Basins of Attraction of Phase Patterns in Pulse-Driven Star-Coupled Wien-Bridge Oscillators with Parameter Deviations

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Abstract—In this study, we investigate the basins of attraction of the phase patterns and clarify the effect of parameter deviations in pulse-driven star-coupled Wien-bridge oscillators with parameter deviations. From the simulation results, it is shown that some phase pattern can be seen easily and the others can be hardly seen because of the deviations. Such phenomena can separate the preferred patterns from undesirable patterns and it is convenient for the use of the system as some kinds of neural networks.

1. Introduction

There have been many investigations of mutual synchronization and multimode oscillation in coupled oscillators [1]–[4]. In particular, we have reported synchronization phenomena observed from \( N \) oscillators with the same natural frequency mutually coupled by one resistor [3, 4]. In LC oscillators systems, we have confirmed that natural frequency mutually coupled by one resistor [3, 4].

In such systems, it is considered that the symmetry of the system is collapsed by the parameter deviation. In this study, we calculate the basins of attraction of the phase patterns in the star-coupled Wien-bridge oscillators system stimulated by pulse using SPICE and clarify the effect of parameter deviations. To derive such basin structures can be the preparation for controlling the phase patterns in coupled oscillators systems by using these parameters as the control parameters.

2. Circuit Models

The circuit models are shown in Fig. 1. In this study, we propose the following two models.

Model 1 The switch unit is connected to Osc 4.

Model 2 The switch unit is connected to the coupling resistor \( r \).

In each model, the switch unit stimulates the star-coupled Wien-bridge oscillators. In this case, the switch closes \( \Delta t \) seconds in every \( T \) seconds, and the periodic pulse stimulation with period \( T \) is added to the system. \( T \) should be sufficiently large to achieve the synchronization within the period. The construction of the subcircuits is shown in Fig. 1 (c).

In both systems, the parameter deviations are provided by the different capacitance in each \( C_1 \sim C_4 \). The capacitance \( C_k \) is described as follows,

\[
C_k = C + (k - 1) \Delta C
\]

where \( C \) is the capacitance of the capacitor in subcircuit and \( \Delta C \) is the deviation parameter. If \( \Delta C \) is larger, the difference of the natural frequency of each oscillator becomes larger.
Figure 1: Circuit models. (a) Switch unit is connected to an oscillator (Model 1). (b) Switch unit is connected to the coupling resistor (Model 2). (c) Construction of subcircuit. (d) Schematic of the system.

3. Simulation Results

In this section, we show the simulation results in proposed models by standard circuit simulator package SPICE. In this study, we use the following circuit parameters: $R = 10\,k\Omega$, $C = 0.015\mu F$, $r = 200\Omega$, $R_f = 14.7\,k\Omega$, $R_i = 4.7\,k\Omega$, $\Delta t = 50\mu sec$, $T = 100\,msec$. In the following results, $A$, $B$ and $C$ indicate in-phase, $+120^\circ$ and $-120^\circ$ phase shift with respect to the phase of Osc 1, respectively.

Figures 2 and 3 show the results for Model 1 and Figures 4 and 5 show the results for Model 2 when $\Delta C = 10^{-3}\mu F$. In these models, we can see successive phase pattern switching of the multiple oscillators due to the parameter deviations. From the results, it is shown that the in-phase synchronization of Osc 1 and Osc 4 is hardly seen. It is considered that this is because the natural frequencies of these oscillators are more different than the other combinations of the oscillators. Such parameter deviations make the system asymmetric and they affect the system dynamics and the derived phase patterns.

Next, we show the precise results of phase pattern...
switching. In the following results, the possible $27 = 3^{4-1}$ phase patterns are indicated as the notation from \( a \) to \( z \) and \( IP \) as shown in Fig. 6. In this case, \( \Delta C = 10^{-3}\mu F \) and the single pulse (i.e., not periodic) is added to the system. Tables 1 and 2 show the phase patterns when the timing of the pulse and pulse voltage are changed. In these cases, the phase pattern before adding the pulse is \( w \). In both models, not so many patterns appear after the pulses are added. As stated in Table 1, for Model 1, only three patterns \( v, w, \) and \( y \) are seen. In particular, pattern \( y \) is more frequently seen after the switching from \( w \) than the other patterns. In this case, note that only the phases of Osc 3 and Osc 4 change after stimulation.

On the other hand, in Model 2, the phase pattern diagram is different from one of Model 1. In particular, there are some cases where the phase of Osc 2 is changed to \( A \) or \( C \). It suggest that the pulses affect the system more globally in Model 2. Note that the pattern \( y \) is much more frequently seen than the other patterns. From these results, the pattern \( y \) is more stable than the other patterns because of the parameter deviations. Therefore, it can be considered that the
deviation of the parameter can be the control parameter of the phase patterns.

4. Conclusions

In this paper, we show the frequency of appearance of the phase patterns in pulse-driven star-coupled Wien-bridge oscillators with the parameter deviations. The symmetry of the system is collapsed by such parameter deviations, and they affect the system dynamics and the phase patterns. From the results, it is shown that some phase pattern can be seen easily and that some phase patterns can be hardly seen because of the deviations. Such phenomena can separate the preferred patterns from undesirable patterns and it is convenient for the use of the system as some kinds of neural networks and associative memories.

References


