

Hybrid SPM to investigate scattered field from rough surface under tapered wave incidence

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Abstract—We propose a novel time-efficient analytical model to investigate the near-zone scattered field from a conducting random rough surface under tapered wave incidence. The proposed method is based on the small perturbation method (SPM), and combined with the principle of stationary phase (POSP), the Lagrange polynomial and the Monte Carlo simulation. By comparing with the method of moment (MoM), the running time of proposed method is about 200 times less than that of MoM under the same accuracy, indicating that the proposed method is suitable for applications where computational time is urgent.

I. INTRODUCTION

The analysis of electromagnetic wave scattered from random rough surfaces is of great interest in many applications including radio communications and remote sensing. Small perturbation method (SPM) [1] is one of the most commonly used analytical methods. It is valid to a surface that is slightly rough ($kh \ll 1$) and whose surface slope is smaller than unity, where k is the free-space wavenumber, and h is the root-mean-squared (RMS) height of the random rough surface. In numerical simulations, rough surface is truncated at edges where $x = \pm L/2$, which means that surface current is forced to be zero for $|x| > L/2$ [2]. To prevent current discontinuity at edges, a tapered incident wave was introduced [3, 4] in spatial or spectral domain.

Different from previous studies, which was focused on far-zone field, in this paper we present a time-efficient analysis of near-zone scattered field from a random rough surface. Two coefficients are defined, namely the coherent scattered taper function T_{coh} and incoherent one T_{incoh} . $T_{incoh} \cdot T_{coh}$ is modified for coherent wave by Lagrange polynomial, and for the incoherent wave, and the POSP is used to deal with an infinite complex integral. Finally a Monte Carlo simulation is used to determine the scattered

field statistics as a function of different rough surface realizations.

II. DESCRIPTION OF THE METHOD

The geometry of the problem is shown in Figure 1. A TE tapered wave, $\bar{\psi}_{inc}(\bar{r})$ [3, 5] is incident with time dependence $\exp(-j\omega t)$, impinging upon a 1-D conducting Gaussian-distributed rough surface, with a random height profile $z = f(x)$, correlation length l , surface length L , RMS height h , and the spectral density $W(k_x) = h^2 l / (2\sqrt{\pi}) \cdot \exp(-k_x^2 l^2 / 4)$. The expression for $\bar{\psi}_{inc}(\bar{r})$ is [4]

$$\begin{aligned} \bar{\psi}_{inc}(x, z) &= T(x, z) \exp(-j\vec{k}_i \cdot \bar{r}) \hat{y} \\ &\equiv T(x, z) \exp[-j(k_{ix}x - k_{iz}z)] \hat{y}, \end{aligned} \quad (1)$$

where $k_{ix} = k \sin \theta_i$, $k_{iz} = k \cos \theta_i$, θ_i is the incident angle, and $T(x, z)$ is the incident taper function

$$T(x, z) = \exp\left[i(k_{ix}x - k_{iz}z)w(\bar{r}) - (x + z \tan \theta_i)^2 / g^2\right] \quad (2)$$

where g is the tapering parameter, and $w(\bar{r}) = \left\{2\left[(x + z \tan \theta_i)^2 - 1\right] / g^2 - 1\right\} / (kg \cos \theta_i)^2$.

The height of the random rough surface is used as a small parameter. For the scattered electric field, we write it as a perturbation series [1], $\psi_s = \psi_s^{(0)} + \psi_s^{(1)} + \psi_s^{(2)} + \dots$. Since the Dirichlet boundary condition requires that on

the PEC surface $z = f(x)$, $\psi_{inc} + \psi_s = 0$, and there is a taper function in the incident wave expression in (1), therefore a scattered taper function should be considered

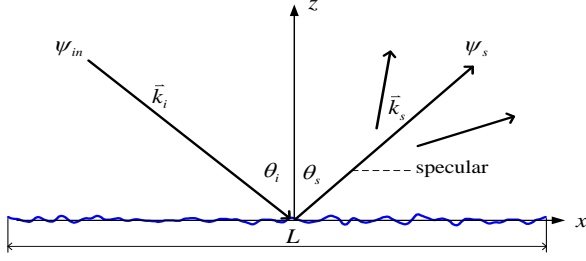


Figure 1 Geometry for wave scattering by a randomly rough.

into the expression of scattered wave. In the following, two different scattered taper functions are introduced

$$\begin{aligned} \psi_s(\vec{r}) &= \psi_{coherent}(\vec{r}) + \psi_{incoherent}(\vec{r}) \\ &= T_{coherent} \psi'_{coherent}(\vec{r}) + T_{incoherent} \psi'_{incoherent}(\vec{r}) \end{aligned} \quad (3)$$

where $\psi'_{coherent}(\vec{r})$ and $\psi'_{incoherent}(\vec{r})$ are the coherent and incoherent wave under plane wave incidence using the traditional SPM. $T_{coherent}$ and $T_{incoherent}$ are defined as the coherent and incoherent taper function, respectively. Their expressions are $T_{coherent} = \Delta \cdot T$ and $T_{incoherent} = T$, where Δ is a polynomial that only related to the incident angles. From [1], the coherent and incoherent wave for the tradition SPM are

$$\psi'_{coherent}(\vec{r}) = -e^{ik_x x + ik_z z} \left(1 - 2k_{iz} \int_{-k}^k dk_x k_z W(k_x - k_x) \right) \quad (4)$$

and

$$\psi'_{incoherent}(\vec{r}) = \int_{-\infty}^{\infty} dk_x e^{ik_x x + ik_z z} 2ik_{iz} F(k_x - k_{ix}) \quad (5)$$

where $k_z = \sqrt{k^2 - k_x^2}$, and $F(k_x)$ is the Fourier transform of $f(x)$. With the two scattered taper functions, the scattered field under tapered wave incidence satisfies the wave equation to order of $O[1/(k^2 g^2)]$.

Since (5) involves complex integral, and the wavenumber k is chosen to be large enough in our case, the POSP is used[7]. The points of stationary phase in (5) are $k_{x0} = \pm xk / \sqrt{f^2 + x^2}$. The integral (5) can be approximately evaluated by considering the sum of the contributions from each of these points, which is $I(k)$:

$$\begin{cases} I_1(k; x, z) \\ \sim 2ik_{iz} F(k_{x0} - k_{ix}) \cdot \exp[i(k_{x0}x + k_{z0}z - \pi/4)] \\ \cdot \sqrt{2\pi / \left| k^2 / (k^2 - k_{x0}^2) \right|^{3/2}} \\ I_2(k; x, z) \\ \sim 2ik_{iz} F(-k_{x0} - k_{ix}) \cdot \exp[i(-k_{x0}x + k_{z0}z - \pi/4)] \\ \cdot \sqrt{2\pi / \left| k^2 / (k^2 - k_{x0}^2) \right|^{3/2}} \end{cases} \quad (6)$$

$$I(k) = I_1(k) + I_2(k). \quad (7)$$

TABLE I. THE RELATIONSHIP BETWEEN Δ AND θ_i

θ_i	10°	20°	30°	40°	50°	60°	70°	80°
Δ	1.18	1.17	1.14	1.109	1.105	1.04	1.02	0.991

At first, when we simply set the coherent wave tapered function as T , the results of the scattered field from different incident angles always have the same shape but with differ magnitudes compared with that of MoM (see Figure 2). It turns out that a relationship between the incident angles θ_i and the coherent scattered field amplitude Δ leads to this difference (see Table I). Then, the additional coefficients for the coherent wave were taken into account. The Lagrange polynomial [8]

$\Delta(\theta_i) = \sum_{n=0}^7 a_n \theta_i^n$ is used to fit this relationship at knots $\theta_{i(i=0-7)} = [10^\circ, 20^\circ, 30^\circ, 40^\circ, 50^\circ, 60^\circ, 70^\circ, 80^\circ]$, yielding coefficients $a_{i(i=0-7)} = [0.8590, 0.7703, -0.6920, 0.3105, -0.0782, 0.0111, -8.25e-4, 2.5e-05]/10$. Later we test its reliability in a large group of different incident angles, the results show that this polynomial approximation yields error less than 0.8% compared with MoM data.

To obtain the statistical average of our method, Monte Carlo simulation is used [6]. In each simulation trial, the scattered field is a function with respect to each sampled rough surface. The statistical average of the field is determined by

$$\psi_{average}^m = (\psi^m + \psi_{average}^{m-1} \cdot (m-1)) / m, \quad (1 \leq m \leq M) \quad (8)$$

where m is the index Monte Carlo trial, and M is the total number of trials. From (8), the convergence rate for

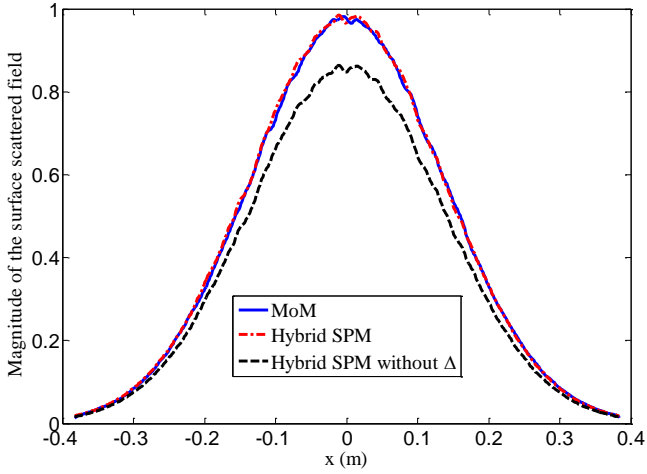


Figure 2 Magnitude of the surface scattered field with the incident wave angle of 30° .

the scattered field can be estimated by

$$\text{convergence}(m) = (\psi_{\text{average}}^{m+1} - \psi_{\text{average}}^m) / \psi_{\text{average}}^m \quad (9)$$

III. RESULTS

The programs were written in Matlab, and ran on a PC with 3.20GHz Intel Core(TM) i3 and 1.74GB memory. The wavelength is $\lambda = 0.03m$ in our case. The parameters of the random rough surface are selected as: $L = 25.6\lambda$, $h = 0.05\lambda$, $l = 0.35\lambda$. The tapering parameter is set to be $g = L/4$. The total number of points is $N = 528$, and N denote the number of times of Monte Carlo trials.

We compare the surface electric fields using the hybrid SPM with that obtained from the MoM. Such comparison results are given in Figure 2 and Figure 3 with incident angles $\theta_i = 30^\circ, 40^\circ, 60^\circ$. It is shown that the two curves obtained using hybrid SPM and MoM overlap each other well. The comparisons for computation time and accuracy are given in Table 2, where accuracy is calculated as

Accuracy=

$$\max \left[\left| \frac{|E_{sc(MoM)}| - |E_{sc(Hybrid\ SPM)}|}{|E_{sc(MoM)}|} \right| \cdot 100 \right] \quad (10)$$

It is shown that our proposed method can accelerate the computation with a factor of 200 over MoM. This is because the adoption of the POSP into our procedure makes the computation more efficient.

TABLE II. COMPARISONS OF COMPUTATION TIME (IN SECOND) AND ACCURACY BETWEEN MOM AND HYBRID SPM

θ_i	10°	20°	30°	40°
MoM (s)	29.8632	30.0314	30.1508	30.1236
Hybrid SPM (s)	0.1462	0.1416	0.1491	0.1651
Accuracy(%)	2.3e-03	0.42	0.39	0.9
θ_i	50°	60°	70°	80°
MoM (s)	30.0075	30.1129	30.0138	30.2286
Hybrid SPM (s)	0.15670	0.1435	0.1389	0.1557
Accuracy(%)	0.29	1.7e-03	3.9e-02	0.52

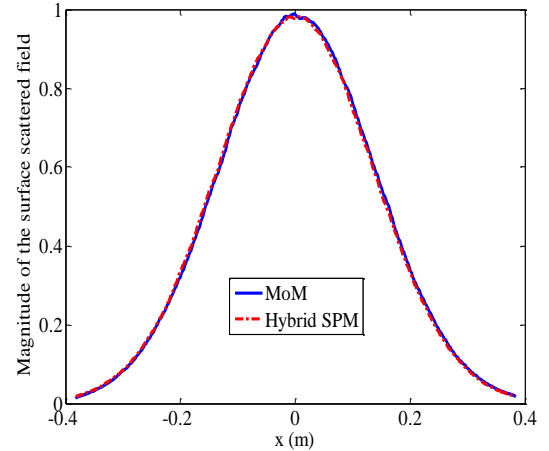
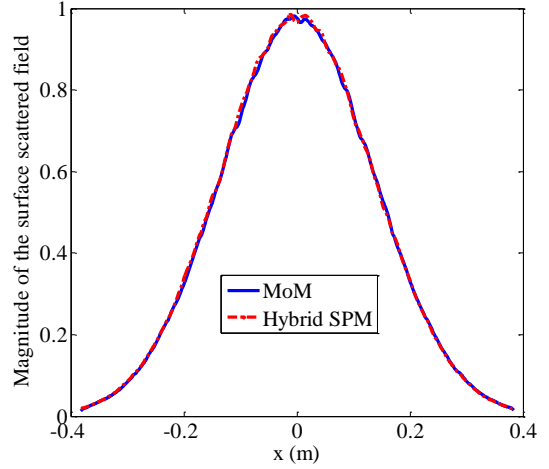


Figure 3 Scattered field with different incident angles (left figure $\theta_i = 40^\circ$, right one $\theta_i = 60^\circ$)

To check the convergence of the Monte Carlo simulation, 50 Gaussian random rough surfaces were generated, and the convergence rate is calculated using (9), and simulation results show that the computations converge after about 25 Monte Carlo realizations for both methods.

IV. CONCLUSION

In this paper, a hybrid model using SPM combined with POSP, Lagrange polynomial and Monte Carlo simulation is proposed for investigating the near field scattered by a conducting random rough surface under tapered wave incidence. The method is validated by comparing with MoM data in terms of computation time and accuracy, and the convergence rate has also been studied. It is shown that our proposed method can accelerate the computation with a factor of 200 over MoM. Therefore it is expected that the proposed method might be useful for fast and accurate computing for applications such as subsurface investigations involving a random rough surface. The future work could be extension to the two-dimensional model.

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