Coupled-Mode Analysis of Two-Parallel Post-Wall Waveguides

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Abstract- **The guided modes supported in a coupled two-parallel post-wall waveguides are analyzed by using a coupled-mode theory for coupled two-dimensional photonic crystal wave guides. The coupled-mode equations, which govern the evolution of the modal amplitude of individual post-wall waveguides, are derived in self-contained way on the basis of eigenmode fields of each single waveguide in isolation. Numerical examples show that for various configurations of post-walls, the solutions of the coupled-mode equations are in very close agreement with those obtained by the rigorous numerical analysis.**

I. INTRODUCTION

Post-wall waveguides [1], also called laminated waveguides [2] or substrate integrated waveguides [3], have received a growing attention because of their promising applications to planar circuit components operating in the microwave and millimeter wave frequency range. The modal properties of post-wall waveguides have been extensively investigated in the past years using various numerical or analytical techniques [1-5]. A number of components based on post-wall wave-guides have been also proposed and demonstrated, such as filters, couplers, and slot array antennas.

The post-wall waveguides are integrated waveguide-like structures composed of periodic rows of circular metallic posts in a grounded dielectric substrate. The structures are quite similar to those of two-dimensional photonic crystal wave-guides [6] consisting of layered periodic arrays of circular metallic cylinders with infinite length. Taking into account this similarity, we have recently proposed [7] a novel analytical model of post-wall waveguides based on the model of two-dimensional photonic crystal waveguides.

In this paper, we shall use the proposed model to analyze a coupled two-parallel post-wall waveguides which is a basic component to be used for a directional coupler. The coupled-mode theory, which has been developed [8] for dealing with coupled two-dimensional photonic crystal waveguides, is reformulated for the post-wall waveguide structures. The coupled-mode equations, which describe the evolution of the modal amplitude of individual post-wall waveguides, are derived in self-contained way on the basis of the eigenmode fields of each single waveguide in isolation. Numerical examples show that for various configurations of post-walls, the solutions of the coupled-mode equations are in very close agreement with those obtained by the rigorous numerical analysis.

II. FORMULATION OF THE PROBLEM

The post-wall waveguide, as illustrated in Fig. 1, is composed of periodic arrays of conducting circular posts embedded in a dielectric substrate that connect two parallel conducting plates separated by a distance d. The radius of the posts is r, the pitch of the periodic arrangement of posts in the z-direction is h, and the material constants of the dielectric substrate are ε_s and μ_0. Although the post arrays in both sides may be N-layered, Fig. 1 shows the structure formed by a single layer array. The waveguide width in the x direction is defined by the separation distance w between the two innermost post arrays. Since the substrate is very thin ($d \ll \lambda$), the electric and magnetic fields are uniform ($\partial/\partial y = 0$) in the y direction. Hence this periodic waveguide is quite similar to a two-dimensional photonic crystal waveguide [6] formed by parallel circular rods which are infinitely long in the y direction.

The transversal view in the x-z plane of a post-wall waveguide bounded by N-layered post arrays is illustrated in Fig. 2. If we assume an even TE mode whose E_y field is symmetric with respect to $x = 0$, the guided field in the post-wall waveguide is expressed as follows:

$$E_y(x,z) = \sum_{m=-\infty}^{\infty} a_m \cos(\kappa_m x) e^{i\beta_m z} \tag{1}$$

where $\beta_m = \beta + 2m\pi/h$, $\kappa_m = \sqrt{k_s^2 - \beta_m^2}$, $k_s = \omega\sqrt{\varepsilon_s \mu_0}$, ω is the angular frequency, β is the mode propagation constant, and $\{a_m\}$ are unknowns. Let us define the column vector $\boldsymbol{a}$ whose

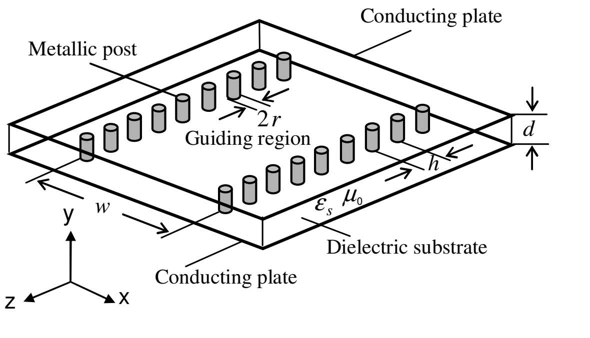

Fig. 1. Schematic of a post-wall waveguide.

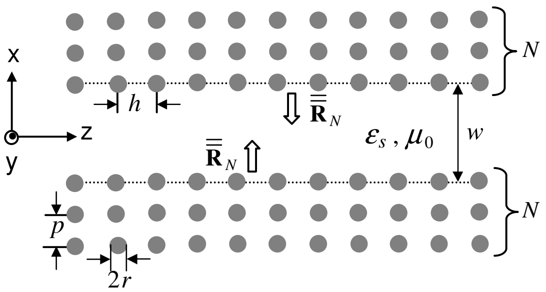

Fig. 2. Transversal view in the *x-z* plane for the post-wall waveguide bounded by *N*-layered post arrays.

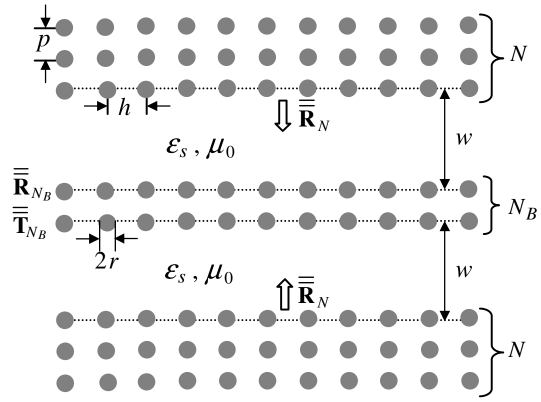

Fig. 3. Two-parallel post-wall waveguides "*a*" and "*b*" coupled through N_B -layered post-wall barrier.

elements are $\{a_m\}$. Using the model of two-dimensional photonic crystal waveguide [6] for Eq.(1), the linear equation to solve $\boldsymbol{a}$ is obtained as follows:

$$[\mathbf{I}-\mathbf{W}(\beta,\omega)\overline{\overline{\mathbf{R}}}_N(\beta,\omega)]\cdot\boldsymbol{a}=0 \tag{2}$$

with

$$\mathbf{W}(\beta,\omega)=[\mathrm{e}^{i\kappa_m w}\,\delta_{mn}] \tag{3}$$

where $\overline{\overline{\mathbf{R}}}_N(\beta,\omega)$ is the generalized reflection matrix of *N*-layered post arrays viewed from the guiding region to the outward direction. $\overline{\overline{\mathbf{R}}}_N(\beta,\omega)$ can be calculated [9] using the T-matrix for a circular rod of perfect conductor and the lattice sums. From Eq.(2) the transcendental equation to determine the propagation constant β is obtained as

$$\det[\mathbf{I}-\mathbf{W}(\beta,\omega)\overline{\overline{\mathbf{R}}}_N(\beta,\omega)]=0\,. \tag{4}$$

The solution to β obtained from Eq.(4) is substituted into Eq.(2) to calculate the amplitude vector $\boldsymbol{a}$. Finally, the mode solution to the post-wall waveguide is expressed as follows:

$$\boldsymbol{a}=A\mathrm{e}^{i\beta_0 z}\boldsymbol{f}(z) \tag{5}$$

where β_0 is a solution of Eq.(4), A is the mode amplitude and $\boldsymbol{f}(z)$ is the normalized mode eigenvector, which is a periodic function of z with period $2\pi/h$.

Figure 3 shows the transversal view of two-identical post-wall waveguides "*a*" and "*b*" which are situated in parallel and coupled through the N_B -layered post-wall barrier. Since the post-wall waveguide is a laterally open structure, a part of the electromagnetic energy may leak out through the gaps between adjacent posts. If the gap length $h-2r$ is chosen to be sufficiently smaller than the wavelength and the post-wall is multilayered in the *x* direction, the leakage effect becomes negligible. However such a situation is not suitable for designing a post-wall waveguide coupler. We assume here that the two-parallel post-wall waveguides are bounded by the enough number of post-array layers in the upper and lower regions but are separated by a barrier consisting of a small number of post-array layers with $N_B<N$. In this case, the leakage of the guided field into the upper and lower half space is strongly suppressed, whereas the two guided modes supported by waveguides "*a*" and "*b*" can interact efficiently to attain the expected power transfer between two waveguides.

Using the model [6] of two-dimensional photonic crystal waveguides, the rigorous dispersion equation for the coupled post-wall waveguides shown in Fig. 3 is derived as follows:

$$\det\Big\{\mathbf{I}-\mathbf{W}(\beta,\omega)\Big[\overline{\overline{\mathbf{R}}}_{N_B}(\beta,\omega)+\overline{\overline{\mathbf{T}}}_{N_B}(\beta,\omega)\Big]\times\mathbf{W}(\beta,\omega)\overline{\overline{\mathbf{R}}}_N(\beta,\omega)\Big\}=0 \quad \text{for } even \text{ mode} \tag{6}$$

$$\det\Big\{\mathbf{I}-\mathbf{W}(\beta,\omega)\Big[\overline{\overline{\mathbf{R}}}_{N_B}(\beta,\omega)-\overline{\overline{\mathbf{T}}}_{N_B}(\beta,\omega)\Big]\times\mathbf{W}(\beta,\omega)\overline{\overline{\mathbf{R}}}_N(\beta,\omega)\Big\}=0 \quad \text{for } odd \text{ mode} \tag{7}$$

where $\overline{\overline{\mathbf{R}}}_{N_B}(\beta,\omega)$ and $\overline{\overline{\mathbf{T}}}_{N_B}(\beta,\omega)$ denote the generalized reflection and transmission matrices of the N_B -layered post-wall barrier. We may directly solve Eqs.(6) and (7) with use of roots searching algorithm for transcendental equations. However such a direct method is very time consuming. If the transmission of guided fields through the barrier layer is weak enough, Eqs.(6) and (7) are approximated by the coupled-mode equations based on the eigenmode field of each single waveguide in isolation as discussed in what follows.

III. Coupled-Mode Equations

When two waveguides "*a*" and "*b*" are well separated, each waveguide behaves as a single waveguide and supports the guided mode independently in the form of Eq.(5). If the waveguides are placed in close proximity, the two guided modes interact through the barrier layer and their mode amplitudes slowly vary along the propagation in the *z* direction. Under this situation, we express the amplitude vectors for guided field modes as follows:

$$\boldsymbol{a}=A(z)\mathrm{e}^{i\beta_0 z}\boldsymbol{f}(z) \quad \text{for waveguide "}a\text{"} \tag{8}$$

$$\boldsymbol{b}=B(z)\mathrm{e}^{i\beta_0 z}\boldsymbol{f}(z) \quad \text{for waveguide "b"} \tag{9}$$

where the slowly varying amplitudes $A(z)$ and $B(z)$ describe a small perturbation in the mode propagation constant and $|dA(z)/dz|,\ |dB(z)/dz| \ll \beta_0,\ 2\pi/h.$ The first-order perturbation analysis under the weak coupling through $\overline{\overline{\mathbf{T}}}_{N_B}(\omega,\beta)$ is applied to Eqs.(6) and (7) with use of Eqs.(8) and (9). Following the same analytical procedure developed in [8], the coupled-mode equations for $A(z)$ and $B(z)$ are derived as follows:

$$\frac{d}{dz}A(z) = i\kappa B(z) \tag{10}$$

$$\frac{d}{dz}B(z) = i\kappa A(z) \tag{11}$$

with

$$\kappa = \frac{\boldsymbol{g}^T \cdot \mathbf{D}_{ab}(\beta_0,\omega)\cdot \boldsymbol{f}}{\boldsymbol{g}^T \cdot \dfrac{\partial \mathbf{D}(\beta_0,\omega)}{\partial \beta_0}\cdot \boldsymbol{f}} \tag{12}$$

$$\mathbf{D}(\beta,\omega) = \mathbf{I} - \mathbf{W}(\beta,\omega)\overline{\overline{\mathbf{R}}}_N(\beta,\omega) \tag{13}$$

$$\mathbf{D}_{ab}(\beta,\omega) = \mathbf{W}(\beta,\omega)\overline{\overline{\mathbf{R}}}_N(\beta,\omega)\mathbf{W}(\beta,\omega)\overline{\overline{\mathbf{T}}}_{N_B}(\beta,\omega) \tag{14}$$

where κ denotes the coupling coefficient, $\boldsymbol{g}$ is the right eigenvector of $\mathbf{D}(\beta_0,\omega)$ that satisfies $\mathbf{D}^T(\omega,\beta_0)\cdot \boldsymbol{g} = \mathbf{0}$ and the superscript T denotes the transpose of the indicated vector and matrix.

Equations (10) and (11) govern the evolution of the mode amplitudes in individual post-wall waveguides. Their solutions describe the power transfer characteristics between two-parallel post-wall waveguides "*a*" and "*b*" coupled through the N_B -layered barrier. From Eqs.(10) and (11) we have two solutions $\Delta\beta = \pm\kappa$ for the perturbed mode propagation constants due to the coupling. Then the propagation constants for the *even* and *odd* modes in the coupled waveguide system are obtained as follows:

$$\beta_{even} = \beta_0 + \kappa \quad \text{for } even \text{ mode} \tag{15}$$

$$\beta_{odd} = \beta_0 - \kappa \quad \text{for } odd \text{ mode} . \tag{16}$$

IV. Numerical Results

To validate the coupled-mode analysis, we performed numerical computations on Eqs.(10)-(14). Although a substantial number of numerical examples could be generated, we consider here only a few examples of the coupled post-wall waveguides. We assume a square lattice with h=p for the upper and lower post-walls in Fig. 3. The numbers of post-wall layers are fixed to be N=3 for the upper and lower layers and N_B=1 for the barrier layer. Then the structure of post-wall waveguides is characterized by four parameters h, r/h, w/h, and $\varepsilon_s/\varepsilon_0$.

TABLE I shows the comparison of the normalized propagation constants $\beta_{even}h/2\pi$ and $\beta_{odd}h/2\pi$ calculated by the coupled-mode equations (10) and (11) with those obtained from a rigorous mode analysis for Eqs.(6) and (7). The results are tabulated from (*a*) to (*e*) for five different structural parameters. The propagation constants of the single waveguide have been analyzed in [3-5] for the structures (*a*), (*b*), (*c*), and (*d*). The propagation constant and the coupling coefficient for the two-parallel waveguides has been reported

TABLE I
NORMALIZED PROPAGATION CONSTANTS $\beta h/2\pi$ OF *EVEN* AND *ODD* MODES CALCULATED FOR COUPLED TWO-PARALLEL POST-WAVEGUIDES WITH $N=3$ AND $N_B=1$.

(a) $h = 2mm,\ r/h = 0.2,\ w/h = 3.6,\ \varepsilon_s/\varepsilon_0 = 2.33$

	Coupled-mode analysis (10), (11)		Rigorous analysis (6), (7)	
f [GHz]	$\beta_{even}h/2\pi$	$\beta_{odd}h/2\pi$	$\beta_{even}h/2\pi$	$\beta_{odd}h/2\pi$
15	0.04589	0.04051	0.04584	0.04045
20	0.14225	0.14054	0.14226	0.14055
25	0.20869	0.20747	0.20870	0.20748
30	0.26860	0.26760	0.26861	0.26761

(b) $h = 5.165mm,\ r/h = 0.075,\ w/h = 2.2767,\ \varepsilon_s/\varepsilon_0 = 2.2$

	Coupled-mode analysis (10), (11)		Rigorous analysis (6), (7)	
f [GHz]	$\beta_{even}h/2\pi$	$\beta_{odd}h/2\pi$	$\beta_{even}h/2\pi$	$\beta_{odd}h/2\pi$
8	0.09068	———	0.07510	———
10	0.17252	0.14697	0.17158	0.14489
12	0.24278	0.22483	0.24226	0.22356
14	0.30601	0.29143	0.30564	0.29034

(c) $h = 1.016mm,\ r/h = 0.3125,\ w/h = 3.9075,\ \varepsilon_s/\varepsilon_0 = 9.9$

	Coupled-mode analysis (10), (11)		Rigorous analysis (6), (7)	
f [GHz]	$\beta_{even}h/2\pi$	$\beta_{odd}h/2\pi$	$\beta_{even}h/2\pi$	$\beta_{odd}h/2\pi$
15	0.06742	0.06721	0.06741	0.06721
19	0.14152	0.14141	0.14152	0.14142
23	0.19782	0.19775	0.19783	0.19775
27	0.24878	0.24871	0.24878	0.24872

(d) $h = 3.556mm,\ r/h = 0.25,\ w/h = 2.3088,\ \varepsilon_s/\varepsilon_0 = 1.0$

	Coupled-mode analysis (10), (11)		Rigorous analysis (6), (7)	
f [GHz]	$\beta_{even}h/2\pi$	$\beta_{odd}h/2\pi$	$\beta_{even}h/2\pi$	$\beta_{odd}h/2\pi$
25	0.16143	0.15850	0.16146	0.15846
30	0.25502	0.25301	0.25501	0.25301
35	0.33343	0.33170	0.33346	0.33176
40	0.40574	0.40405	0.40569	0.40399

(e) $h = 12.5mm,\ r/h = 0.2,\ w/h = 2.0,\ \varepsilon_s/\varepsilon_0 = 1.0$

	Coupled-mode analysis (10), (11)		Rigorous analysis (6), (7)	
f [GHz]	$\beta_{even}h/2\pi$	$\beta_{odd}h/2\pi$	$\beta_{even}h/2\pi$	$\beta_{odd}h/2\pi$
9	0.26083	0.25334	0.26077	0.25325
10	0.31900	0.31237	0.31894	0.31229
11	0.37308	0.36675	0.37302	0.36668
12	0.42534	0.41877	0.42528	0.41868

in [10] for the structure (e).

From TABLE I we can see that for all structures the results obtained by the coupled-mode analysis are in close agreement with those of the rigorous mode analysis over a broad range of frequency. For the structure (*e*), since the *odd* mode of the rigorous analysis enters in a cutoff region at $f = 8.0 GHz$, we have discarded the corresponding propagation constant of the coupled-mode analysis. Note that the coupled-mode analysis always yields two propagation constants whenever the single post-wall waveguide in isolation supports one guided mode. The results given for the structure (*e*) agree well with the theoretical and experimental results reported in [10].

The coupling length L_c, which is the characteristic length for the complete power transfer from one waveguide to another, is given by

$$L_c = \frac{\pi}{\beta_{even} - \beta_{odd}} = \frac{\pi}{2\kappa}. \tag{17}$$

TABLE I demonstrates that the coupling length tends to increase as r/h increases and hence the gap width between the adjacent posts decreases. In order to design the directional coupler within a practical device length, we need to optimize the value r/h for the assumed frequency band.

V. Conclusion

We have presented a self-contained coupled-mode analysis for two-parallel post-wall waveguides based on the model of two-dimensional photonic crystal waveguides. The first-order coupled-mode equations have been systematically derived, which govern the evolution of the modal amplitudes in each individual post-wall waveguide. The coupling coefficients have been calculated by using the propagation constants and eigenmode solutions of the single post-wall waveguide in isolation. The proposed formulation provides a useful analytical and numerical technique for approximating the coupling between post-wall waveguides in close proximity with a good physical justification.

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