Simplified Wavelength Calculations for Fast and Slow Wave Metamaterial Ridged Waveguides and their Application to Array Antenna Design

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***Abstract*- Simple equations for calculating the wavelengths of the Fast and Slow waves in combined-mode metamaterial ridged waveguides are derived. The transverse resonance method is used in analyzing the fast wave and a distributed transmission line with series inductive reactance is used for the slow wave. The equations are used to show how the wavelength characteristics can be controlled and to find the angles of undesired lobes of array antennas in actual examples.**

I. INTRODUCTION

Metamaterial Ridged Waveguides (MRW) [1][2] and GAP waveguides [3][4] have the advantages of ease of manufacture and the reliability of non-contacting metal structures compared to ordinary hollow waveguides. In this paper, we use an analytical method to derive simple equations for the characteristics of MRWs.

The most significant parameter in designing a waveguide is the wavelength. Longer wavelengths than those in free space are preferable for electronic circuits in order to apply a uniform voltage over the terminals, and shorter ones than those in free space are preferable for feeding the lines of array antennas in order to suppress undesired lobes. For these flexible requirements, the fast and slow wave combined mode MRWs presented in the literature [5][6] provide adequate solutions.

On the other hand, fast and slow wave combined mode MRWs have complicated structures compared to ordinary waveguides, and require an EM-simulator for circuit design. However, in practice, it is hard to justify the time taken and cost of EM-simulation for calculating the wavelengths. For this reason, we propose a simple analysis and equations for calculating the wavelengths of fast and slow wave combined mode MRWs, which reduce the calculation time and improve design efficiency. Using these equations, the variation in wavelength with the physical size of the structure can be evaluated much faster than with EM-simulators. As an example to demonstrate their use, at each end of sections II and III, we apply the equations to an actual design in order to change the angles of undesired lobes.

In addition, we think this paper is beneficial for new researchers of MRWs, in that it provides an understanding of the physical phenomena involved in the propagation modes in MRWs with complex structures.

II. FAST WAVE

A. *Analysis of the fast wave*

Fig. 1 shows the structure and parameters of a fast and slow wave combined mode MRW. In the metamaterial region, the metal rods are $\lambda/4$ in height. The upper and lower plates are not in contact with each other with the gap between them selected to be $\lambda/8$ (the essential condition is that this is less than $\lambda/4$) in order to prevent parallel plate mode propagation and to confine the electro-magnetic energy to the ridge.

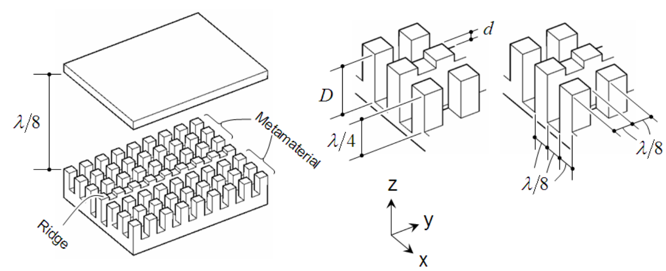

Figure 1. Structure and parameters of the fast and slow wave combined mode MRW.

The transverse resonance method is a well known conventional method for analysing the fast wave. As shown in the literature [5][6], the wavelength of the fast wave λ_f can be described by the following equation, where ϕ is the phase of the reflection from the EMB (Equivalent Magnetic Boundary) of a TEM wave injected from the centre of the ridge in the transverse direction (x-direction in Fig. 1), and where the reflection coefficient at the EMB is 1.

$$\frac{\lambda_f}{\lambda} = \sqrt{\frac{1}{1-(2\pi/\phi)^2}} \tag{1}$$

where λ is the wavelength in free space.

If the characteristic impedance at the centre of the ridge is Z_o, and the input impedance in the transverse direction from the edge of the ridge is Z_t, then ϕ can be described by the following equation.

$$\phi = Ang\left(\frac{Z_t/Z_o - 1}{Z_t/Z_o + 1}\right) \tag{2}$$

Note that an equivalent circuit for the transverse resonance from the centre of ridge can be constructed, as shown in Fig. 2. There are parasitic reactances at the corners of the ridge as shown in Fig. 3, which can be calculated by equations obtained from the literature [7], with the results shown in Table 1. We assume the position of EMB is as shown in Fig. 2 and therefore Z_m is the impedance of an open-ended-line, $\lambda/8$ in length. Note that Z_t is the impedance containing Z_m, a short-ended-line of length D (see Fig. 1) and a line of length $\lambda/16$, therefore Z_t can be written as follows.

$$Z_t / Z_o = \left(Z + jZ_o \tan\frac{\pi}{8} \right) \Big/ \left(Z_o + jZ \tan\frac{\pi}{8} \right) \qquad (3)$$

$$Z = jZ_o \left(-\cot\frac{\pi}{4} + \tan 2\pi\frac{D}{\lambda} \right) \qquad (4)$$

These equations mean that the wavelength ratio of fast wave to TEM wave, λ_f/λ, can be calculated from D/λ. It is necessary to include parasitic reactances, such as shown in Fig. 3, and select angles of $-720<\phi<-360$ before substituting into equation 1, because the waveguide boundary is at EMB.

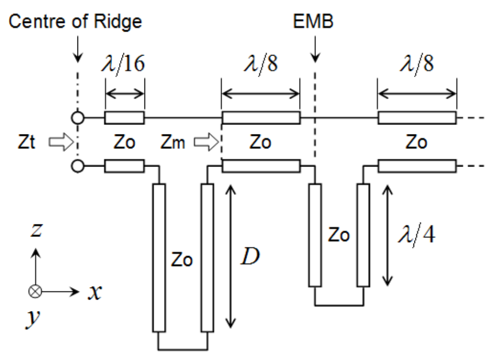

Figure 2. Equivalent circuit in transverse resonance direction.

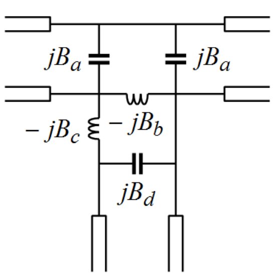

Figure 3. Parasitic reactance at a corner.

TABLE I
Parasitic reactance values.

B_a	B_b	B_c	B_d
$\frac{0.333}{Z_o}$	$\frac{0.109}{Z_o}$	$\frac{1.273}{Z_o}$	$\frac{0.196}{Z_o}$

B. Analytical result

Fig. 4 shows the wavelength ratio λ_f/λ as a function of trench depth D, calculated using the analytical method presented in this paper. This is compared with the results from an EM-simulator. The values are shown on the left ordinate, where λ_{fo} is the wavelength for D=4λ/16. As can be seen in Fig. 4, the two plots are similar, both decreasing as D increases. The difference between the results is less than 5% at D=6λ/16, which means the analytical method presented in this paper is sufficiently accurate for approximating the wavelength in a MRW.

The curve produced by the analytical method can be represented by the following 3rd order polynomial.

$$\frac{\lambda_f}{\lambda_{fo}} = -16.79\left(\frac{D}{\lambda}\right)^3 + 21.40\left(\frac{D}{\lambda}\right)^2 - 9.95\left(\frac{D}{\lambda}\right) + 2.41 \qquad (5)$$

Using equation 5, the variation in wavelength ratio can be obtained much more quickly and easily than with an EM-simulator.

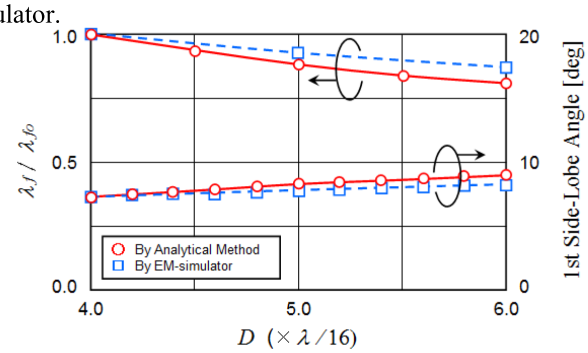

Figure 4. Wavelength of the fast wave and the angle of the 1st side lobe of a 10 element array with a resonant feed line calculated using equation 5.

C. Design example

A design example using the equations derived above is given in this section. One of the applications that adopts the fast and slow wave combined mode MRW is the feed line of an array antenna, as mentioned at the beginning of the paper. There are two types of feed line for array antennas, one is a travelling wave type and the other is a resonance type. The resonance type was selected because its waveguide structure is the same over the whole circuit for narrow bandwidth systems [1][2]. For automotive radar applications, as described in [1][2], the performance of undesired lobes is very important for evaluating reflected waves. Therefore, in our work, the variation in the angle of the 1st side lobe was calculated for the wavelengths obtained using equation 5 and the EM-simulator. The results are shown in Fig. 4 with the 1st side lobe angle given on the right ordinate. Fig. 5 shows the side-lobe characteristics as a function of the depth D for a 10 element liner array antenna. In Figs. 4 and 5, the value of λ_{fo} for D=4λ/16 is taken from simulation; the values of λ_f are calculated from equation 5.

As can be seen in Fig. 4, for 4λ/16 <D< 6λ/16, the angle of the 1st side lobe from equation 5 and the EM-simulator are in good agreement, showing that there is no significant difference between the analytical method and EM-simulation.

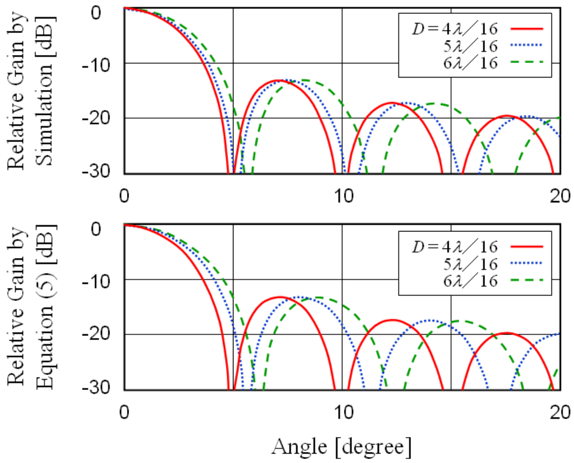

Figure 5. Side-lobe characteristics as a function of the depth D for 10 elements liner array antenna.

III. SLOW WAVE

A. *Analysis of the slow wave*

In general a slow wave effect can be realized by having a periodic stepped structure to increase the equivalent length of the waveguide. This is equivalent to inserting inductive impedances in series with the waveguide. Fig. 6 shows cross sections of the MRW with and without periodic steps and their equivalent circuits. As shown in Fig. 6b, the periodic steps with depth d ($d<\lambda/4$: see Fig. 1) work as short-ended-lines with impedance Z_s. In series with the waveguide these produce a waveguide with a longer equivalent length.

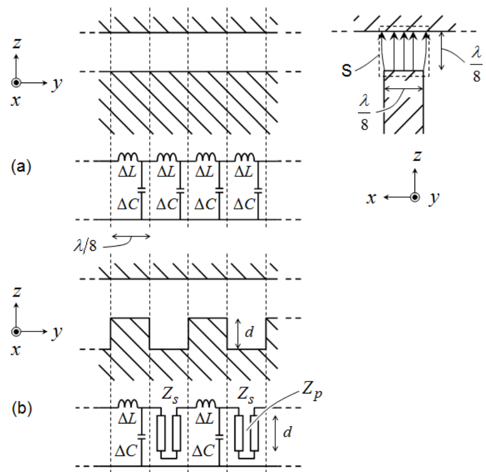

Figure 6. Cross sections of the MRW with and without periodic steps and their equivalent circuits.

Analysis of the slow wave proceeds as follows. The equivalent circuit for each $\lambda/8$ section has a small series inductance, ΔL, and a small shunt capacitance, ΔC. With periodic steps ($d\neq0$), alternate $\lambda/8$ sections are replaced by Z_s, as shown in Fig. 6b. Therefore the ratio of the wavelengths of the combined fast and slow wave mode ($d\neq0$) to the fast wave (d=0), λ_{fs}/λ_f, is expressed as

$$\frac{\lambda_{fs}}{\lambda_f}=\sqrt{\frac{\Delta L\Delta C}{\left(\frac{\Delta L}{2}+\frac{Z_s}{\omega}\right)\frac{\Delta C}{2}}}=2\sqrt{\frac{\Delta L}{\Delta L+\frac{Z_s\lambda}{\pi v_c}}} \tag{6}$$

where v_c is the velocity of light.

Equation 6 means that λ_{fs}/λ_f can be obtained from ΔL and Z_s. ΔL is obtained as follows. In the MRW structure, it can be assumed that most of the electric field is confined to the region S, shown on the upper-right hand side of Fig. 6. Therefore, ΔC is equal to the ideal parallel plate capacitance of a $\lambda/8$ cube, given by

$$\Delta C=\varepsilon\frac{(\lambda/8)^2}{\lambda/8}=\varepsilon\frac{\lambda}{8} \tag{7}$$

where ε is the permittivity of air.

Define the characteristic impedance of the ridge for d=0 as Z_f

$$Z_f=\sqrt{\frac{\Delta L}{\Delta C}} \tag{8}$$

Since the height and width of region S are equal ($\lambda/8$), the characteristic impedance of the ridge for a TEM wave propagating in region S is 120π. However, a fast wave (TE wave) propagates on the ridge, so the magnetic field normal to the axis of the waveguide, H', can be given in terms of the wavelength ratio of the fast wave to that in free space, λ_f/λ, by

$$H'=H\frac{\lambda}{\lambda_f} \tag{9}$$

This means that Z_f can expressed as

$$Z_f=\frac{E}{H'}=\frac{E}{H}\frac{\lambda_f}{\lambda}=120\pi\frac{\lambda_f}{\lambda} \tag{10}$$

From equations 7, 8 and 10

$$\Delta L=\varepsilon\frac{\lambda}{8}\left(120\pi\frac{\lambda_f}{\lambda}\right)^2 \tag{11}$$

Z_s is obtained as follows. The periodic step is equivalent to a short-ended-line with a cross section of $\lambda/8$ square and no waveguide wall in the $-z$ direction, as shown in Fig. 6. Therefore, a TEM wave propagates in it. Its characteristic impedance, Z_p, (see Fig. 6) is 120π, so that the impedance of the periodic step, Z_s, is

$$Z_s=Z_p\tan\frac{2\pi d}{\lambda}=120\pi\tan\frac{2\pi d}{\lambda} \tag{12}$$

With ΔL and Z_s the wavelength ratio of combined mode of the fast and slow wave to the fast wave, λ_{fs}/λ_f, can be calculated from equation 6. The parasitic reactances at the corners, with the same values given in Table 1, also need to be included.

B. Analytical result

Fig. 7 shows the wavelength ratio λ_{fs}/λ_f as a function of step depth d, calculated using the analytical method presented in this paper and this is compared with the results from an EM-simulator. The values are shown on the left ordinate, where λ_{fso} is the wavelength for $d=\lambda/16$. The value of λ_f in equation 9 was calculated with $D=\lambda/4$. As can be seen immediately from Fig. 7, the EM-simulations didn't converge at higher values of d, and therefore no results were obtained for $d>2\lambda/16$, which sometimes occurs for low loss structures in time domain simulators. This has a serious effect on the design efficiency. On the other hand, the analytical method presented here not only gives results which are in accordance with the simulated results for $d<2\lambda/16$, but also calculates the wavelength ratio for $d>2\lambda/16$, showing that the wavelength ratio is reduced by around 50% over the whole range.

The curve produced by the analytical method can be represented by the following 3rd order polynomial.

$$\frac{\lambda_{fs}}{\lambda_{fso}} = -124.93\left(\frac{d}{\lambda}\right)^3 + 43.99\left(\frac{d}{\lambda}\right)^2 - 8.56\left(\frac{d}{\lambda}\right) + 1.39 \quad (13)$$

Using equation 13, the variation in wavelength ratio can be obtained much more quickly and easily than with an EM-simulator.

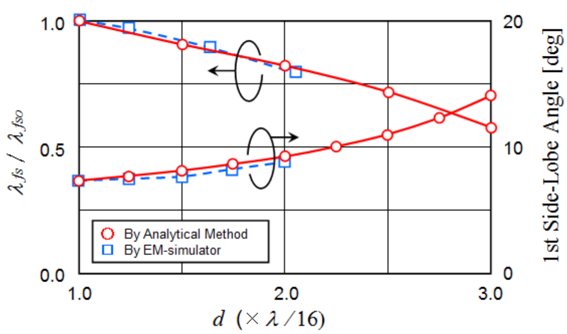

Figure 7. Wavelength of the fast and slow wave combined mode, and the 1st side lobe angle of a 10 element array with a resonant feed line calculated using equation 13.

C. Design example

As was done for the fast wave, the variation in the angle of the 1st side lobe was calculated for the wavelengths obtained using equation 13 and the EM-simulator. The results are shown in Fig. 7, with the values shown on the right ordinate. Fig. 8 shows the side-lobe characteristics as a function of the depth d for a 10 element liner array antenna. In Figs. 7 and 8, the value of λ_{fso} for $d=\lambda/16$ is taken from simulation; the values of λ_{fs} are calculated from equation 13.

As shown in Fig. 7, the EM-simulator cannot evaluate the 1st side lobe angle for $d>2\lambda/16$ because the simulations didn't convergence, whereas the analytical method can evaluate this, and shows that the 1st side lobe angle changes from 7^{o} to 14^{o} when the depth d varies from $\lambda/16$ to $3\lambda/16$.

The above study demonstrates that the analytical method presented in this paper can accurately calculate wavelength changes not only for the fast wave but also for the combined mode much more quickly and easily than EM-simulation and also evaluate undesired lobe characteristics of array antennas.

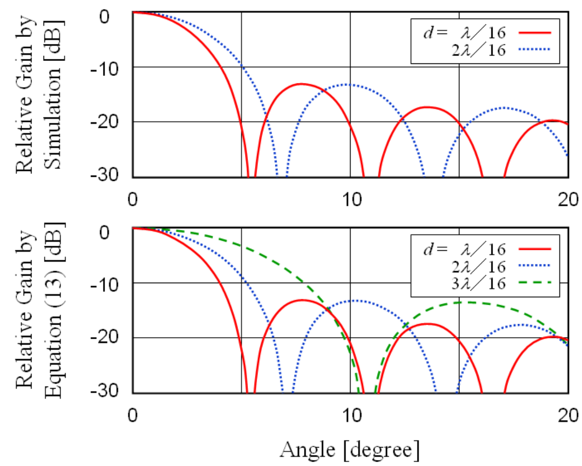

Figure 8. Side-lobe characteristics as a function of the depth d for a 10 element linear array antenna.

IV. CONCLUSION

Simple equations for the wavelengths of Fast and Slow wave combined-mode MRWs were proposed. The transverse resonance method was used to analyze the fast wave and a distributed transmission line with series inductive reactance was used for the slow wave.

Whereas the EM-simulations didn't converge for $d>2\lambda/16$, the analytical equations covered the range $\lambda/16<d<3\lambda/16$ and revealed the characteristics of the undesirable lobes of array antennas in actual examples.

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