カオスと情報
-IoTからIoBへ-

IEICE 第8回　Net Science研究会＠関西学院大学梅田キャンパス

京都大学大学院 情報学研究科
梅野 健

* カオスは計算できるのか？

* カオスで計算できるのか？

本研究の動機

* カオスと情報、常に関係ある。

傍証. ●R. Shaw's Information Flow (1980),

●Ruelle's 不等式 ($\lambda \geqq h_{KS}$), Pesin's等式($\lambda = h_{KS}$)

カオスと情報, cf. 式部マップ(「カオスとその周辺」研究会)

cf. 「複雑系研究会」と「NetScience研究会」

* 脳の中のカオスの役割

—CDMA型脳Project(1997@理研)—

→ Outputs:可解カオス,カオスモンテカルロ(カオス揺らぎの一般(スペクトル)理論)

* カオスを信号として用いるカオス通信

—脳型CDMA通信Project(1998@CRL)—

実環境での通信の実験から逆に使える通信用カオス符号を学ぶ (1998-2012)

* IoT(Internet of Things)には, Thingsからの情報を伝える通信方法のアイデアが欠落している。
(2012-現在@KU)

* IoB(Internet of Brains):究極の脳内カオスコードのカオス通信コンセプト

(2014-現在実験・解析中@KU)

The most important moment for communication and original motivation of my research through 1998-2014 (a little bit sharpen before 1998)

* A connection establishment instant such as key agreement, code acquisition, synchronization of frequency, … ..e.t.c. is vitally important.
* Is chaos and complexity effective for such a practical communication/computation purpose?
* Investigate Brain function issue such as the binding problem through practical CDMA(code division multiple access)/OFDM(orthogonal frequency division

multiplexing) and their modified modulation shceme with "chaos code" (my original motivation since 1997).

最近の研究(with 大久保(D1)):
Generalized Boole Transformation

$$X_{n+1} = \alpha X_n - \frac{\beta}{X_n},\ X \in \mathbb{R},\ 0 < \alpha < 1,\ 0 < \beta$$

Invariant Measure of the generalized Boole Transformations

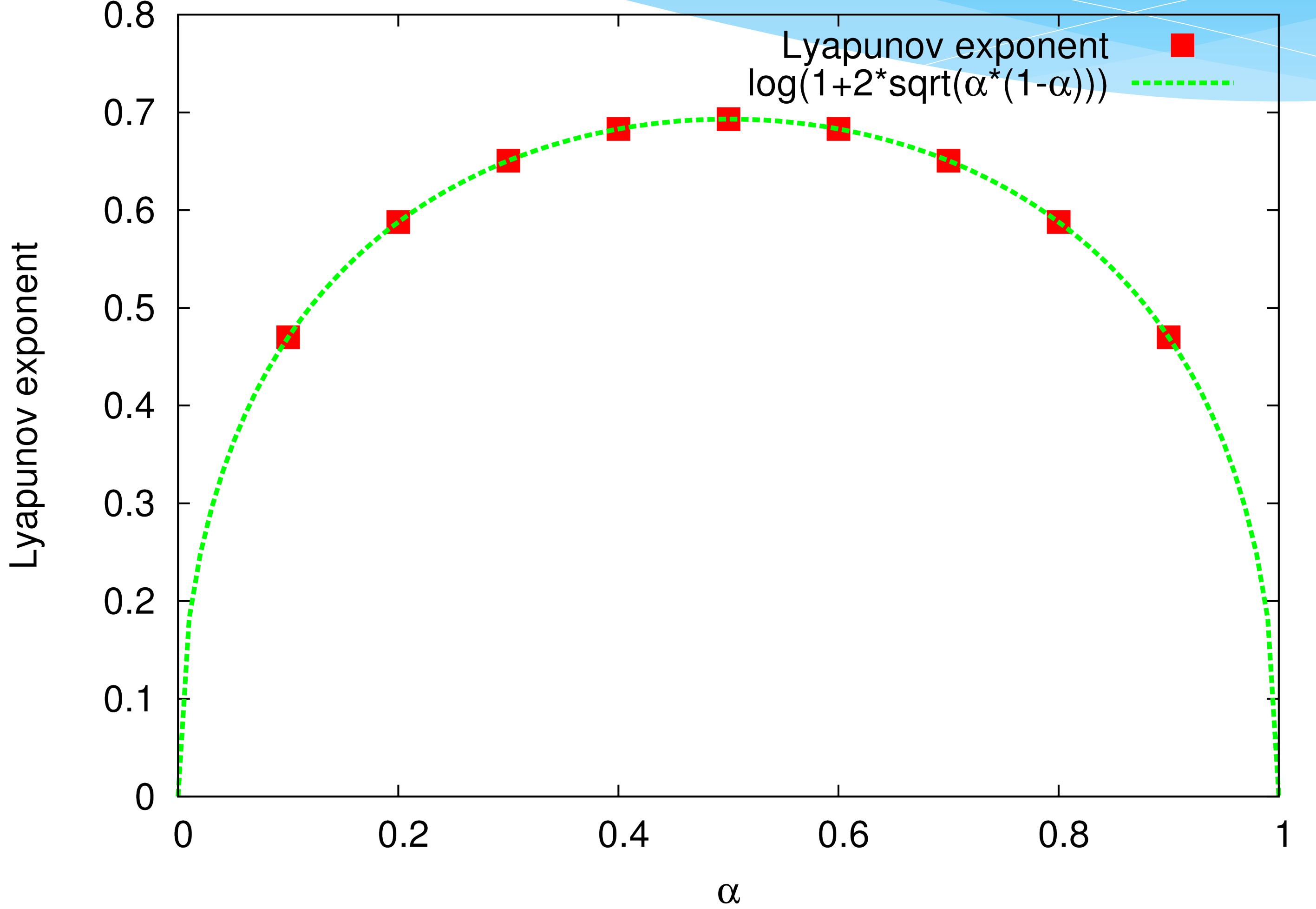
$$d\mu(x) = \frac{\sqrt{\beta(1-\alpha)}}{x^2(1-\alpha)+\beta}dx$$

Lyapunov Exponent Analytically Obtained Recently (2014)! (I proposed, Mr. Okubo proved.)

$$\lambda(\alpha) = \log(1 + 2\sqrt{\alpha(1-\alpha)})$$

更に、ルベーグ測度に絶対連続なエルゴード的不変測度を持つ系に対する
Pesinの等式により、コルモゴルフ＝シナイエントロピー＝λ(α)

Lyapunov Exponent (Simulations and Analytical Formula, K. Okubo) in 2014



Maximum of Lyapunov Exponent

* λ attained a maximum value at $\alpha=1/2$.
  $\lambda(\alpha)=\text{Log } 2$ where $|d\lambda/d\alpha| = 0$.

* The system with the maximum Lyapunov exponent is nothing but the solvable chaos whose map is defined by duplication formula of cotangent function given by [1]

$$Y=1/2(x-1/x)$$

[1] KU, "Superposition of chaotic processes with Levy's stable law", Physical Review E,vol. 58(1998).

Variational Principle
for Dynamical Systems F

- This suggests a variational principle for Lyapunov exponent $\lambda[F]$ with respect to a parameter of dynamical system F.
- Relation with thermodynamic formalism for physical measure of dynamical system developed by D. Ruelle.

Infinite Response of Parameter Dependence of Lyapunov Exponent.

* $|D\lambda/d\alpha|_{\alpha=1,\text{or } 0} = \infty$ at the onset of chaos ($\lambda=0$)
* Extremely difficult to compute Lyapunov exponent near the case of $\alpha \approx 1$, original Boole transformation

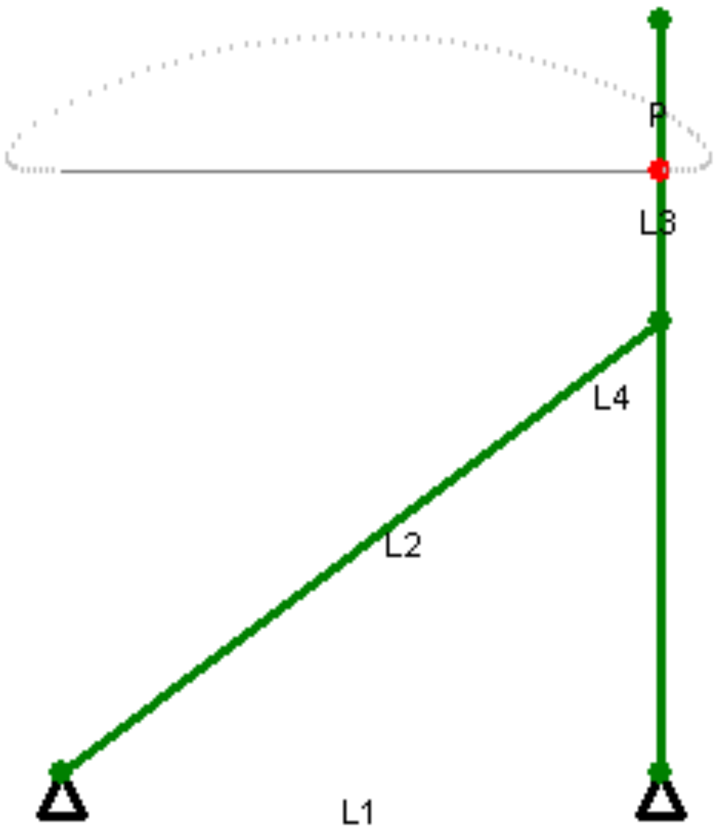
  $Y=X-1/X$.
* This seems to be a typical case of complexity in a simple map.

By a transformation: *cotan* θ→*cos* θ,

- We can get a second-order Chebyshev polynomial as

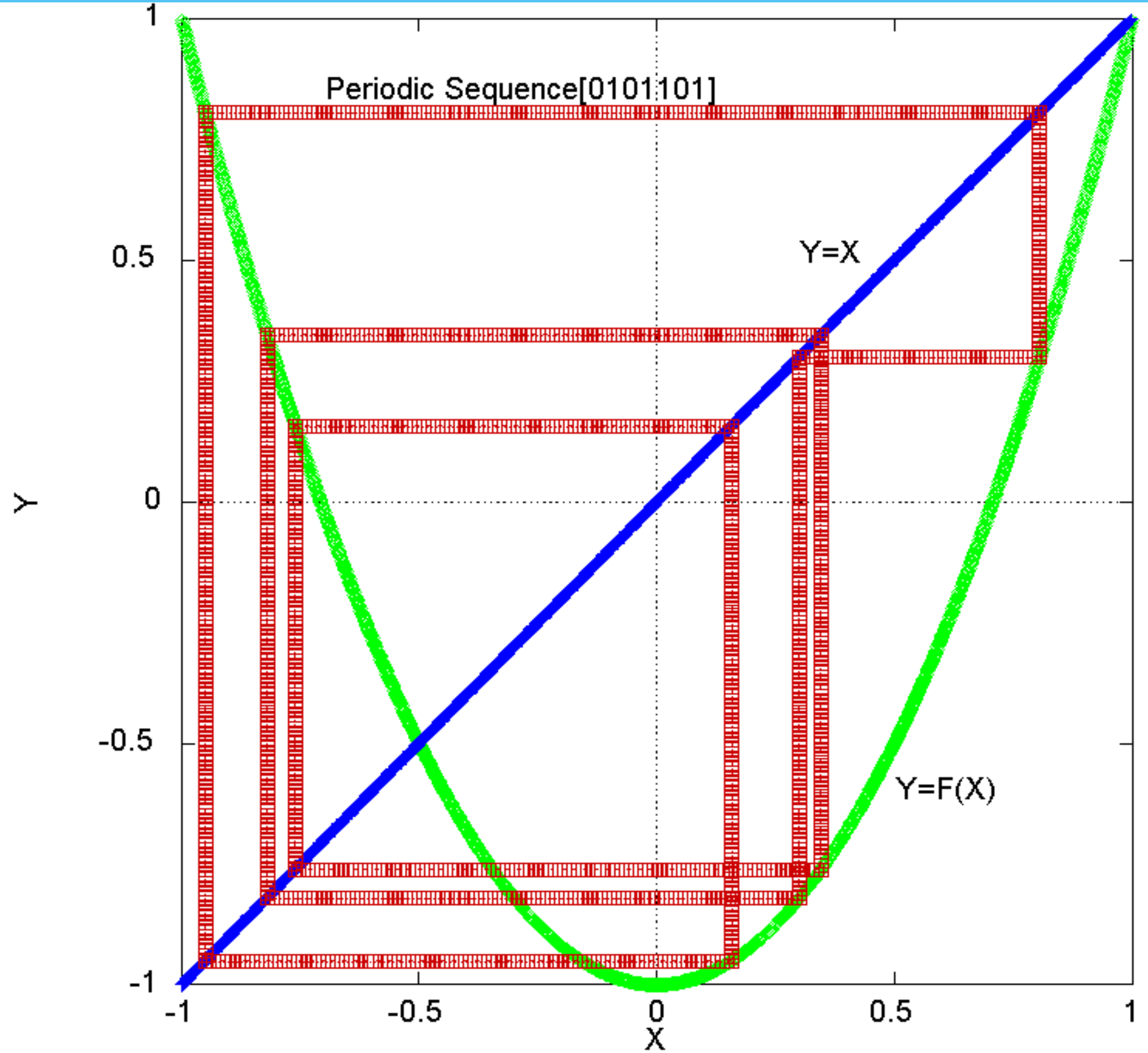
$$Y=2X^2 -1.$$

Researh on Chebyshev Links (for more than 30 years in 19th century) that produce energy from entropy (randomness) → Chebyshev Polynomials

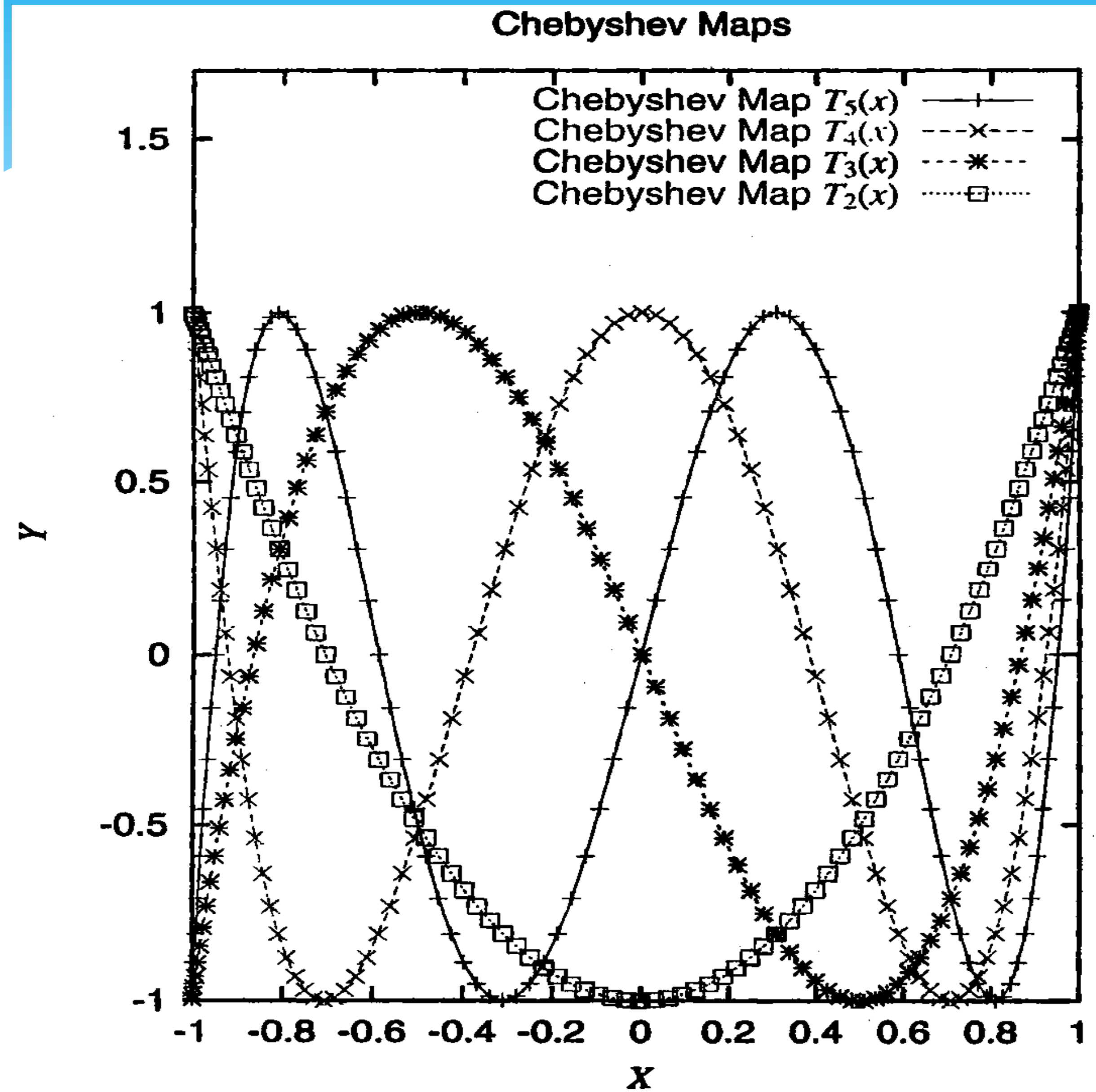


Simple Chaos Code (Chebyshev Polynomial) represented by $Y=2X^2-1$ as $X=\cos(a) \rightarrow Y=\cos(2a)$.

From, K. Umeno and K-I. Kitayama, Electroncis Letters (1999)



Chebyshev Maps as Generator F.



Addition Formula:

$T(a, \cos\theta) = \cos(a\theta)$

$T(0, x) = 1$

$T(1, x) = x$

$T(2, x) = 2x^2 - 1$

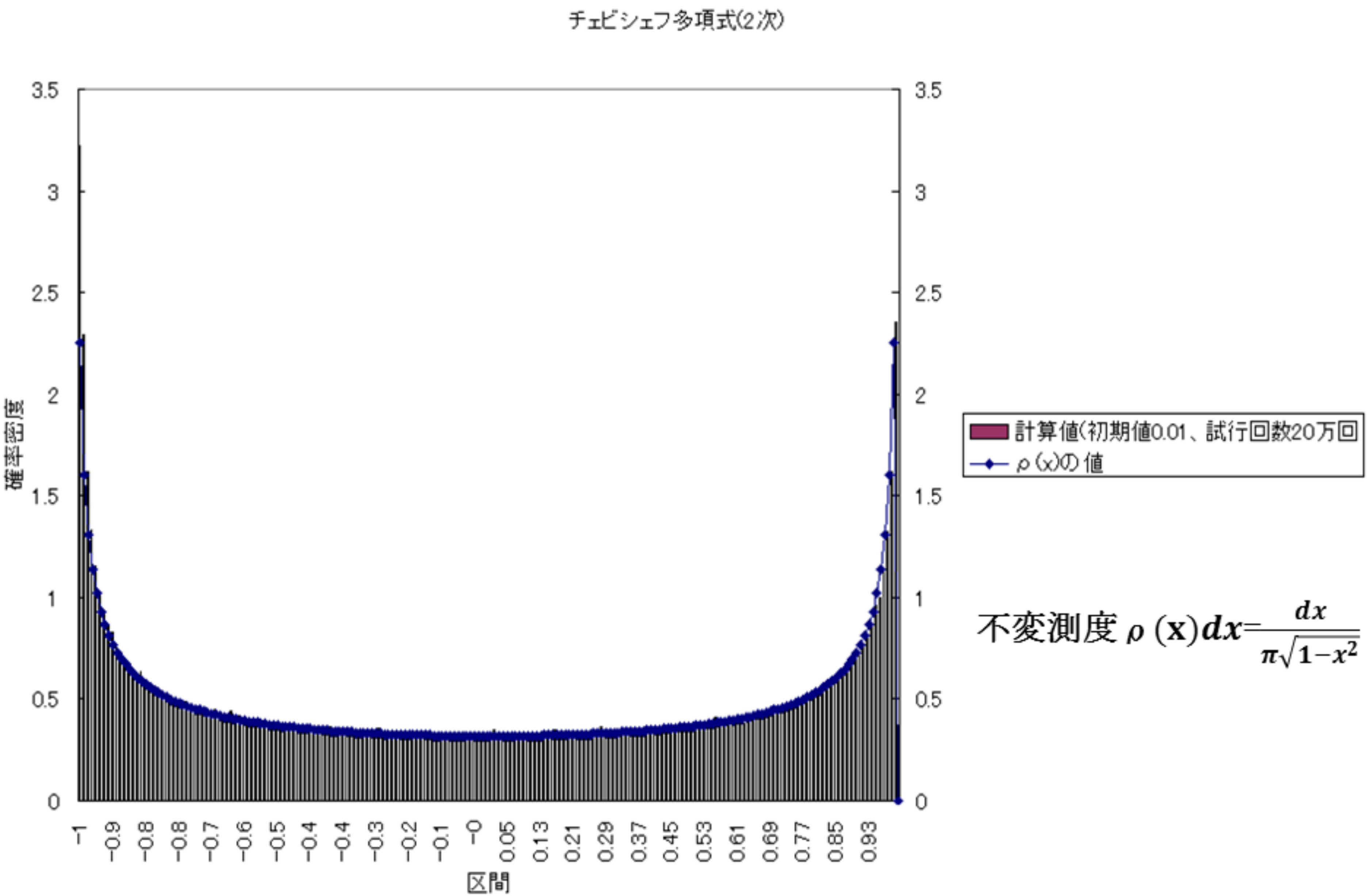
$T(3, x) = 4x^3 - 3x$

Mixing Property:

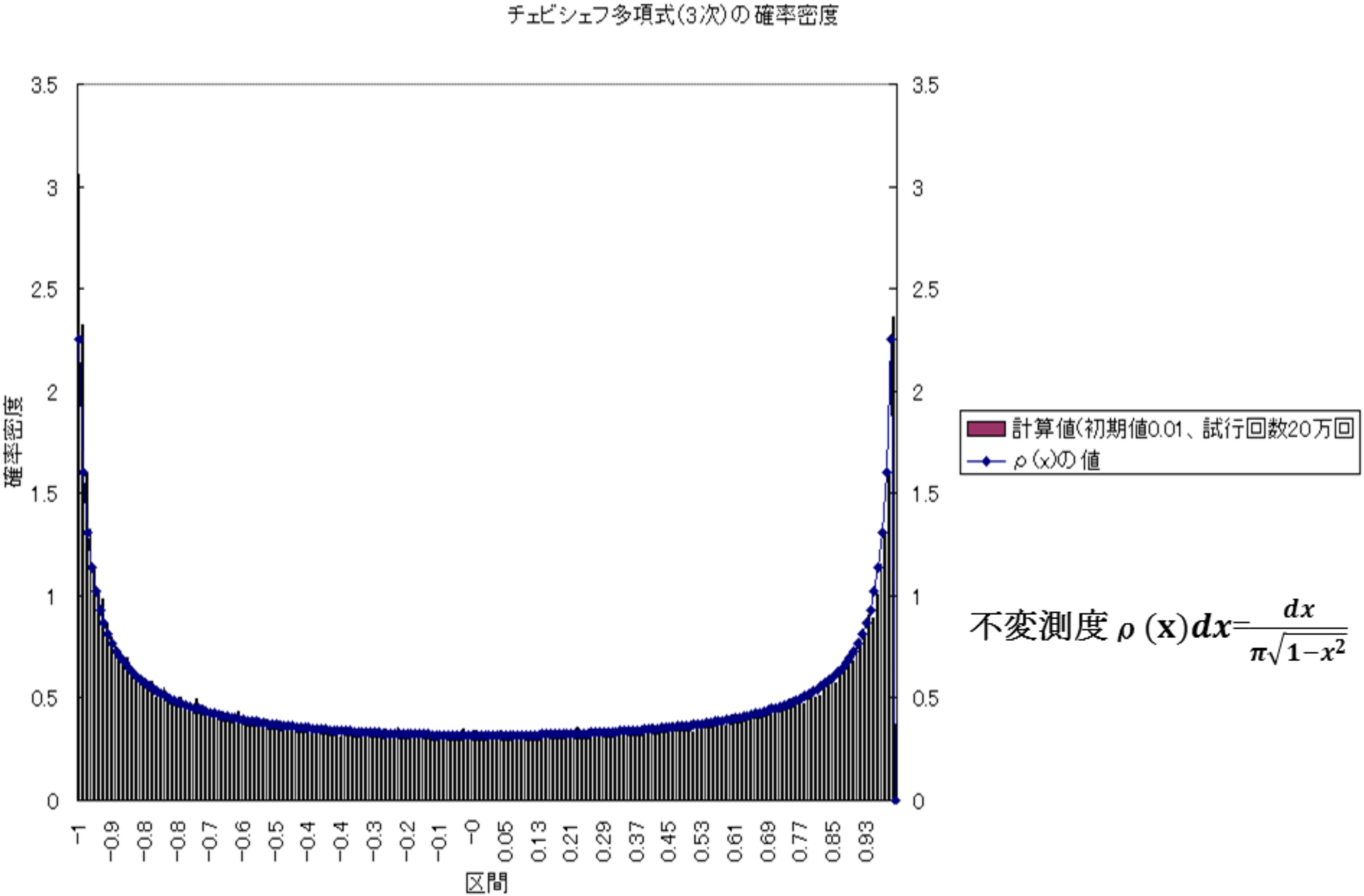
$x_{n+1} = T(x_n)$

$\langle A(x_0), B(x_n) \rangle \rightarrow \langle A \rangle \langle B \rangle$ for $n \rightarrow \infty$.

Mixing→Ergodicity



不変測度 $\rho\,(\mathbf{x})dx = \dfrac{dx}{\pi\sqrt{1-x^2}}$



不変測度 $\rho(\mathbf{x})dx = \dfrac{dx}{\pi\sqrt{1-x^2}}$

Examples of Exactly Solvable Chaos

(1)1947-S. Ulam and J.von Neumann, Bull.Am.Math.Soc.USA Vol.53 Y=4X(1-X) --- Logistic Duplication of $\sin^2(a)$.

$$X = \sin^2 a\,,\ Y = \sin^2 2a\,,\ \rightarrow\ Y = 4\sin^2 a \cos^2 a = 4X(1-X)\,,$$

(2)1964- Adler and Rivlin, Proc. Am. Math. Soc. vol. 15

Chebyshev polynomials $Y=T_m(X)$ Multiplication of cos(a)

(3)1985 Katsura and Fukuda, Physica A Duplication formula of Jacobian elliptic function (no expression of ρ(x))

(4)1997 –Umeno Phys. Rev. E vol. 55 Addition theorems of elliptic functions and a relation between ρ(x) and its elliptic integrals

(5)1998 –Umeno Phys. Rev. E vol. 58 Multiplication of $\tan^{-\alpha}$ generating general Levy's stable laws ρ(x) including the Cauchy distribution

(6)2005 Grammaticos, Ramani and Viallet, Physics Letters A Reversible maps generalization of (4) (no expression of ρ(x))

(7)2008 Kajiwara, Ultradiscretization of (4) (no expression of ρ(x))

Generalized Formula for $F_3(x)$ with complex $l, m = \bar{l}$

$$
\begin{aligned}
&-\Big(x\,\big(-3 + 4x + 8 r x \,\text{Cos}[a] - 12 r x^2 \,\text{Cos}[a] - 6 r^2 x^2 \,\text{Cos}[a]^2 + 12 r^2 x^3 \,\text{Cos}[a]^2 - 4 r^3 x^4 \,\text{Cos}[a]^3 + \\
&\qquad r^4 x^4 \,\text{Cos}[a]^4 - 6 r^2 x^2 \,\text{Sin}[a]^2 + 12 r^2 x^3 \,\text{Sin}[a]^2 - 4 r^2 x^4 \,\text{Sin}[a]^2 - \\
&\qquad 4 r^3 x^4 \,\text{Cos}[a]\,\text{Sin}[a]^2 + 2 r^4 x^4 \,\text{Cos}[a]^2\,\text{Sin}[a]^2 + r^4 x^4 \,\text{Sin}[a]^4\big)^2\Big) \Big/ \\
&\quad \big(-1 + 24 r x^2 \,\text{Cos}[a] - 16 r x^3 \,\text{Cos}[a] + 12 r^2 x^2 \,\text{Cos}[a]^2 - 136 r^2 x^3 \,\text{Cos}[a]^2 + \\
&\quad 96 r^2 x^4 \,\text{Cos}[a]^2 - 16 r^3 x^3 \,\text{Cos}[a]^3 + 312 r^3 x^4 \,\text{Cos}[a]^3 - 240 r^3 x^5 \,\text{Cos}[a]^3 - \\
&\quad 30 r^4 x^4 \,\text{Cos}[a]^4 - 360 r^4 x^5 \,\text{Cos}[a]^4 + 320 r^4 x^6 \,\text{Cos}[a]^4 + 96 r^5 x^5 \,\text{Cos}[a]^5 + \\
&\quad 200 r^5 x^6 \,\text{Cos}[a]^5 - 240 r^5 x^7 \,\text{Cos}[a]^5 - 100 r^6 x^6 \,\text{Cos}[a]^6 - 24 r^6 x^7 \,\text{Cos}[a]^6 + \\
&\quad 96 r^6 x^8 \,\text{Cos}[a]^6 + 48 r^7 x^7 \,\text{Cos}[a]^7 - 24 r^7 x^8 \,\text{Cos}[a]^7 - 16 r^7 x^9 \,\text{Cos}[a]^7 - 9 r^8 x^8 \,\text{Cos}[a]^8 + \\
&\quad 8 r^8 x^9 \,\text{Cos}[a]^8 + 12 r^2 x^2 \,\text{Sin}[a]^2 - 104 r^2 x^3 \,\text{Sin}[a]^2 + 216 r^2 x^4 \,\text{Sin}[a]^2 - \\
&\quad 192 r^2 x^5 \,\text{Sin}[a]^2 + 64 r^2 x^6 \,\text{Sin}[a]^2 - 16 r^3 x^3 \,\text{Cos}[a]\,\text{Sin}[a]^2 + 312 r^3 x^4 \,\text{Cos}[a]\,\text{Sin}[a]^2 - \\
&\quad 624 r^3 x^5 \,\text{Cos}[a]\,\text{Sin}[a]^2 + 544 r^3 x^6 \,\text{Cos}[a]\,\text{Sin}[a]^2 - 192 r^3 x^7 \,\text{Cos}[a]\,\text{Sin}[a]^2 - \\
&\quad 60 r^4 x^4 \,\text{Cos}[a]^2\,\text{Sin}[a]^2 - 336 r^4 x^5 \,\text{Cos}[a]^2\,\text{Sin}[a]^2 + 624 r^4 x^6 \,\text{Cos}[a]^2\,\text{Sin}[a]^2 - \\
&\quad 480 r^4 x^7 \,\text{Cos}[a]^2\,\text{Sin}[a]^2 + 192 r^4 x^8 \,\text{Cos}[a]^2\,\text{Sin}[a]^2 + 192 r^5 x^5 \,\text{Cos}[a]^3\,\text{Sin}[a]^2 + \\
&\quad 144 r^5 x^6 \,\text{Cos}[a]^3\,\text{Sin}[a]^2 - 288 r^5 x^7 \,\text{Cos}[a]^3\,\text{Sin}[a]^2 + 96 r^5 x^8 \,\text{Cos}[a]^3\,\text{Sin}[a]^2 - \\
&\quad 64 r^5 x^9 \,\text{Cos}[a]^3\,\text{Sin}[a]^2 - 236 r^6 x^6 \,\text{Cos}[a]^4\,\text{Sin}[a]^2 + 24 r^6 x^7 \,\text{Cos}[a]^4\,\text{Sin}[a]^2 + \\
&\quad 120 r^6 x^8 \,\text{Cos}[a]^4\,\text{Sin}[a]^2 + 32 r^6 x^9 \,\text{Cos}[a]^4\,\text{Sin}[a]^2 + 144 r^7 x^7 \,\text{Cos}[a]^5\,\text{Sin}[a]^2 - \\
&\quad 72 r^7 x^8 \,\text{Cos}[a]^5\,\text{Sin}[a]^2 - 48 r^7 x^9 \,\text{Cos}[a]^5\,\text{Sin}[a]^2 - 36 r^8 x^8 \,\text{Cos}[a]^6\,\text{Sin}[a]^2 + \\
&\quad 32 r^8 x^9 \,\text{Cos}[a]^6\,\text{Sin}[a]^2 - 30 r^4 x^4 \,\text{Sin}[a]^4 + 24 r^4 x^5 \,\text{Sin}[a]^4 + 48 r^4 x^6 \,\text{Sin}[a]^4 - \\
&\quad 96 r^4 x^7 \,\text{Sin}[a]^4 + 48 r^4 x^8 \,\text{Sin}[a]^4 + 96 r^5 x^5 \,\text{Cos}[a]\,\text{Sin}[a]^4 - 56 r^5 x^6 \,\text{Cos}[a]\,\text{Sin}[a]^4 - \\
&\quad 48 r^5 x^7 \,\text{Cos}[a]\,\text{Sin}[a]^4 + 96 r^5 x^8 \,\text{Cos}[a]\,\text{Sin}[a]^4 - 64 r^5 x^9 \,\text{Cos}[a]\,\text{Sin}[a]^4 - \\
&\quad 172 r^6 x^6 \,\text{Cos}[a]^2\,\text{Sin}[a]^4 + 120 r^6 x^7 \,\text{Cos}[a]^2\,\text{Sin}[a]^4 - 48 r^6 x^8 \,\text{Cos}[a]^2\,\text{Sin}[a]^4 + \\
&\quad 64 r^6 x^9 \,\text{Cos}[a]^2\,\text{Sin}[a]^4 + 144 r^7 x^7 \,\text{Cos}[a]^3\,\text{Sin}[a]^4 - 72 r^7 x^8 \,\text{Cos}[a]^3\,\text{Sin}[a]^4 - \\
&\quad 48 r^7 x^9 \,\text{Cos}[a]^3\,\text{Sin}[a]^4 - 54 r^8 x^8 \,\text{Cos}[a]^4\,\text{Sin}[a]^4 + 48 r^8 x^9 \,\text{Cos}[a]^4\,\text{Sin}[a]^4 - \\
&\quad 36 r^6 x^6 \,\text{Sin}[a]^6 + 72 r^6 x^7 \,\text{Sin}[a]^6 - 72 r^6 x^8 \,\text{Sin}[a]^6 + 32 r^6 x^9 \,\text{Sin}[a]^6 + \\
&\quad 48 r^7 x^7 \,\text{Cos}[a]\,\text{Sin}[a]^6 - 24 r^7 x^8 \,\text{Cos}[a]\,\text{Sin}[a]^6 - 16 r^7 x^9 \,\text{Cos}[a]\,\text{Sin}[a]^6 - \\
&\quad 36 r^8 x^8 \,\text{Cos}[a]^2\,\text{Sin}[a]^6 + 32 r^8 x^9 \,\text{Cos}[a]^2\,\text{Sin}[a]^6 - 9 r^8 x^8 \,\text{Sin}[a]^8 + 8 r^8 x^9 \,\text{Sin}[a]^8\big)
\end{aligned}
$$

Real shape of chaotic maps (recent ones)

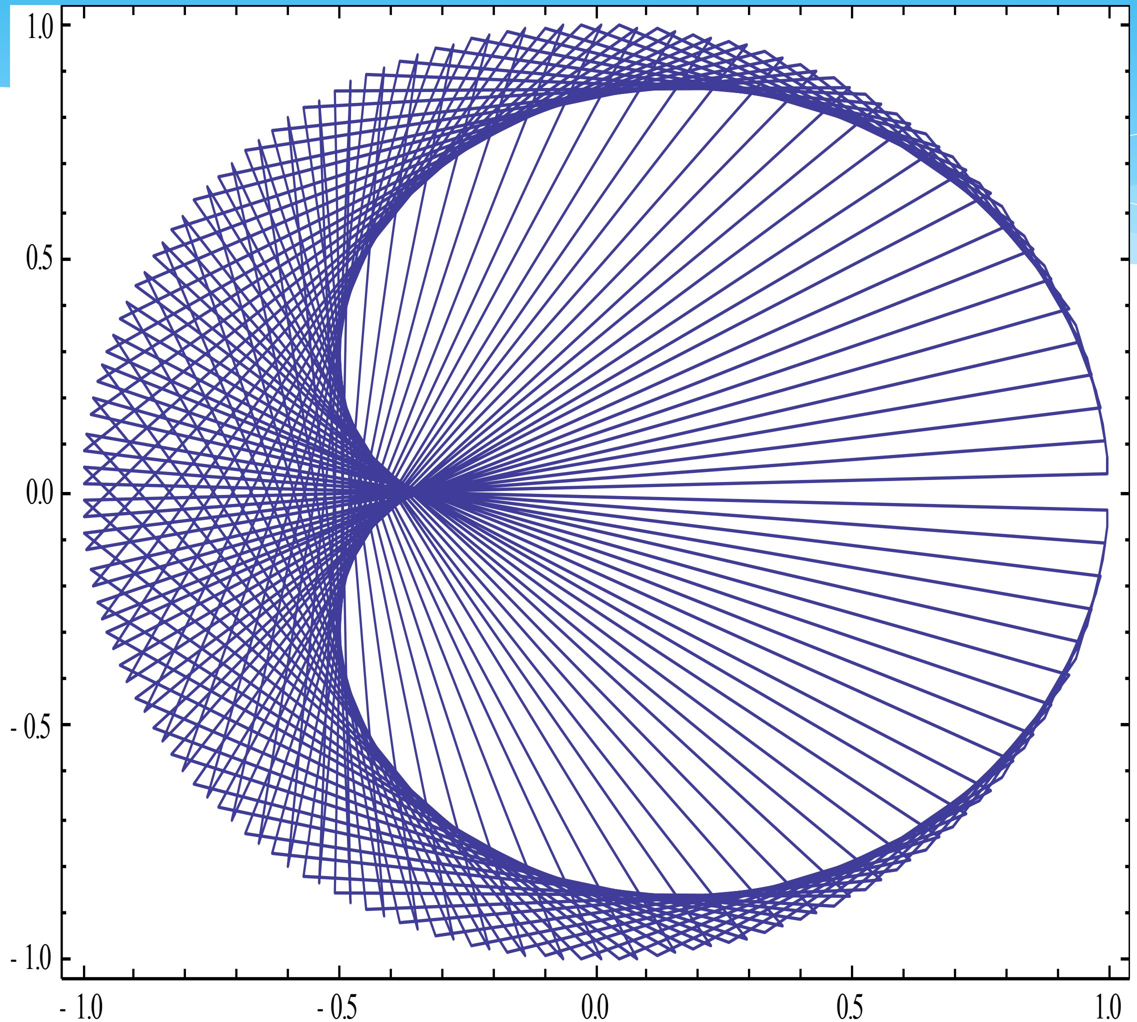
The first communication experiment with Chebyshev maps in 2004 was unsuccessful.

- Was not successful.

- But after adding the constant power preservation law condition with 2D chaotic codes, we can finally obtain communication link with much better quality (this research is under for 5G with chaos.)

(Chaos Codes with Constant Power、KU08)

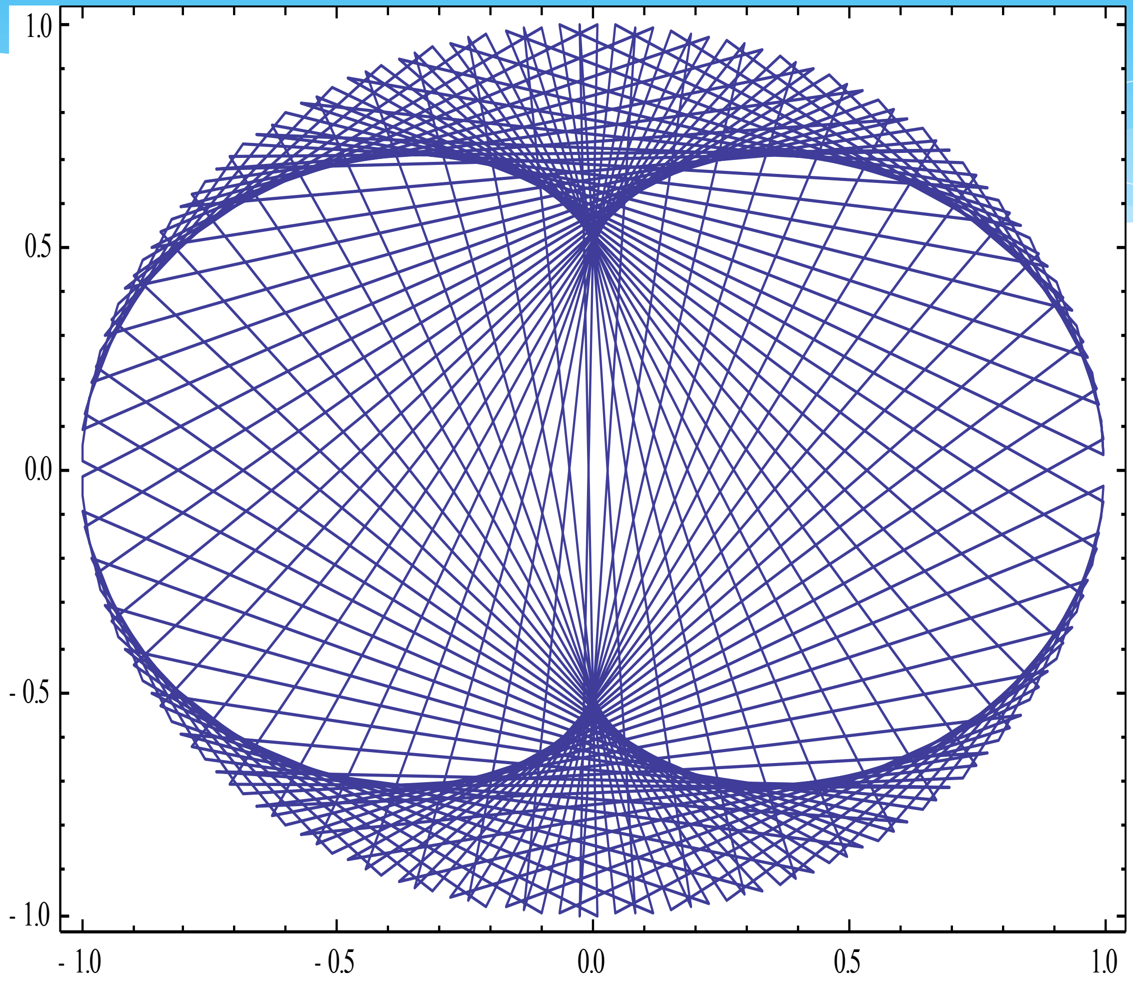
*



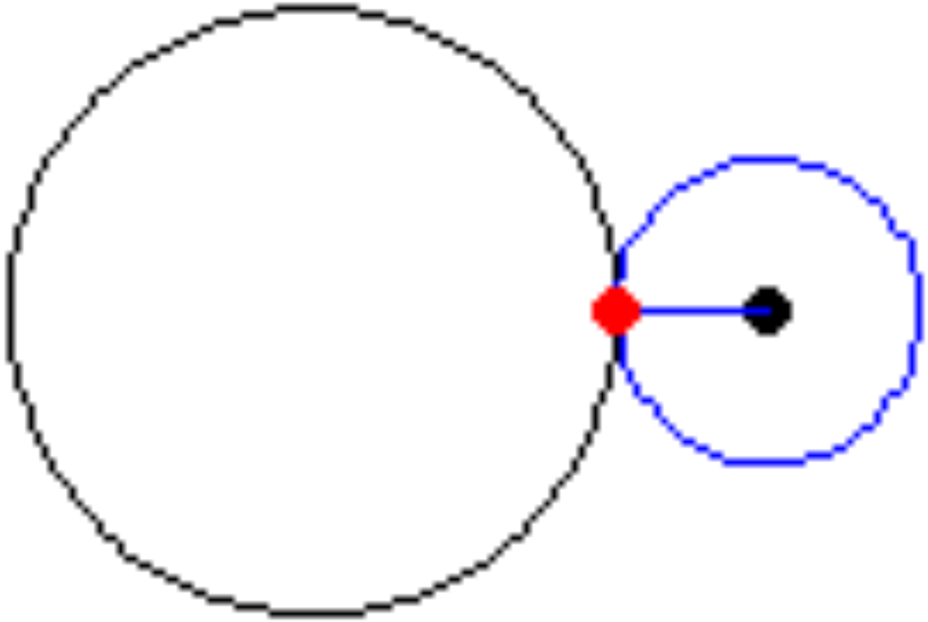
p（Prime Number ）=173,q(Primitive root)=2

Another Chaos Code Pattern: Nephroid (KU08)

*



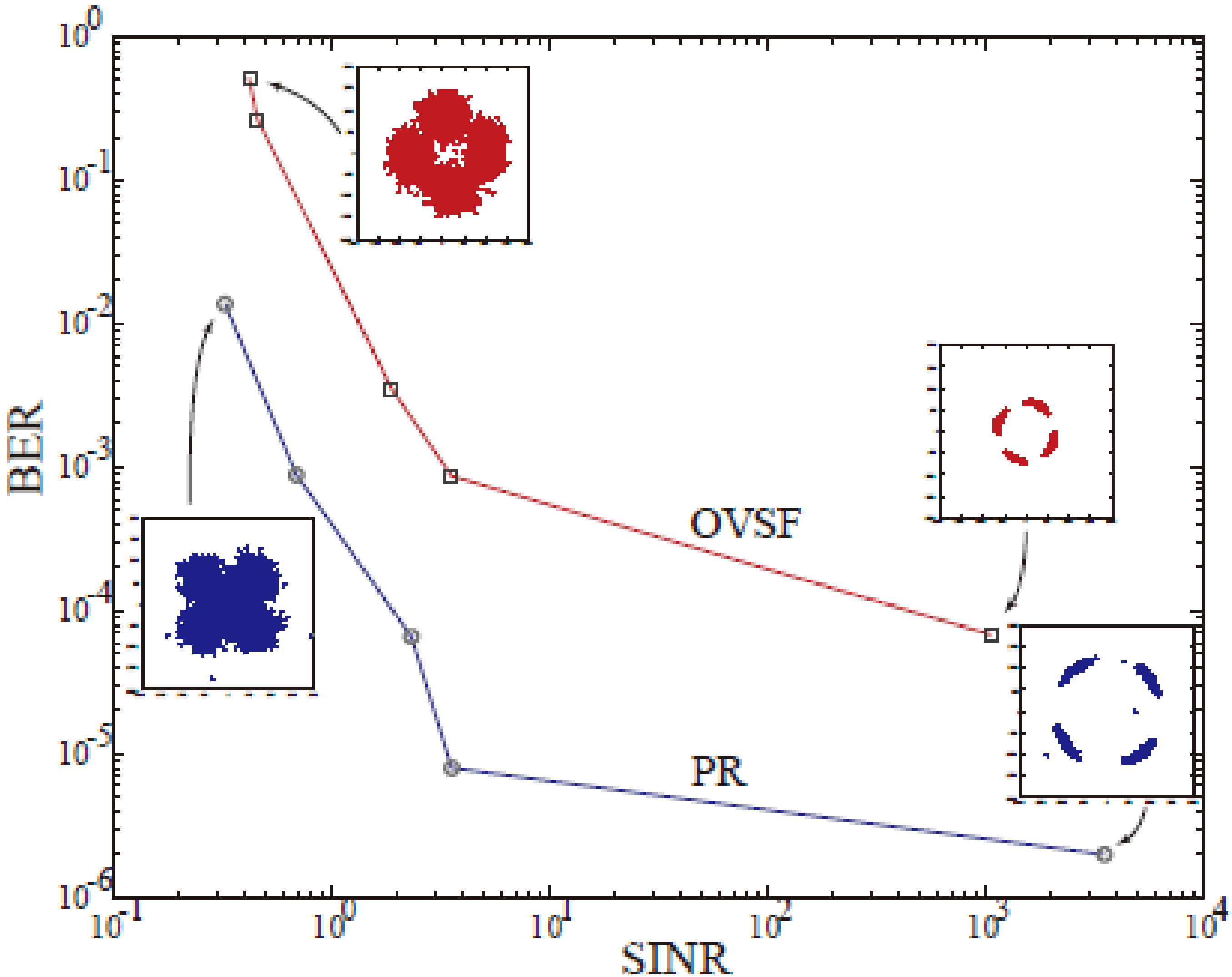
p(素数)=173,q(原始根)=3



RF Experiments

(2nd G Chaos-CDMA with Primitive Root codes)

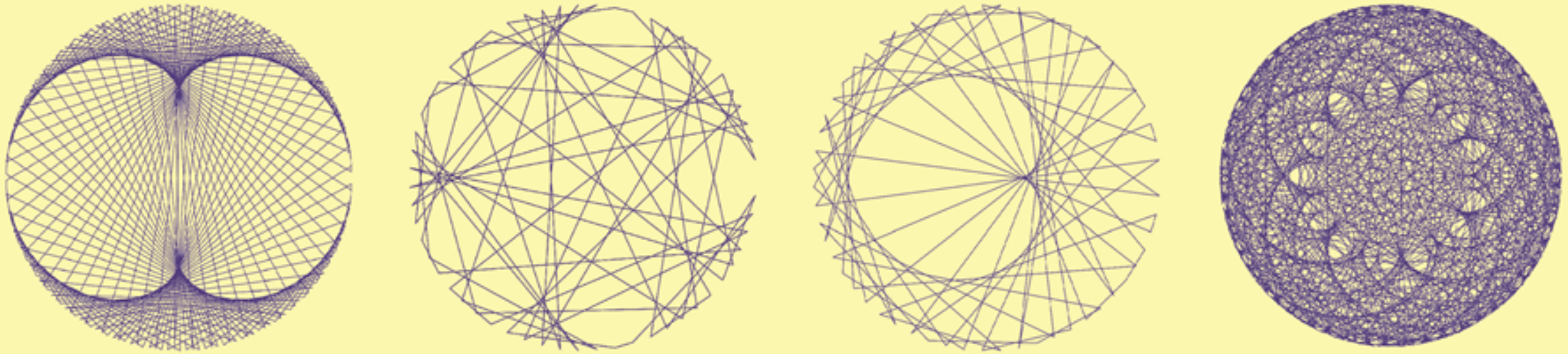
Experimental Results: Chaos code is <u>10 times better</u> than the conventional orthogonal codes.



Chaos Approach to IoT

See ITU News (2013) 12.

Key: Chaos codes can be an infinite number of base (orthogonal) signals for spectrum sharing



P=173, q=3 P=59, q=11 P=59, q=32 P=563, q=54

P is a prime number and q is its associated primitive root number. (There are φ(P-1) primitive roots for each prime P.)

Optimal Strategy for Communications

* Minimize Interference Noise with respect to modulation scheme under the condition such that modulation power must be constant.

Monte Carlo Computation

- Monte Carlo Approach (Randomized Algorithm) for Multi-dimensional Integration by Ulam-Metropolis (1949)

  $Var(error)=O(1/N)$ Central Limit Theorem

(which is independent on r(r *times differentiability*) and *d(dimensionality)*))

- Bakhalov Approach with effective use of "smoothness" r of integrand functions (1959)

  $Var(error)=O(1/N^{(1+2r/d)})$

- KU's Chaotic Monte Carlo computation Algorithm (invented in 1996, published in 2000)

  Superefficiency (firstly found on Oct. 7, 1998, reporting in my paper (KU, 2000))

  $Var(error)=O(1/N^2)$

  (which is also independent on r and *d*)

- **Ex.** For **LDS**(Low Discrepancy Sequence) cases,

  $Var(error)=O((\log N)^{2d}/N)$

Principle of Monte Carlo Computation and Chaotic Monte Carlo Computation

Birkhoff Ergodic Theorem (1931)

$1/N \lim \Sigma_{i=0,..N-1} B(X_i) = \int_\Omega B(x)\rho(x)dx = <B>$,

where $\rho(x)dx$ is an ergodic invariant measure of *a given chaotic map such as* $X_{i+1}=F(X_i)$.

For any integrand $A(x)$ *whose* $A/\rho \in L_2(\Omega, \rho)$,

we can compute the integral

$$\int_\Omega A(x)dx$$

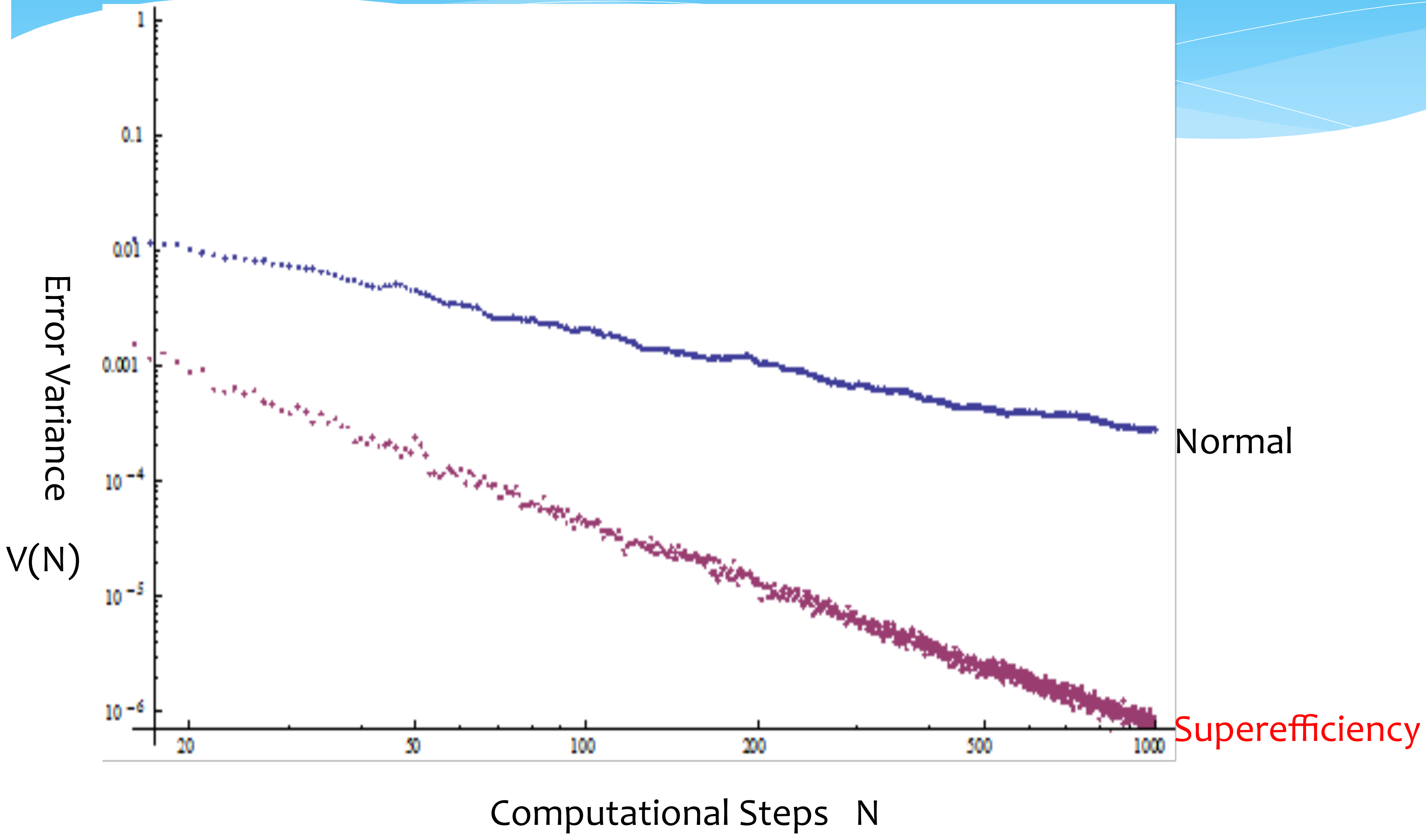
by computing the time average (Monte Carlo Algorithm) :

$1/N\, \Sigma_{i=0,..N-1} B(X_i)$, where $B(x)=A(x)/\rho(x)$.

Superefficient Chaotic Monte Carlo Computation
(Random Number→Chaos)

(What we find in 1998 is that temporally chaotic correlation is important for MC)

(First Observation on 1998.10.7) (Published in 、KU, JJAP in 2000)



Monte Carlo Computation

- Monte Carlo Approach (Randomized Algorithm) for Multi-dimensional Integration by Ulam-Metropolis (1949)

  $Var(error)=O(1/N)$ Central Limit Theorem

(which is independent on r(*r times differentiability*) and *d*(*dimensionality*))

- Bakhalov Approach with effective use of "smoothness" r of integrand functions (1959)

  $Var(error)=O(1/N^{(1+2r/d)})$

- KU's Chaotic Monte Carlo computation Algorithm (invented in 1996, published in 2000)

  Superefficiency (firstly found on Oct. 7, 1998, reporting in my paper (KU, 2000))

  $Var(error)=O(1/N^2)$

  (which is also independent on r and *d*)

- **Ex.** For **LDS**(Low Discrepancy Sequence) cases,

  $Var(error)=O((\log N)^{2d}/N)$

What happened in Superefficiency？

The common problem (to compute an integral)：

* Integrand function : $A[x]=\frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$
* Integral to be obtained→ $\int_{-1}^{1} A[x]\, dx = \mathrm{Erf}[\frac{1}{\sqrt{2}}] = 0.682689....$
* $x_{i+1}=T_2(x_i)=2x_i^2-1$ （Chaotic Iteration）
* $\rho[x]=\frac{1}{\pi\sqrt{1-x^2}}$ (Ergodic Invariant Measure
* $\rho(x)dx$-which is absolutely continuous with respect to the Lebesgue measure dx-)
* $\int_{-1}^{1} A[x]\, dx = \int_{-1}^{1} \frac{B[x]}{\pi\sqrt{1-x^2}} dx \approx 1/N\Sigma_{i=1,..N} B[x_i]$ （Algorithm 1 ）
* $\int_{-1}^{1} A[x]\, dx = \int_{-1}^{1} \frac{B^*[x]}{\pi\sqrt{1-x^2}} dx \approx 1/N\Sigma_{i=1,..N} B^*[x_i]$ (Algorithm 2)

Simple Numerical Experiments: (Chaos: $X_{n+1}=2X_n^2-1=F(X_n)$)

* Integrand function

B(x)= .→ Normal

$B_9(x)=B(x)+\Delta B(x)=$

$B(x)+0.606434279682070 3x$

$+\ 0.033525664829120 08(-3x+4x^3)$

$+0.011882515144670 544(5x-20x^3+16x^5)$

$+0.033525664829120 08(-1+18x^2-48x^4+32x^6)$

$+0.004886099508467573(-7x+56x^3-112x^5+64x^7)$

$+0.002968468501070 7527(9x-120x^3+432x^{5+}576x^7+256x^9)$→**Supereff iciency**

General Formula V(N) [KU,2000]

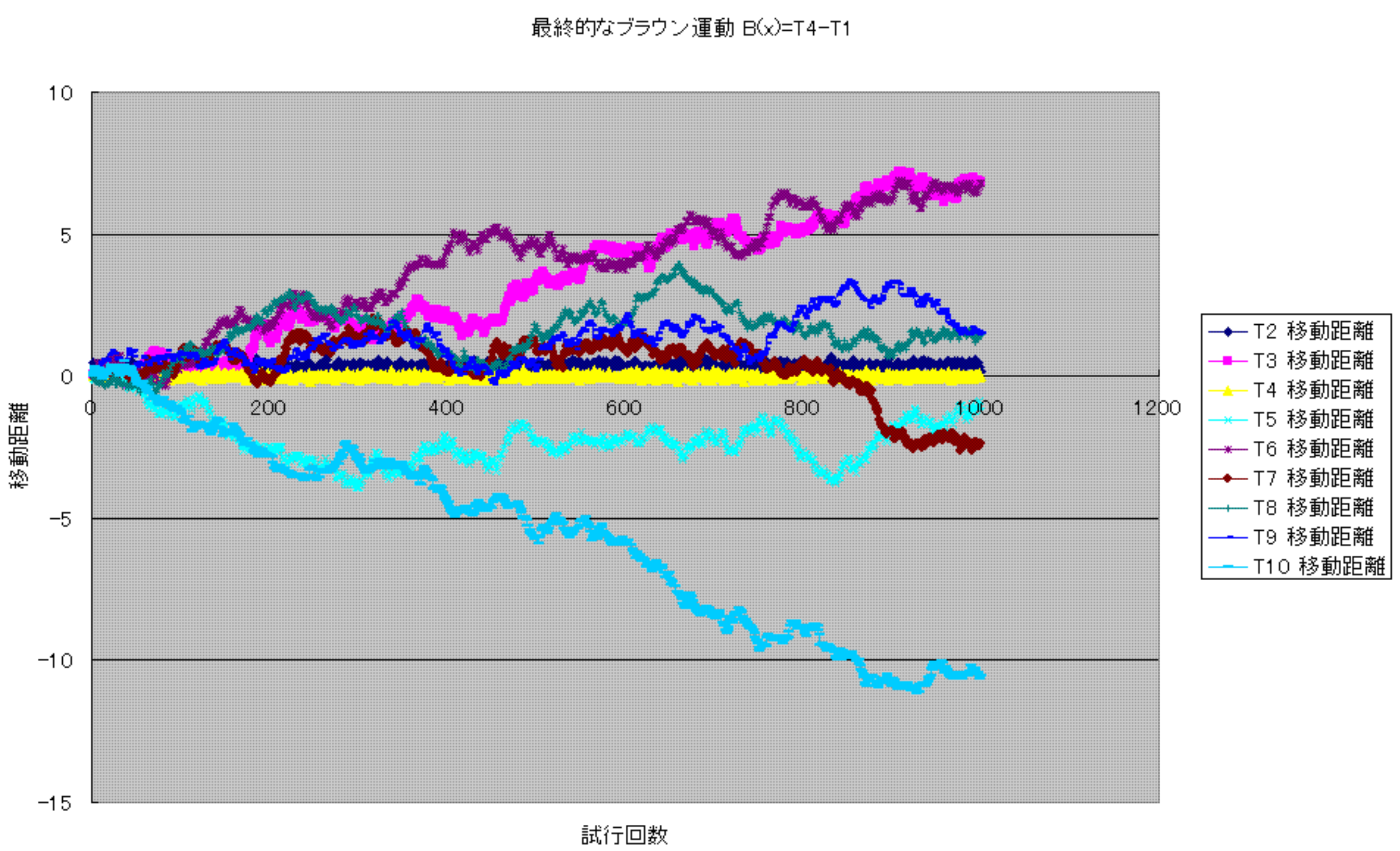
* General Formula : $V(N)=D/N+E/N^2$ for $N\to\infty$

  $D=\langle B^2\rangle - \langle B\rangle^2 \quad +2\,\Sigma_{j=1,..\infty}\{\langle BB_j\rangle-\langle B\rangle^2\}$

  D can be regarded as a Diffusion coefficient of deterministic diffusion with kicked variable $B_j-\langle B\rangle$
  (Discrete time Green-Kubo formula
  －Nonequilibrium・Fluctuation Dissipation Theorem (FDT)—)

  $E=2\,\Sigma_{j=1,..\infty}\, j\{\langle B_0B_j\rangle-\langle B\rangle^2\}$
  （Nonlinear terms outside FDT and
  $|E|<\infty$）

Deterministic Diffusion



Lebesgue Spectrum Analysis Basis Construction (K.Umeno,Nonlinear Analysis,2001)

$F(x)=T_2(x)=2x^2-1$(the second order Chebyshev Polynomial)

$\Phi_{k,j}(x)=\sqrt{2}\ T_{k2^j}(x)$, k is the odd number。

$\Phi_{k,j+1}(x)=\sqrt{2}\ T_{k2^{j+1}}(x)=\Phi_{k,j}[T_2(x)]=\Phi_{k,j}[F(x)]$

So those satisfies the Lebesgue Spectrum condition:

$$\Phi_{k,j+1}(x)=\Phi_{k,j}[F(x)]$$

Lebesgue Spectrum Analysis (Computation of Correlation)

$<B(x_n),B(x_{n+1})>=a_1a_2+a_2a_3+\ldots\ldots\ldots+a_{L-1}a_L$

$<B(x_n),B(x_{n+2})>=a_1a_3+a_2a_4+\ldots\ldots\ldots+a_{L-2}a_L$

…

$<B(x_n),B(x_{n+L-1})>=a_1a_L$

$<B(x_n)^2>=<B^2>= a_1a_1+a_2a_2+\ldots+a_La_L = \Sigma_{1\leqq j\leqq L}a_j^2$

Lebesgue Spectrum Analysis (Computation of D(Diff. Coefficient))

$$D = \langle B^2 \rangle + 2 \Sigma_{j=1,..L-1} \langle B_0 B_j \rangle$$

$$= a_1{}^2 + a_2{}^2 + \ldots + a_L{}^2 + 2(a_1 a_2 + a_2 a_3 + \ldots\ldots\ldots + a_{L-1} a_L)$$

$$+ 2(a_1 a_3 + a_2 a_4 + \ldots\ldots\ldots + a_{L-2} a_L)$$

$$+ \ldots\ldots$$

$$+ 2\, a_1 a_L$$

$$= (a_1 + a_2 + \ldots + a_L)^2 = (\Sigma_{1 \leqq j \leqq L} a_j)^2 \quad \geqq 0$$

Superefficiency Condition：

$$D = 0 \ 《=》\ a_1 + a_2 + \ldots + a_L = \Sigma_{1 \leqq j \leqq L} a_j \ = 0$$

Lebesgue Spectrum Analysis (General Form of D(Diffusion Coefficient))

D(General) : $D = \Sigma_{k \in \Lambda} (\Sigma_{j \geqq 0} a_{k,j})^2 \geqq 0$

$D(k) = (\Sigma_{j \geqq 0} a_{k,j})^2$ …….

So, we obtain

$D = \Sigma_{k \in \Lambda} D(k)$ …..Lebesgue Spectrum Decomposition of Diffusion Constant

(D=Distant with the Optimal Chaotic Algorithm)

Lebesgue Spectrum Analysis (General Superefficiency)

* We consider $\Delta B(x) \in H$. In this case, we obtain new integrand function

$$B'(x)=B(x)+\Delta B(x) \in H$$

where、$<\Delta B(x)>=0$. Therefore, we have

$$<B'(x)>=<B(x)+\Delta B(x)>=<B(x)>$$

Computational Cost for General Superefficiency

* Preprocessing（ΔB(x)'s Evaluation）－N step

$$A_k = \Sigma_{j \geqq 0} a_{k,j} = \Sigma_{j \geqq 0} \langle B(x),\ \Phi_{k,j}(x) \rangle$$

By evaluation of A_k by normal Monte Carlo computation with O(1/N) variance、we obtain that B'(x)=B(x)+ΔB(x)'s diffusion coefficient D' is never zero but has an approximate estimate given by O(1/N) as

$$D' = G/N \quad \text{for } N \to \infty$$

* Postprocessing（Approximate Superefficiency computation of B'(x)）－N steps

$V(N)=D'/N+E'/N^2 = G/N^2+E'/N^2=O(1/N^2)$ at 2N steps

→**Superefficiency Again!!**

Again, Variational Principle exists for an optimal strategy with chaos

- Variance of Errors obeys the general formula
  $V(\delta,\Delta B)=D[\Delta B]\delta+E\delta^2$ where $\delta=1/N$, random number has a mixing property.

- Miminize computational diffusion coefficient $D[\Delta B]$
  with respect to ΔB (additional Integrand functional) maintaining a condition such that an integral of $\Delta B=0$.

What I saw a superefficiency in MC with chaos in 1998 is that the optimal strategy $D[\Delta B]=0$ really exists.
What I conclude here is that optimal strategy $D[\Delta B]=0$
always exists for arbitrary multidimensional integrand B.

モンテカルロ計算と Kユーザー多重通信混雑解消問題との関係

モンテカルロ計算　v.s.　Kユーザー多重通信

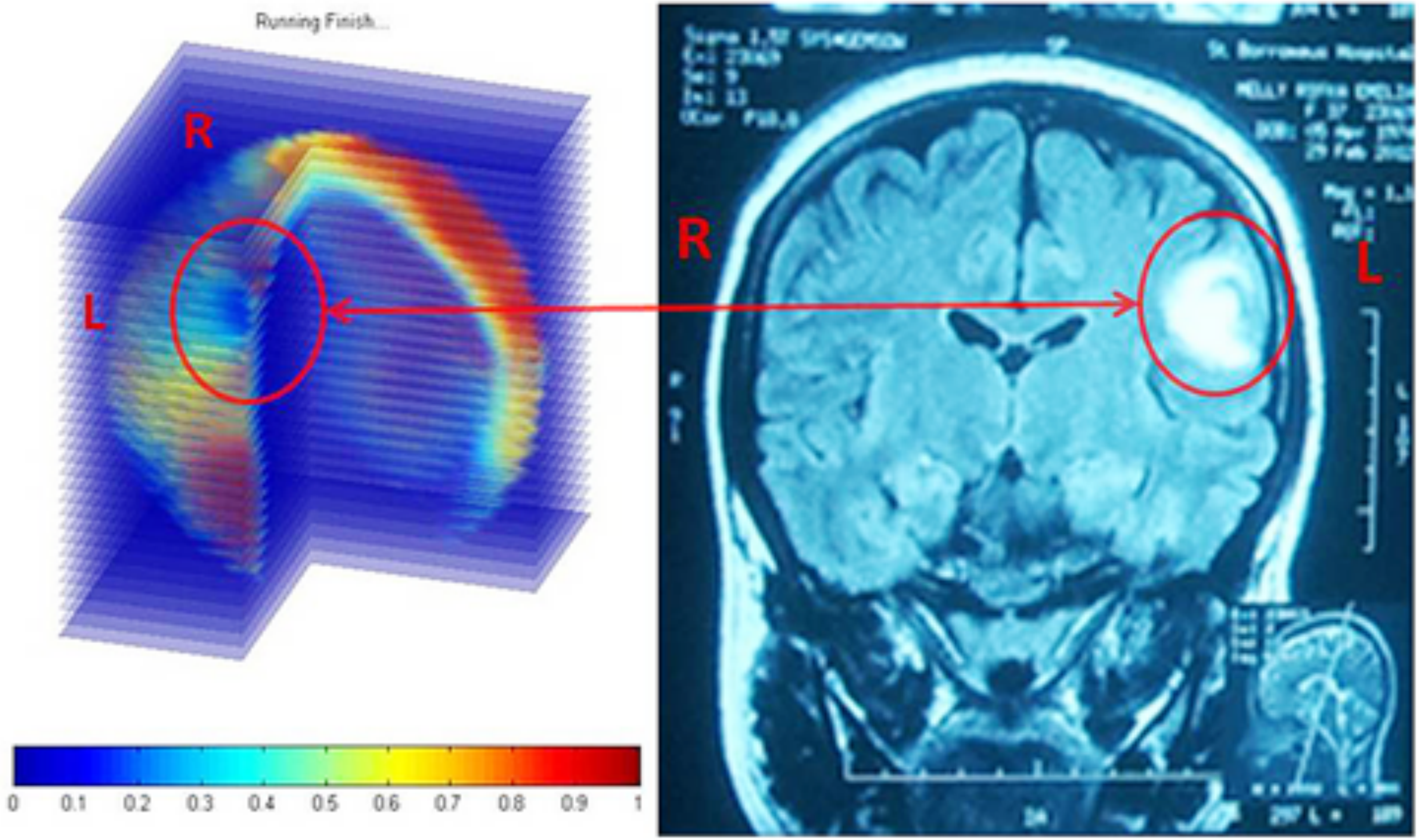
	モンテカルロ計算	Kユーザー多重通信
	誤差分散 $V(N)=O(1/N^{\beta})$	干渉雑音の分散 $I(K)=O(K^{\beta-2})$
通常（中心極限定理）	$V(N)=O(1/N)$ ✔	$I(K)=O(K)$ ✔
Superefficiency	$V(N)=O(1/N2)$ ✔	$I(K)=O(1)$ （構成可能か？）

Superefficiency =ユーザー数Kが、拡散利得Nに対して$O(N^2)$となる符号が構成可能か？

Yes，であることを発見（最も最近の結果）

Dr. Warsito Taruno (CTECH Lab.)

Our Natural Temporal Chaos in Brain

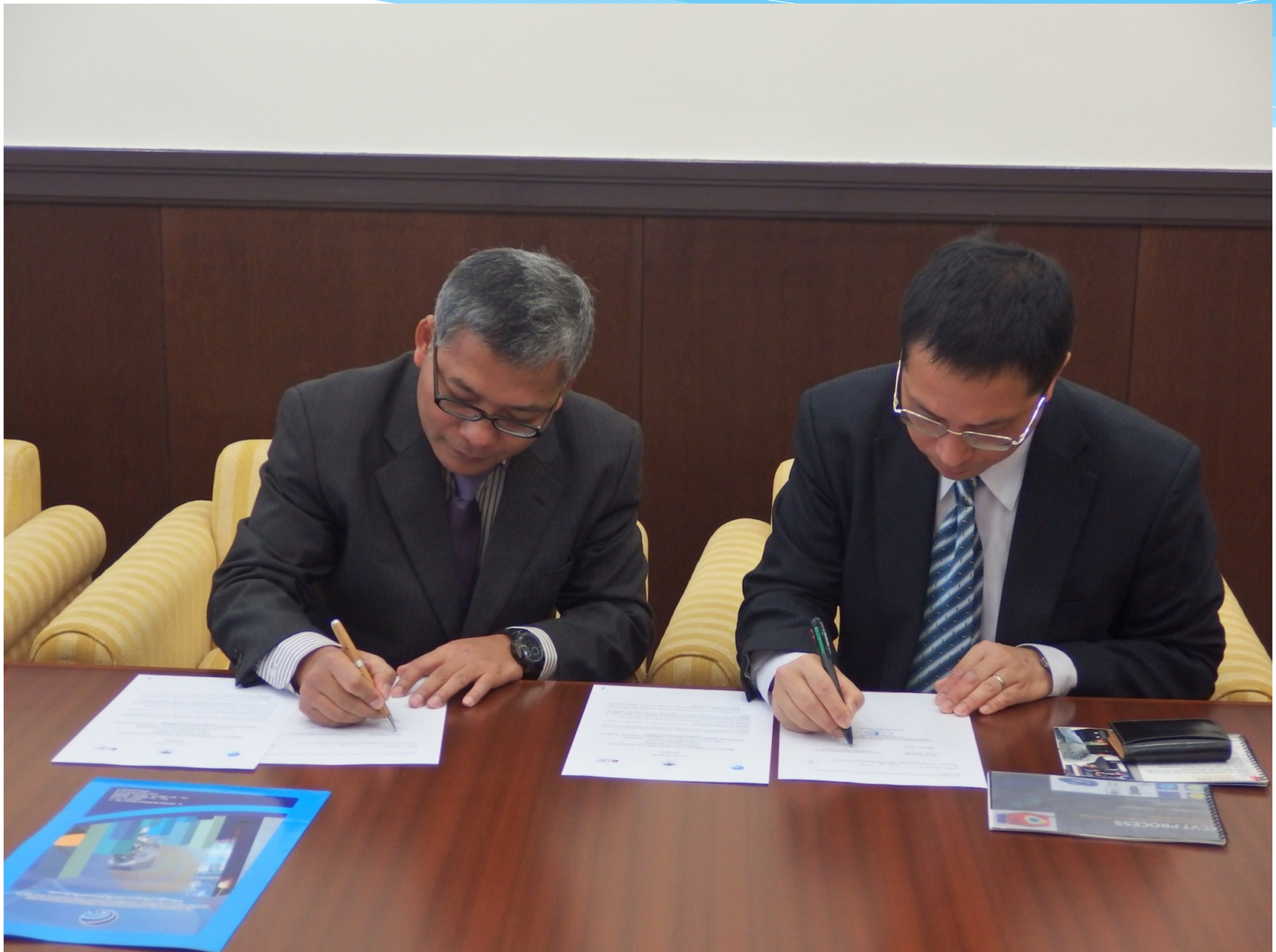


ECVT

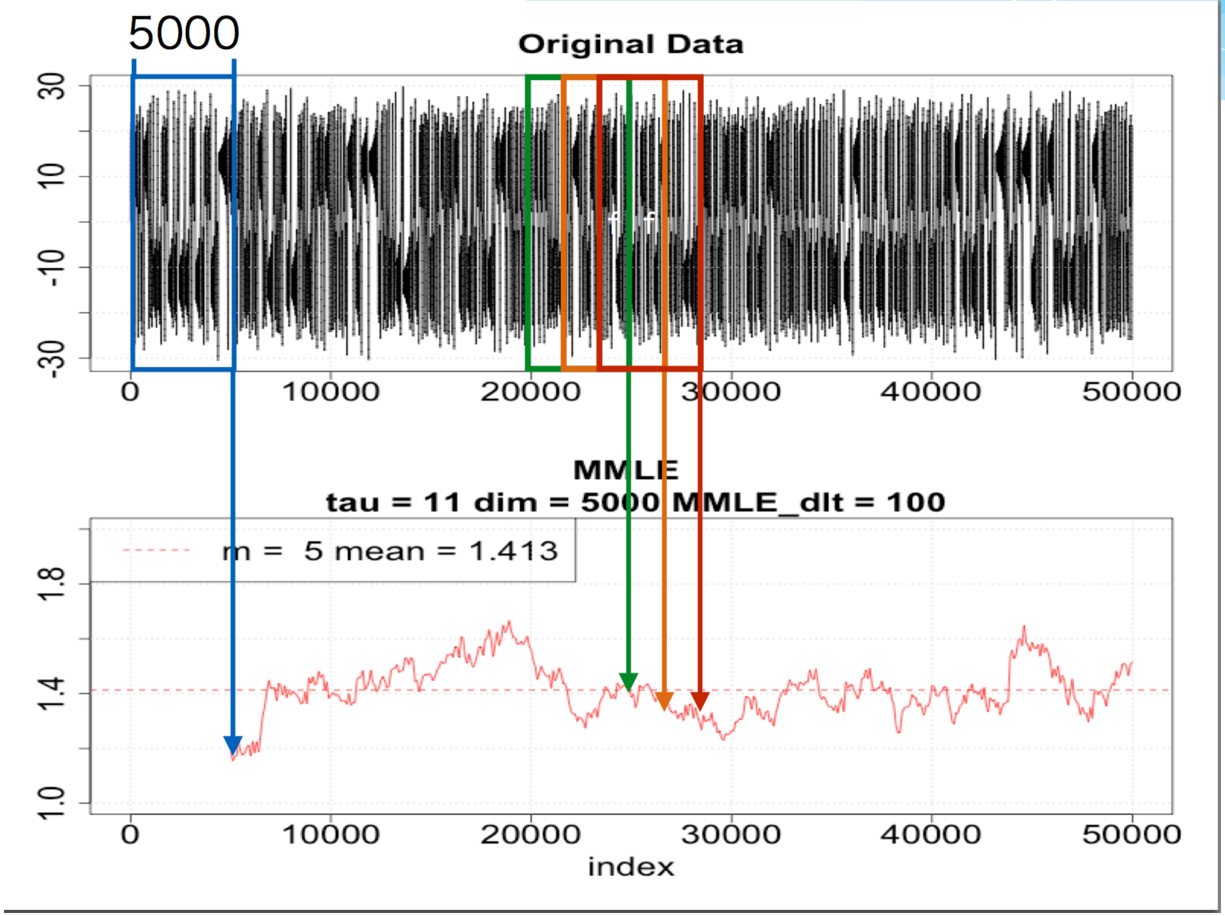
MRI

We agreed at Kyoto for R&D on future Brain-to-Brain Communications

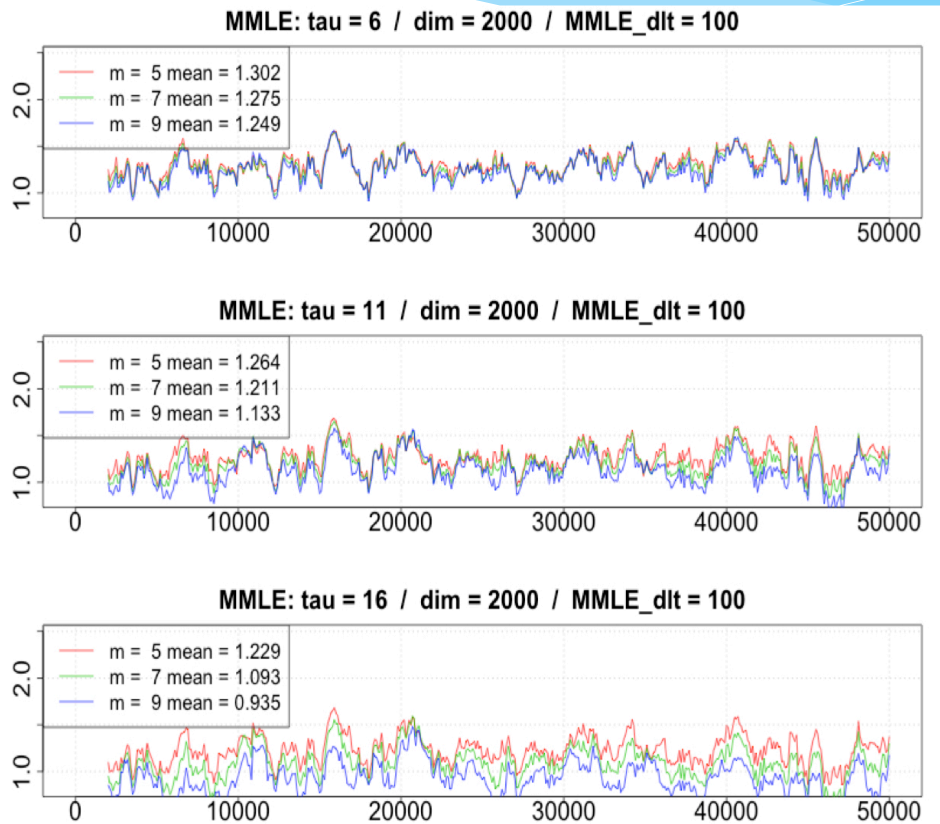
Jakarta Post on May 21, 2014



MMLE in Brain (Moving Maximum Lyapnov Exponent) proposed by Mr. Okada (M2)



脳波のカオス性,
及びその変化を検知に成功



Concluding Remarks

- Chaos has a certain effect in practical communications/ computations.
- Optimal strategy with chaos codes certainly exists for both.
- We can construct optimal algorithm with chaos for general MC (superefficient effect) through a variational principle for computational diffusion.
- There are many kinds of variational principles (which we cannot prove) for communication/computation.
- Superefficient Chaotic MC effect with suitable chaotic correlation has same as in Quantum Search or Quantum Phase estimation with quantum correlation.

(Always $O(N^2) \rightarrow O(N)$ computational complexity effect)

■ Is there a reason for this coincidence (open problem)?

■ Further Investigation of Variational Principles for Computation/ Information.

■Spatio-Temporal chaos codes detected in our Brain with multi-dimensional sensor tomography technique is our current –future research topics in a framework of international research cooperation.