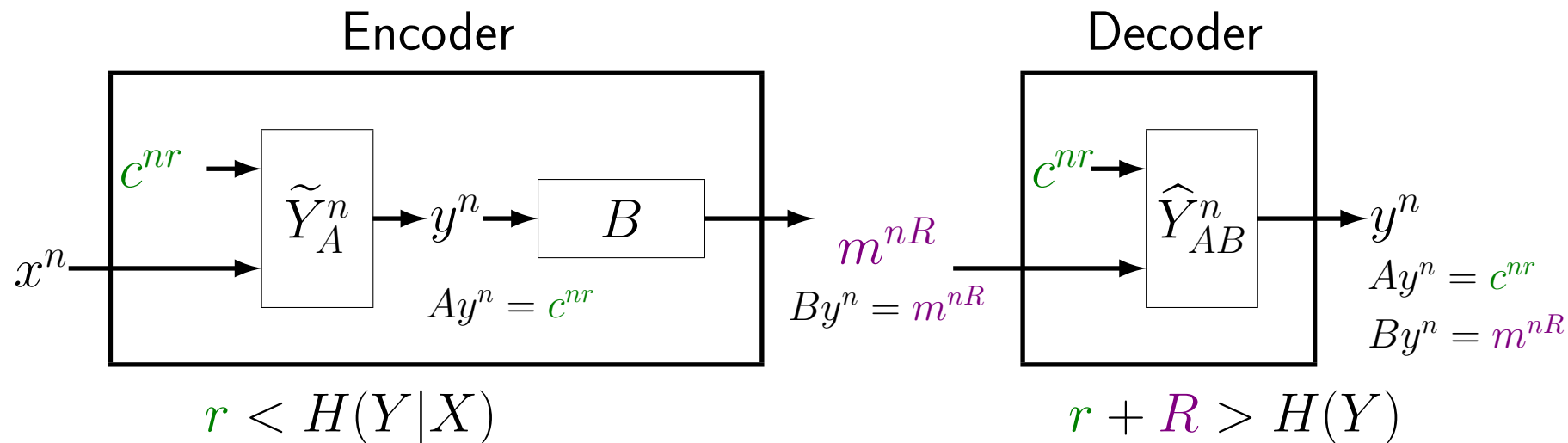
$$\nu_{\tilde{Y}_A^n | X^n C^{nr}}(y^n | x^n, c^{nr}) \equiv \frac{\mu_{Y^n | X^n}(y^n | x^n) \chi(Ay^n = c^{nr})}{\sum_{y^n} \mu_{Y^n | X^n}(y^n | x^n) \chi(Ay^n = c^{nr})} \quad \text{CRNG}$$

$$\nu_{\hat{Y}_{AB}^n | C^{nr} M^{nR}}(y^n | c^{nr}, m^{nR}) \equiv \frac{\mu_{Y^n}(y^n) \chi(Ay^n = c^{nr}, By^n = m^{nR})}{\sum_{y^n} \mu_{Y^n}(y^n) \chi(Ay^n = c^{nr})} \quad \text{CRNG}$$

■ CRNG $\hat{Y}_{AB}^n$ reproduces y^n satisfying $Ay^n = c^{nr}$ and $By^n = m^{nR}$.

► The probability is proportional to $\mu_{Y^n}(y^n) \equiv \sum_{x^n} \mu_{Y^n | X^n}(y^n | x^n) \mu_{X^n}(x^n)$.

Lossy source code (CoCo lossy) [M., 2014]



- When $r < H(Y|X)$, we can generate $c^{nr}(= Ay^n)$ independent of x^n . Then the encoder and the decoder can share a fixed c^{nr} .
- The encoder can generate y^n satisfying $c^{nr} = Ay^n$ and $d(x^n, y^n)/n < D$ with high probability.
- When $r + R > H(Y)$, the decoder can reproduce y^n from $c^{nr} = Ay^n$ and $m^{nR} \equiv By^n$.

Theorem [M., 2014]

■ Assume that $r, R > 0$ satisfy

$$r < H(Y|X) \tag{1}$$

$$r + R > H(Y). \tag{2}$$

Then for any $\delta > 0$ and sufficiently large n there are (sparse) matrices A, B and a vector c^{nr} such that

$$\text{Prob}(\rho_n(X^n, \psi(\varphi(X^n))) \leq D) \geq 1 - \delta.$$

■ For any source X , the rate-distortion function

$$R_X(D) \equiv \min_{\mu_{Y|X}} I(X; Y)$$

is achievable with the proposed code because for any $R > H(Y) - H(Y|X) = I(X; Y)$ there is $r > 0$ satisfying (1) and (2).

Interpretation of $R_X(D)$

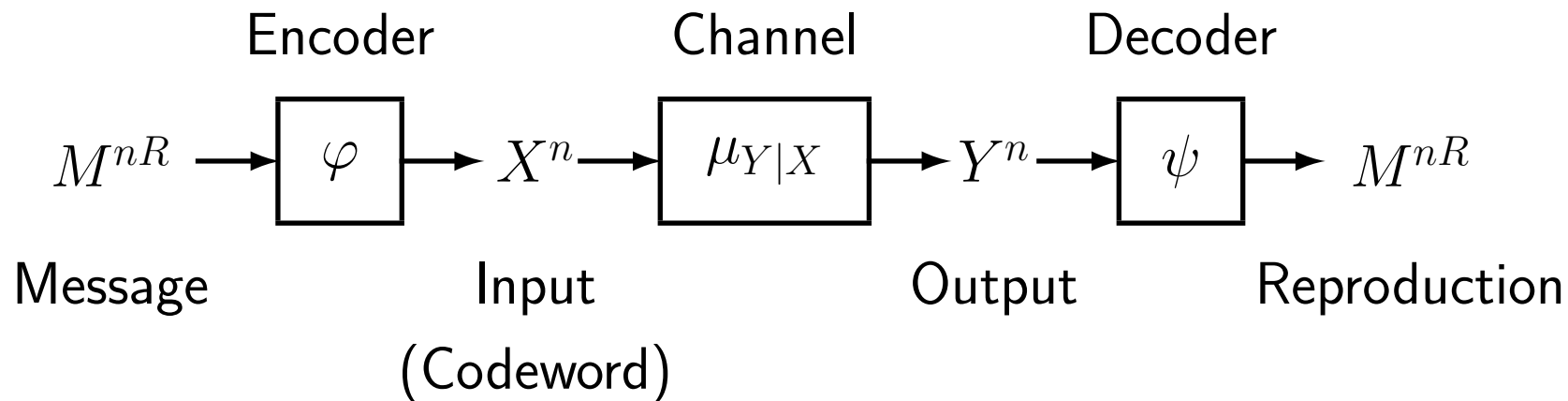
$$\begin{aligned} R_X(D) &= \min_{\substack{\mu_{Y|X} : \\ E[\rho(X, Y)] \leq D}} I(X; Y) \\ &= \min_{\substack{\mu_{Y|X} : \\ E[\rho(X, Y)] \leq D}} \left[\overset{\text{description rate of } Y}{H(Y)} - \underset{\substack{\text{description rate of } Y \text{ independent of } X \\ \text{rate of shared vector}}}{H(Y|X)} \right] \\ &\quad \text{optimizing} \\ &\quad \text{transformation from } X \text{ to } Y \end{aligned}$$

$$\mu_{XY}(x, y) = \mu_{Y|X}(y|x)\mu_X(x)$$

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 - ▶ Constrained-random-number generator
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- Extensions to multi-terminal problems ← New
- Concluding remarks

Channel coding



Channel capacity [Shannon, 1948]

$$C_{Y|X} = \max_{\mu_X} [H(X) - H(X|Y)] = \max_{\mu_X} I(X; Y)$$

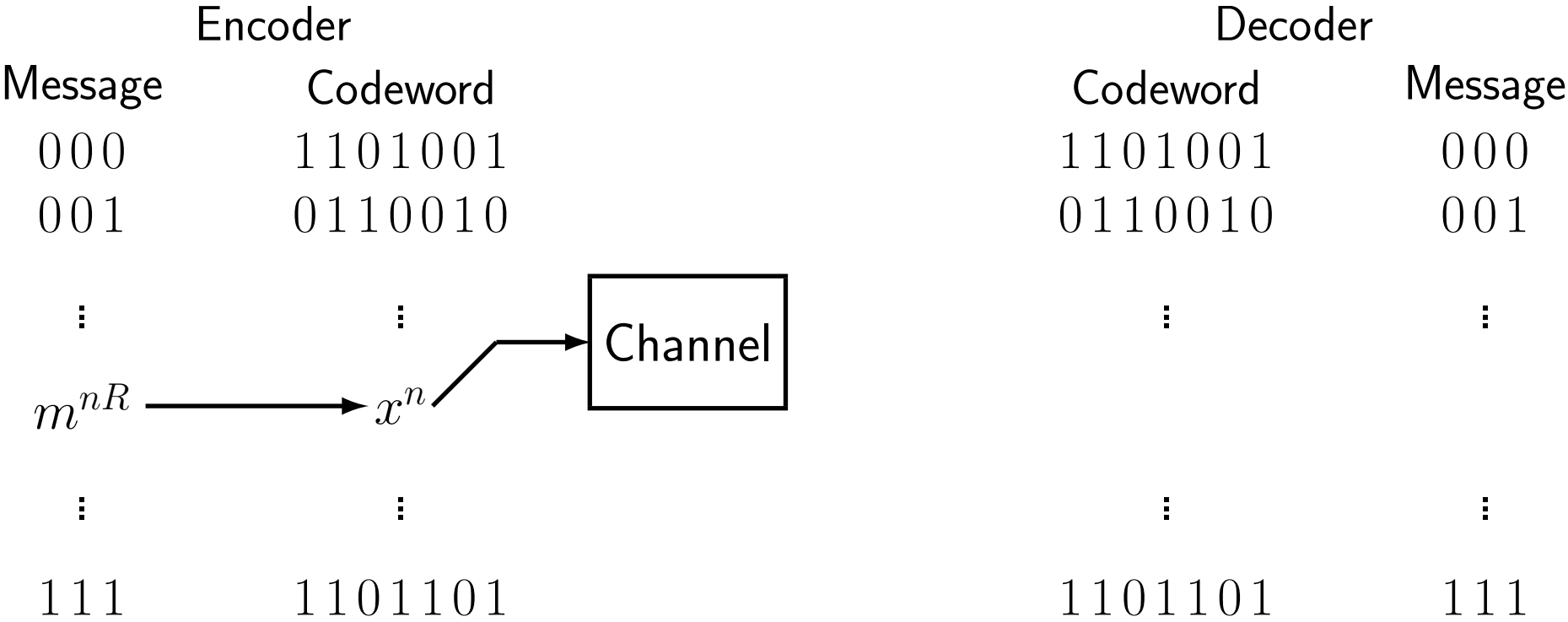
- Optimal input distribution μ_X can be obtained by using Arimoto-Blahut algorithm.

Random codebook generation [Shannon, 1948]

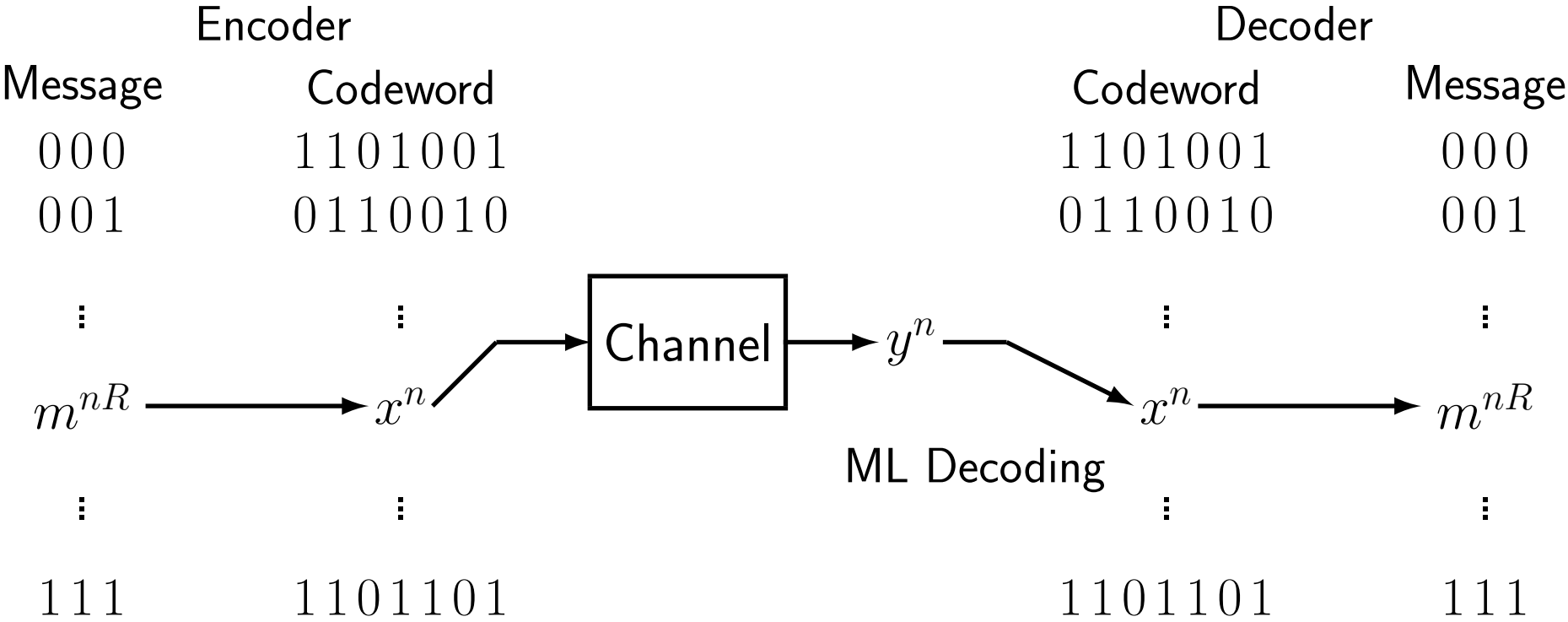
Encoder		Decoder	
Message	Codeword	Codeword	Message
000	1101001	1101001	000
001	0110010	0110010	001
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
111	1101101	1101101	111

Generate
 2^{nR} codewords
subject to μ_X

Random codebook generation [Shannon, 1948]



Random codebook generation [Shannon, 1948]



■ Decoding error probability $\rightarrow 0$ by generating less than $2^{nI(X;Y)}$ codewords.

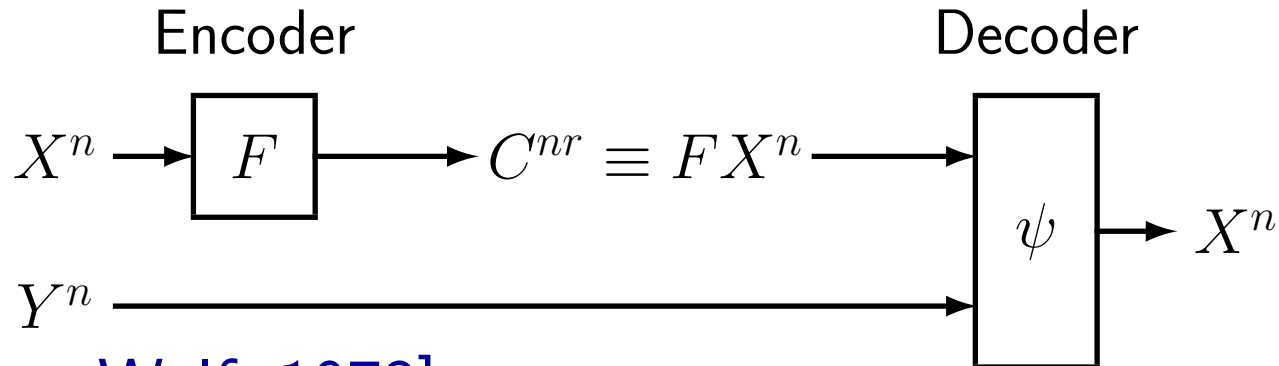
Shortcomings of random codebook generation

- Because a codebook is generated at random,
 - ▶ ML decoding is impractical,
 - ▶ a large lookup table for (message, codeword) is necessary.

LDPC codes [Gallager, 1963], Polar codes [Arıkan, 2009]

- Decoding is practical.
- These codes achieve capacity of a **symmetric** channel.
- An additional technique (e.g. a quantization map) is necessary to achieve capacity of a general **(asymmetric)** channel.

Lossless source coding with side information at decoder



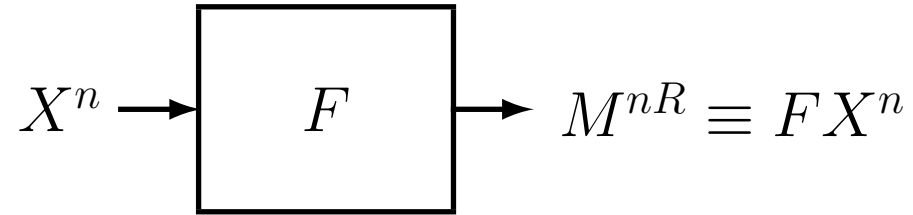
Theorem [Slepian-Wolf, 1973]

- The infimum achievable rate is $H(X|Y)$. $\Rightarrow$ Code exists when $r > H(X|Y)$.

Theorem

- For any (X, Y) , the rate $H(X|Y)$ is achievable by using
 - ▶ random binning [Cover, 1975]
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Uniform random number generation

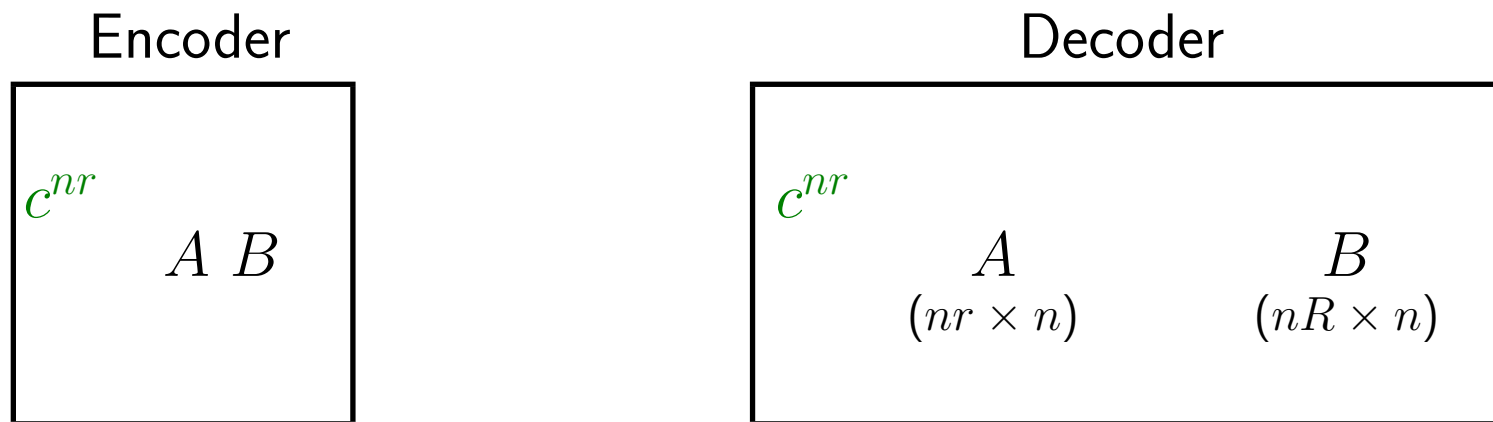


- $nR \times n$ matrix F is operated to a non-uniform random vector X^n .
- M^{nR} is expected to be a uniform random vector.

Theorem [Vembu-Verdú, 1995]

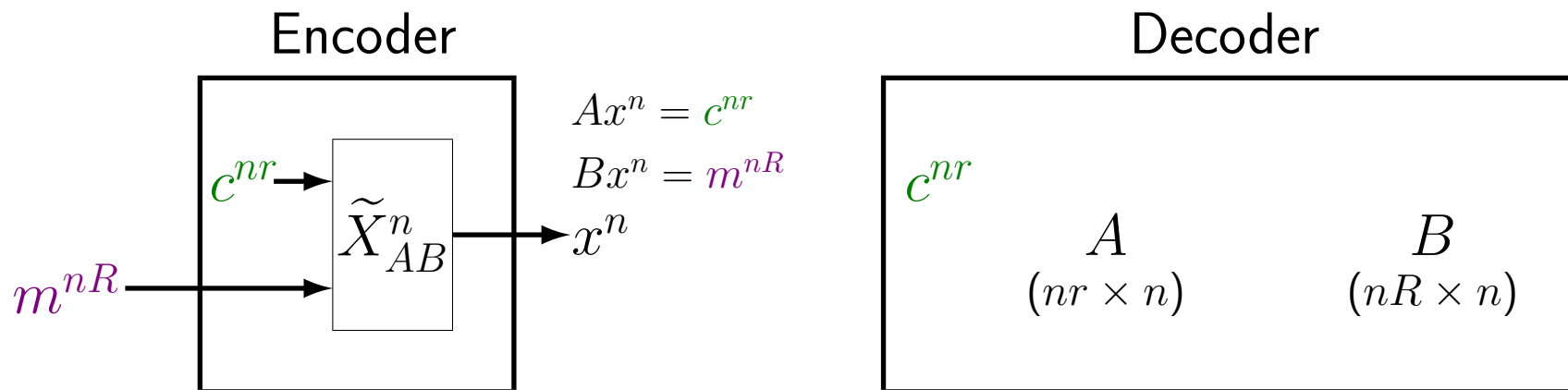
- The supremum achievable rate is $H(X)$. $\Rightarrow$ Code exists when $R < H(X)$.

Channel code (CoCo channel) [M.-Miyake,2019]



- An encoder and a decoder share (sparse) matrices A , B and a vector $c^{nr} \in \text{Im}A$.

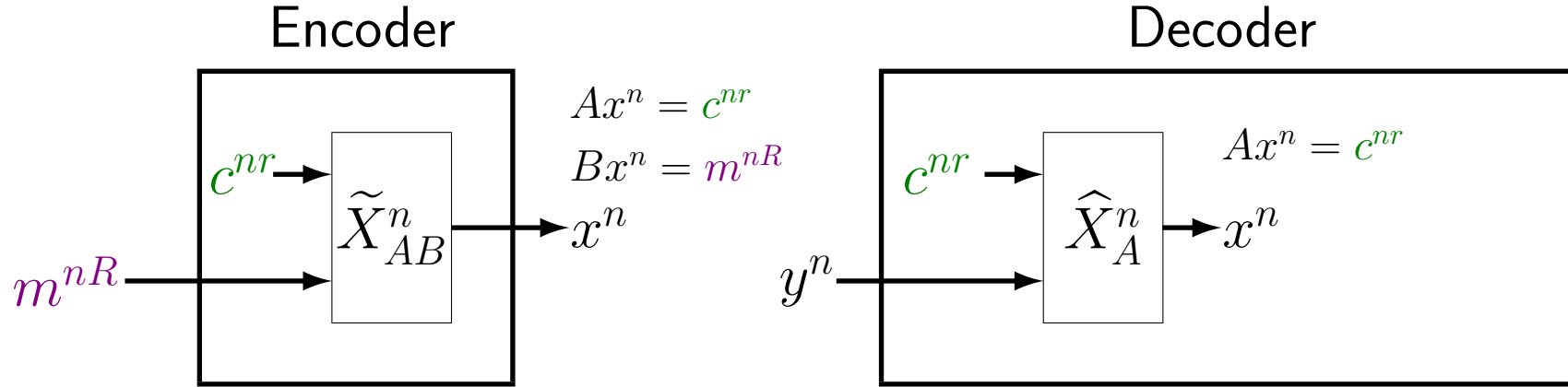
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$$\nu_{\tilde{X}_{AB}^n | C^{nr} M^{nR}}(x^n | c^{nr}, m^{nR}) \equiv \frac{\mu_{X^n}(x^n) \chi(Ax^n = c^{nr}, Bx^n = m^{nR})}{\sum_{x^n} \mu_{X^n}(x^n) \chi(Ax^n = c^{nr}, Bx^n = m^{nR})} \quad \text{CRNG}$$

- CRNG $\tilde{X}_{AB}^n$ generates x^n satisfying $Ax^n = c^{nr}$ and $Bx^n = m^{nR}$.
- The probability is proportional to $\mu_{X^n}(x^n)$.

Channel code (CoCo channel) [M.-Miyake,2019]



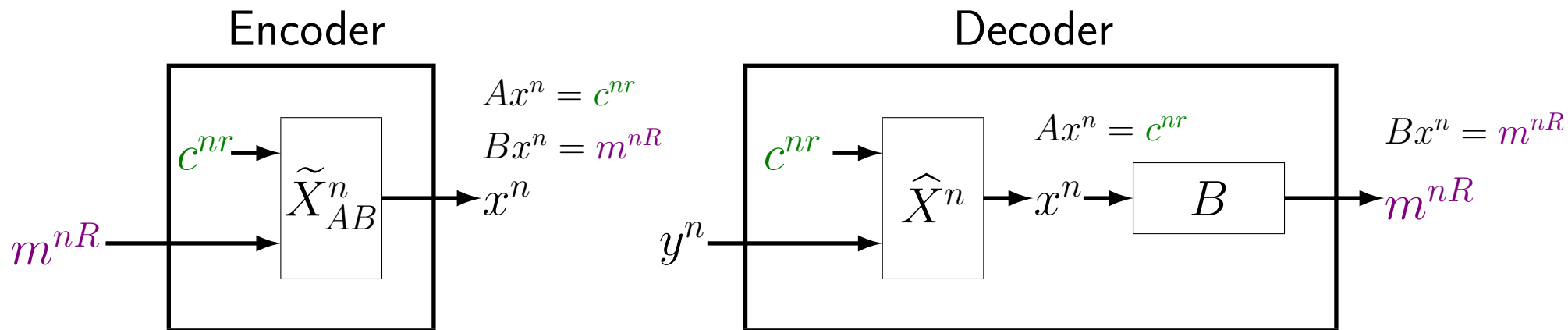
$$\nu_{\tilde{X}_{AB}^n | C^{nr} M^{nR}}(x^n | c^{nr}, m^{nR}) \equiv \frac{\mu_{X^n}(x^n) \chi(Ax^n = c^{nr}, Bx^n = m^{nR})}{\sum_{x^n} \mu_{X^n}(x^n) \chi(Ax^n = c^{nr}, Bx^n = m^{nR})} \quad \text{CRNG}$$

$$\nu_{\hat{X}_A^n | Y^n C^{nr}}(x^n | y^n, c^{nr}) \equiv \frac{\mu_{X^n | Y^n}(x^n | y^n) \chi(Ax^n = c^{nr})}{\sum_{x^n} \mu_{X^n | Y^n}(x^n | y^n) \chi(Ax^n = c^{nr})} \quad \text{CRNG}$$

■ CRNG $\hat{X}_A^n$ reproduces x^n satisfying $Ax^n = c^{nr}$.

► The probability is proportional to $\mu_{Y^n | X^n}(y^n | x^n) \equiv \frac{\mu_{Y^n | X^n}(y^n | x^n) \mu_{X^n}(x^n)}{\sum_{x^n} \mu_{Y^n | X^n}(y^n | x^n) \mu_{X^n}(x^n)}$.

Channel code (CoCo channel) [M.-Miyake, 2019]

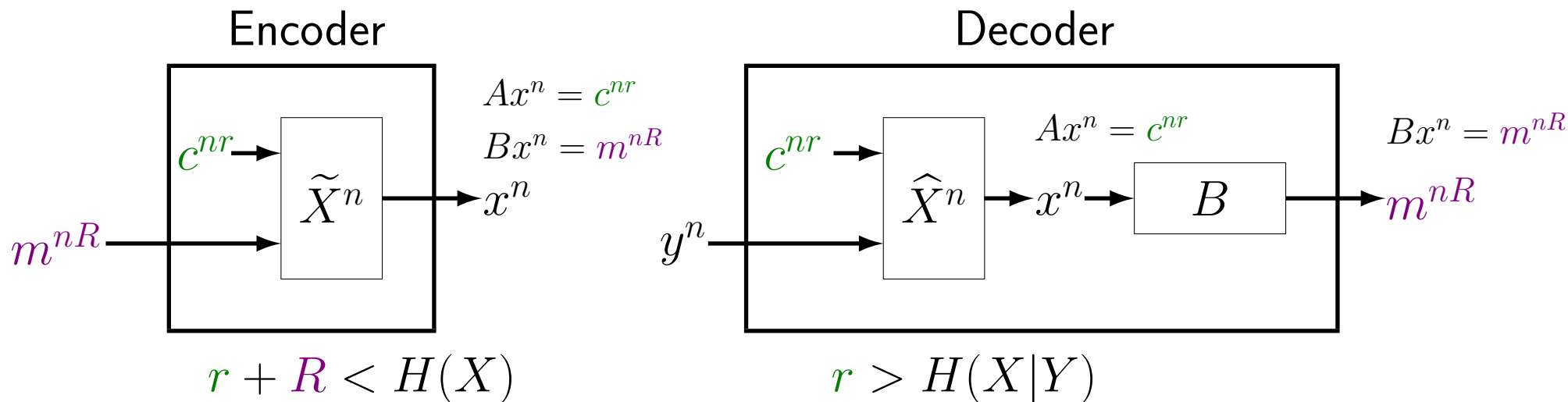


$$\nu_{\tilde{X}^n|C^{nr}M^{nR}}(x^n|c^{nr}, m^{nR}) \equiv \frac{\mu_{X^n}(x^n)\chi(Ax^n = c^{nr}, Bx^n = m^{nR})}{\sum_{x^n} \mu_{X^n}(x^n)\chi(Ax^n = c^{nr}, Bx^n = m^{nR})} \quad \text{CRNG}$$

$$\nu_{\hat{X}^n|Y^n C^{nr}}(x^n|y^n, c^{nr}) \equiv \frac{\mu_{X^n|Y^n}(x^n|y^n)\chi(Ax^n = c^{nr})}{\sum_{x^n} \mu_{X^n|Y^n}(x^n|y^n)\chi(Ax^n = c^{nr})} \quad \text{CRNG}$$

■ The decoder can reproduce $m^{nR} = Bx^n$.

Channel code (CoCo channel) [M.-Miyake, 2019]



- When $r + R < H(X)$, we can generate x^n satisfying $Ax^n = c^{nr}$ and $Bx^n = m^{nR}$ from uniformly generated (c^{nr}, m^{nR}) .
- When $r > H(X|Y)$, the decoder can reproduce x^n from c^{nr} and y^n .

Theorem [M.-Miyake, 2019]

■ Assume that $r, R > 0$ satisfy

$$r > H(X|Y) \tag{3}$$

$$r + R < H(X). \tag{4}$$

Then for any $\delta > 0$ and sufficiently large n there are (sparse) matrices A, B , and a vector c^{nr} such that

$$\text{Error}(A, B, c^{nr}) \leq \delta.$$

■ For any discrete memoryless channel, the capacity

$$C_{Y|X} \equiv \max_{\mu_X} I(X; Y)$$

is achievable with this code by optimizing μ_X , because for any $R < H(X) - H(X|Y) = I(X; Y)$ there is $r > 0$ satisfying (3),(4).

Interpretation of $C_{Y|X}$

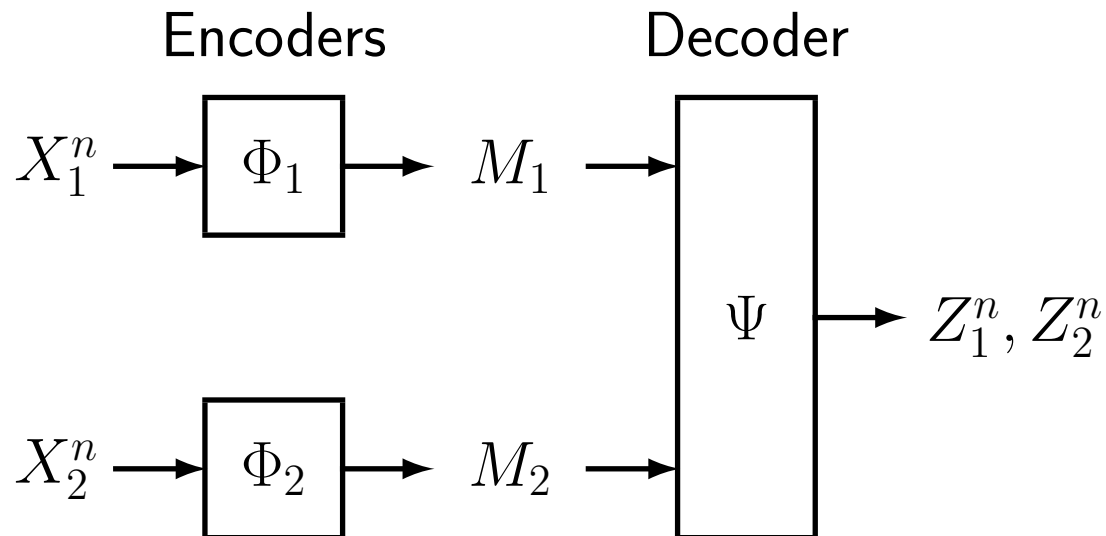
$$\begin{aligned} C_{Y|X} &= \max_{\mu_X} I(X; Y) \\ &= \max_{\mu_X} \left[\overset{\text{description rate of } X}{H(X)} - \underset{\substack{\text{description rate of } X \text{ after observing } Y \\ \text{rate of shared vector}}}{H(X|Y)} \right] \\ &\quad \text{optimizing} \\ &\quad \text{input distribution} \end{aligned}$$

$$\mu_{XY}(x, y) = \mu_{Y|X}(y|x)\mu_X(x)$$

Outline of this talk

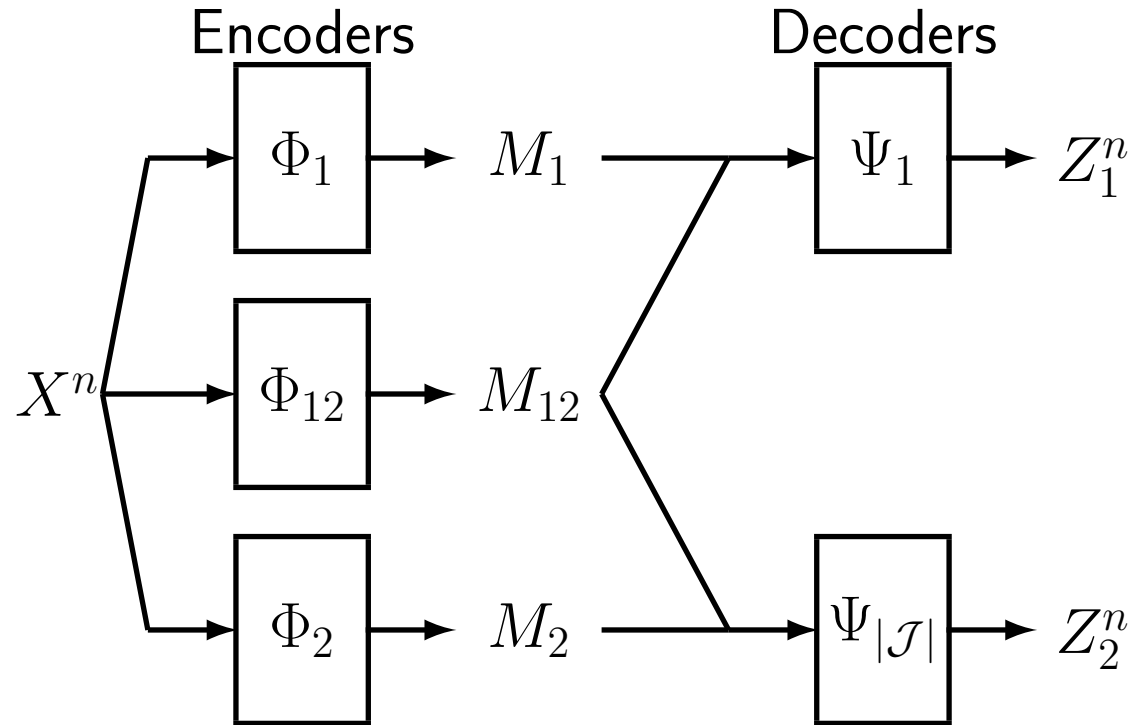
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Distributed source coding [Berger,1978][Tung,1978]



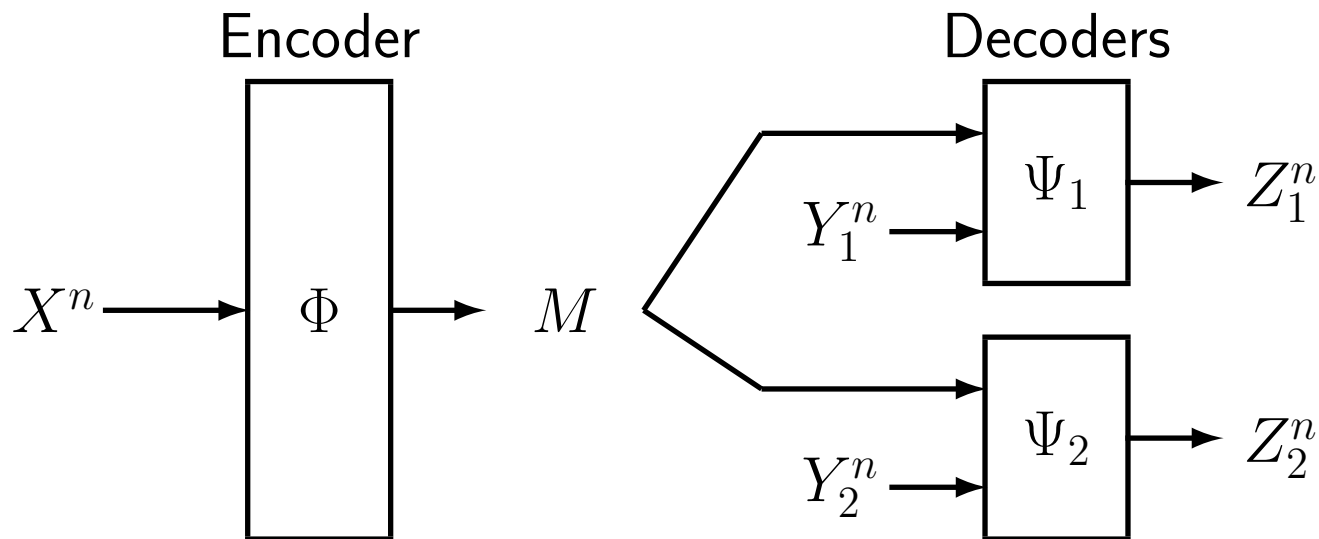
- Sources (X_1^n, X_2^n) are correlated.
- Encoder i transmits codeword M_i
- Decoder reproduces (Z_1^n, Z_2^n) .
- It is expected that $d_i(X_i^n, Z_i^n) \leq D_i$ with probability close to 1 for all i .

Multiple description coding [Witsenhausen,1980][Wolf-Wyner-Ziv,1980]



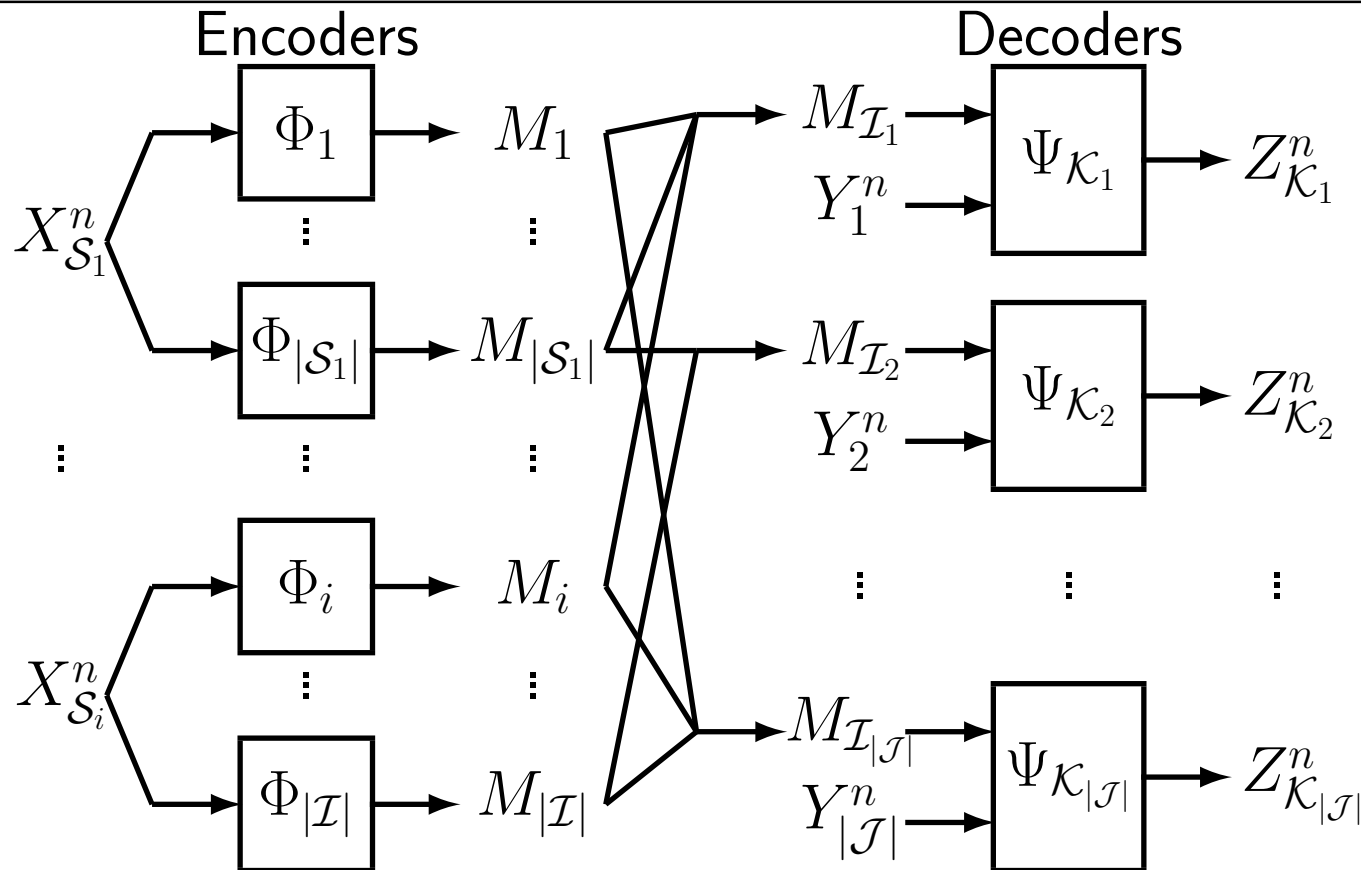
- Encoder i transmits codeword M_i
- Decoder i reproduces Z_1^n from codewords (M_i, M_{12}) .
- It is expected that $d_i(X_i^n, Z_i^n) \leq D_i$ with probability close to 1 for all i .

Source coding with side information at decoders [Heegard-Berger, 1985]



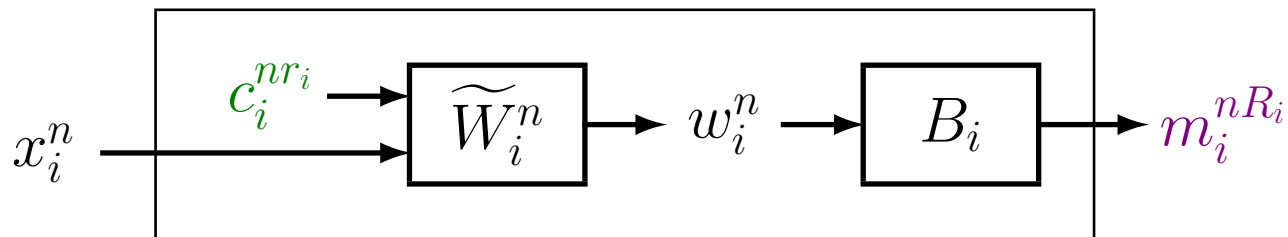
- Encoder i transmits codeword M_i
- Decoder i reproduces Z_i^n from codewords (M_i, M_{12}) .
- It is expected that $d_i(X^n, Z_i^n) \leq D_i$ with probability close to 1 for all i .

Multi-terminal source coding (unified extension) [M., 2025]



■ Optimal multi-letter region is derived in [M., 2025].

Construction of encoder

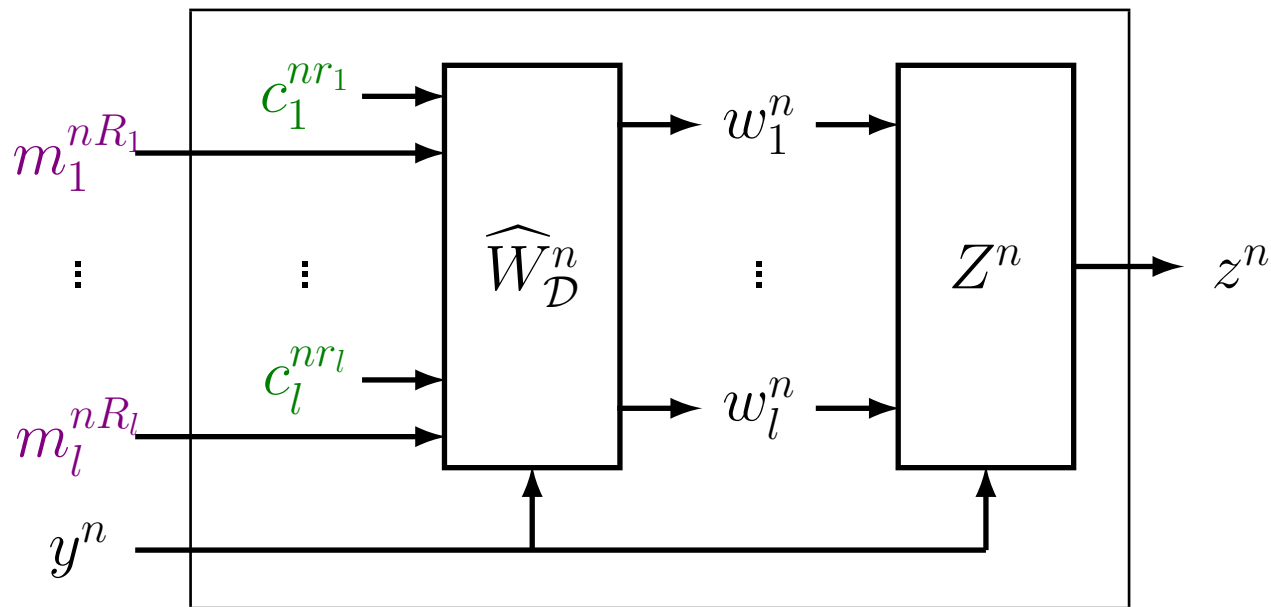


■ w_i^n satisfies $Aw_i^n = c_i^{nr_i}$ and $Bw_i^n = m_i^{nR_i}$.

■ It works when

$$r_i < H(W_i|X_i).$$

Construction of decoder

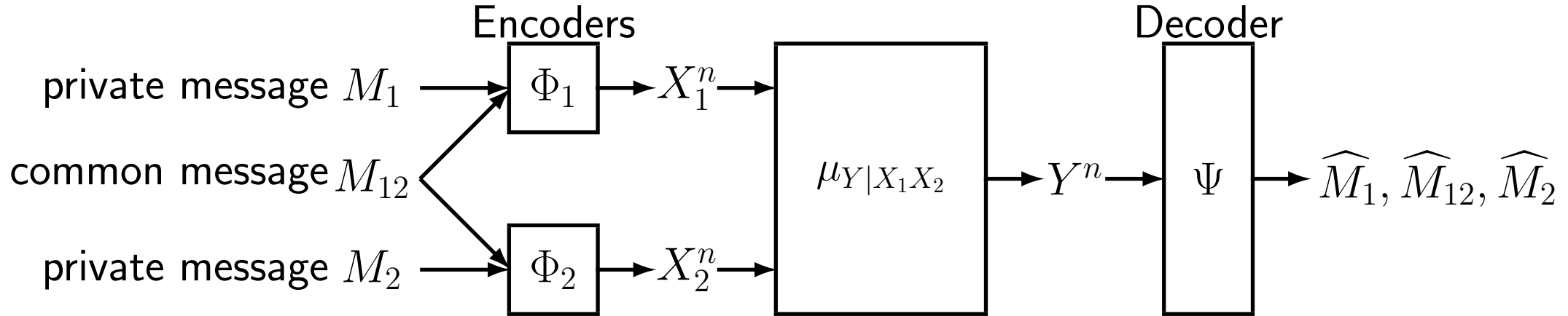


■ w_i^n satisfies $A_i w_i^n = c_i^{nr_i}$ and $B_i w_i^n = m_i^{nR_i}$.

■ It works when

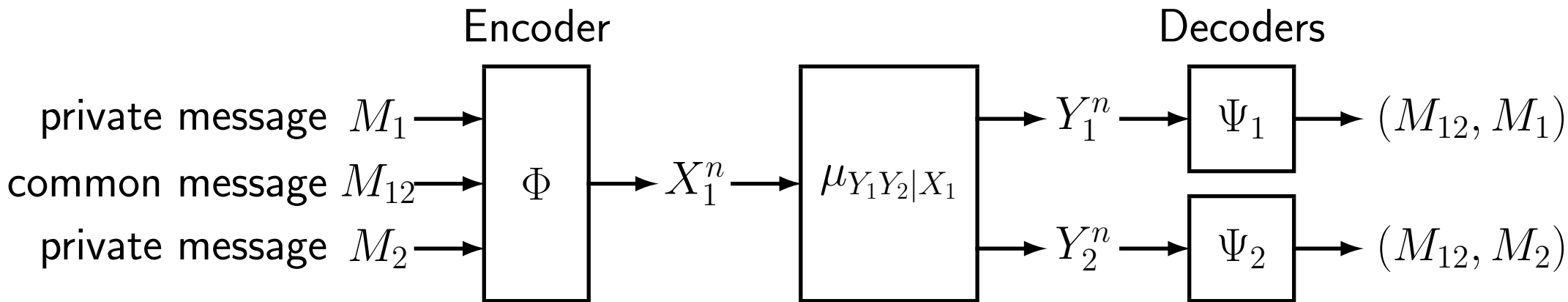
$$\sum_{i \in \mathcal{D}'} [r_i + R_i] > H(W_{\mathcal{D}'}^n | Y) \quad \text{for all } \mathcal{D}' \subset \mathcal{D}.$$

Multiple-access channel coding with common message [Slepian-Wolf,1973]



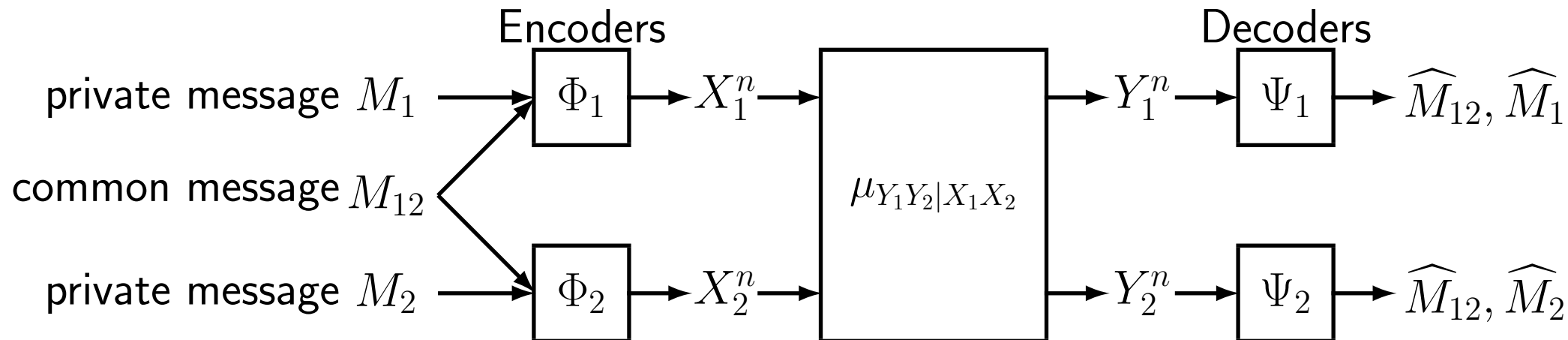
- Encoder i transmits X_i^n generated from a message pair (M_{12}, M_i) .
- Decoder j reproduces (M_{12}, M_j) with arbitrary small error probability.

Broadcast channel coding with common message [Marton,1979]



- Encoder i transmits X_i^n generated from message (M_{12}, M_1, M_2) .
- Decoder j reproduces (M_{12}, M_j) from Y_j^n with arbitrary small error probability.

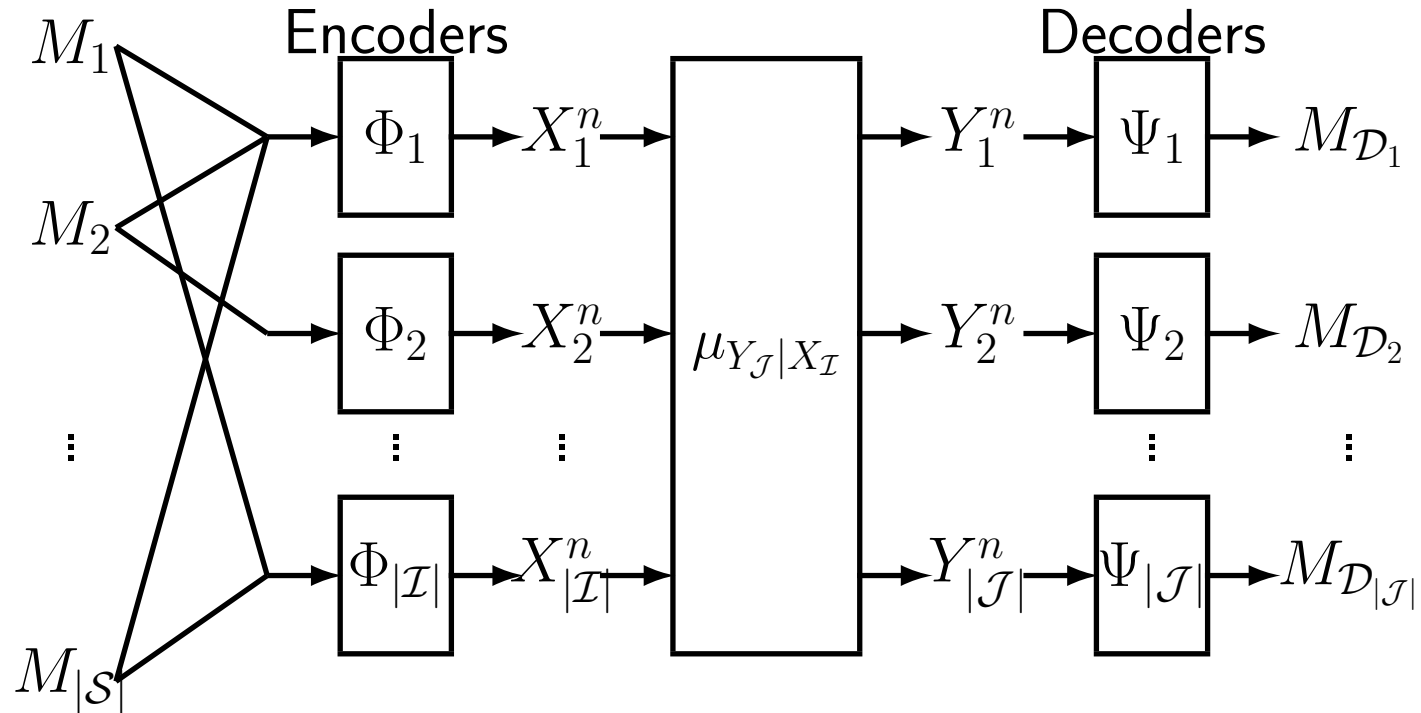
Interference channel coding with common message [Jiang-Xin-Garg,2008]



- Encoder i transmits X_i^n generated from a message pair (M_{12}, M_i) .
- Decoder j reproduces (M_{12}, M_j) with arbitrary small error probability.

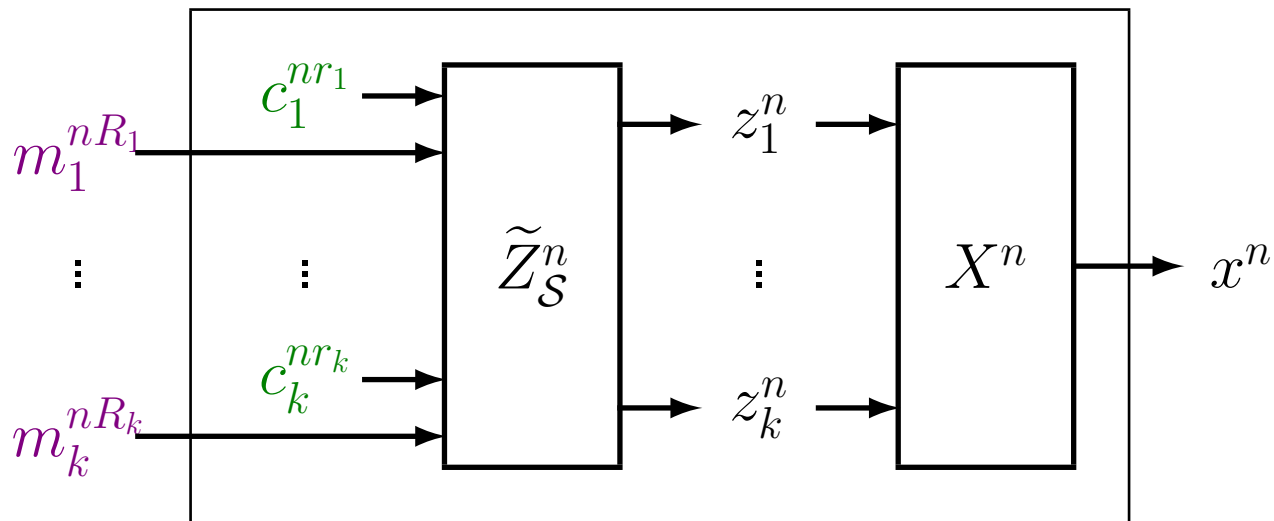
Multi-terminal channel coding (unified extension)

[Somekh-Baruch and Verdú, 2006][M.-Miyake, 2018][M., 2023]



■ Optimal multi-letter region is derived in [M., 2023].

Construction of encoder

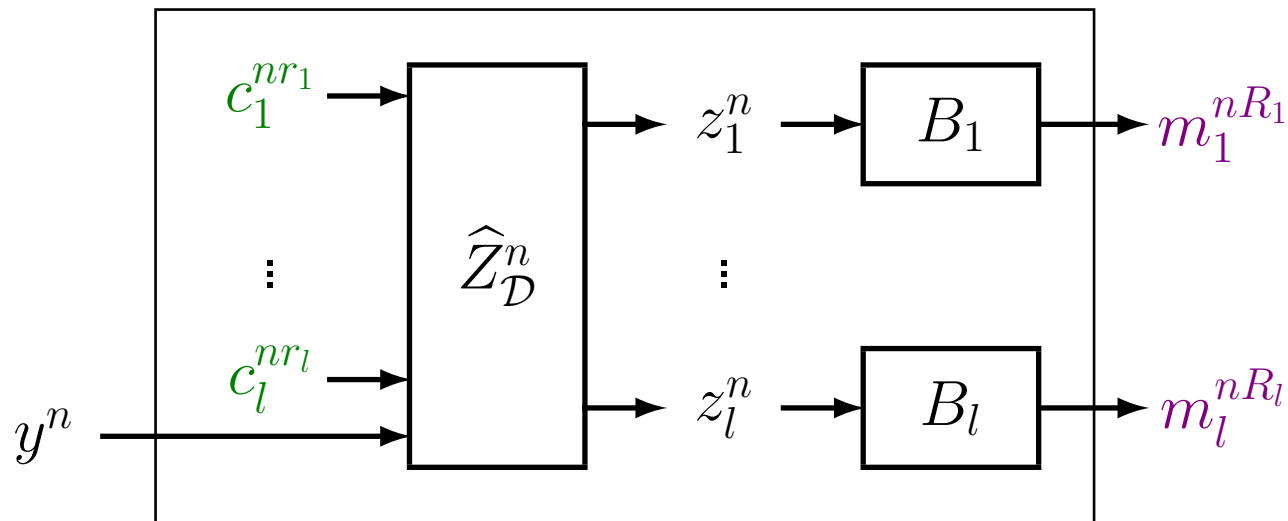


■ z_i^n satisfies $A_i z_i^n = c_i^{nr_i}$ and $B_i z_i^n = m_i^{nR_i}$.

■ It works when

$$\sum_{i \in \mathcal{S}'} [r_i + R_i] < H(Z_{\mathcal{S}'}^n) \quad \text{for all } \mathcal{S}' \subset \mathcal{S}.$$

Construction of decoder



■ z_i^n satisfies $A_i z_i^n = c_i^{nr_i}$ and $B_i z_i^n = m_i^{nR_i}$.

■ It works when

$$\sum_{i \in \mathcal{D}'} r_i > H(Z_{\mathcal{D}'} | Y) \quad \text{for all } \mathcal{D}' \subset \mathcal{D}.$$

Outline of this talk

- Our contributions
 - ▶ Constrained-random-number generator
- Lossy source code ← Review of YouTube lecture
 - ▶ Lossless source coding
 - ▶ Independent uniform random number generation
- Channel code ← Review of YouTube lecture
 - ▶ Lossless source coding with side information at decoder
 - ▶ Uniform random number generation
- Extensions to multi-terminal problems ← New
- Concluding remarks

Concluding remarks

- Lossy source code and channel code are constructed by using **constrained-random-number generators**.
 - ▶ A similar configuration using Polar code is shown in [Honda-Yamamoto, 2013].
- Simple and rigorous proof is given by using
 - ▶ Balanced coloring property [M.-Miyake, 2011]
 - ▶ Collision resistance property [M.-Miyake, 2010]of an ensemble satisfying **(α, β) -hash property** [M.-Miyake, 2010].
- Codes can be extended intuitively to relayless multi-terminal source/channel coding problems [M., 2023][M., 2025].
 - ▶ Codes can be constructed like assembling building blocks.
 - ▶ Single-letter region characterization is an open question.

Thank you for your kind attention.

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