A 3-D FDTD Scheme for the Computation of HPM Propagation in Atmosphere

Ke Xiao, Shunlian Chai, Haisheng Zhang, Huiying Qi, and Ying Liu
College of Electronic Science and Engineering, National University of Defense Technology,
Changsha, Hunan 410073 China

Abstract- **Focus on the simulation of the dynamics of high power microwave (HPM) ionization of air at atmosphere, we analyze the physical mechanism in the formation of plasma. The magnetohydrodynamic formulations, formed by Maxwell equations coupled with simple fluid description, are applied to describe the ionization procedure. Then, we present a three dimensional (3-D) finite-difference time domain (FDTD) scheme to deal with such problems, of which the computational procedure is discussed in detail. At last, numerical experiments are provided, and some parameters about plasma are analyzed. The theory also predicts the pulse breakdown threshold accurately.**

I. INTRODUCTION

Air ionization using high power microwave (HPM) source in a range of pressures and frequencies has been extensively investigated. Most of work was motivated by the need to avoid breakdown in RF transmission systems in waveguides and near antennas, some of the applications involve earth-space communications and creation of artificial ionized layers in the atmosphere, as well as antennas for high-power microwave transmission [1][2].

Early studies [3][4] of air ionization focused on the determination of the breakdown field as a function of several parameters such as pressure, frequency, and pulse duration, the approaches generally use the effective field concept, where it was calculated assuming a simple relationship between momentum transfer collision frequency and pressure, besides, all calculations were made assuming that diffusion is negligible. However, it is difficult for us to use the effective field model to investigate the ionization procedure, since the complex coupling between the wave and the plasma can not be described analytically and numerically.

In order to investigate the mechanism of ionization under HPM, we should describe the procedure using exact physical and computational model based on kinetic theory [5], one is the microcosmic model describing the motion of particles, which is simple but the interaction between particles can not be considered. Another is magnetohydrodynamic equations (MHD) based on macroscopic formulations of plasma. Based on the MHD theory, Yee et al. [6] provided a one-dimensional method based on fluid equations and Maxwell equations to investigate the dynamic behavior of intense electromagnetic pulse propagating through atmosphere. In [7], a simple 1D model consists of a 1D plasma fluid model and a 1D wave equation was developed to investigate the experimental observations. The pattern description, however, needs at least a two-dimensional (2D) approach, such studies includes works by J. P. Boeuf et al. [8][9], where coupling Maxwell equations with plasma fluid equations were applied to describe the formation of plasma under conditions similar to the experiments[10]. Recently, differential solver such as finite-difference time domain (FDTD) method has been proposed to analyze the hybrid equations [9][11]. Briefly, there are always two ways to consider the model of plasma in FDTD, one way is to treat the generated plasma in atmosphere as air filled with induced plasma currents [9][11], another way is the usage of equivalent dielectric medium theory[5].

In this paper, we use the coupling Maxwell equations with plasma fluid equations to describe the procedure of HPM propagating in atmosphere, a continuity equation is also applied to describe the electron density. Then, a 3-D FDTD method is provided to solve the hybrid equations, afterwards, power thresholds for breakdown of incident HPM pulses, as well as plasma parameters (e.g. electron density, plasma frequency) are to be analyzed, and by comparing to the traditionally calculated results, the FDTD method is demonstrated to be more accurate.

II. PHYSICAL AND COMPUTATIONAL MODEL

We know that, under the condition of HPM propagating in atmosphere, the quiver energy of free electrons increases, as well as the collisions between charged particles and neutral molecules, where some kinetic energy is transformed to thermal energy since inelastic collisions exists. In the process, when the quiver energy of bound electron is higher than the energy required for ionizing, the bound electrons in neutral molecules may acquire sufficient energy to escape and become free ones, which we called ionization.

A. Magnetohydrodynamic Formulations

To analyze the process, Maxwell equations are coupled with the Boltzmann equation and air plasma transport equations through the electron current density

$$\nabla\times\vec{E}=-\mu_0\frac{\partial\vec{H}}{\partial t} \tag{1}$$

$$\nabla\times\vec{H}=\varepsilon_0\frac{\partial\vec{E}}{\partial t}+\vec{J}_e \tag{2}$$

$$\vec{J}_e=-eN_e\vec{v}_e \tag{3}$$

where, $\vec{E}$ and $\vec{H}$ represent the electric and magnetic field of HPM, $\vec{J}_e$ is the plasma current density induced by the incident waves, N_e is the electron density, e denotes the electron charge, $\vec{v}_e$ is the electron mean velocity, μ_0 and ε_0 express the magnetic permeability and electric permittivity of vacuum, respectively, where the ion contribution to the current density is neglected.

In order to obtain the plasma current density in (3), N_e and $\vec{v}_e$ should be calculated. From the simplified electron momentum transfer equation as shown in (4), an approximation to the mean electron velocity can be derived.

$$m_e \frac{d\vec{v}_e}{dt} = -e\vec{E} - m_e \vec{v}_e v_m \tag{4}$$

where, m_e is electron mass, v_m represents the momentum transfer collision frequency. The time evolution of the free electrons density N_e in atmosphere can be represented as follows associated with ionization, diffusion, attachment and recombination [9]

$$\frac{dN_e}{dt} = v_i N_e - v_d N_e - v_a N_e - v_r N_e^2 + v_{r0} N_{e0}^2 \approx v_{net} N_e \tag{5}$$

where, v_{net} denotes the net ionization frequency, and we use the approach in [12] to express v_{net} as follows:

$$v_{net} = 9.0\times10^{-9} N_e \frac{\tilde{\varepsilon}}{1+v_m^2/\omega^2} \exp\left[-\sqrt{\frac{2\mathrm{eV}}{\tilde{\varepsilon}/\left(1+v_m^2/\omega^2\right)}} \right] \tag{6}$$

where, v_m is estimated by $3\times10^{-7}N_e$ [12], and $\tilde{\varepsilon}$ represents the quiver energy of an electron:

$$\tilde{\varepsilon} = \frac{1}{2} m_e \frac{e^2 E_0^2}{m_e^{\,2} \omega^2} \tag{7}$$

where E_0 is the peak amplitude of an oscillating electric field.

B. FDTD Solver for Hybrid Equations

To solve the magnetohydrodynamic equations numerically, it is needed to transform them to an equivalent set of equations on discrete time, electromagnetic fields and currents (relate to average velocity of electron and its density). One of the powerful method is the FDTD method, which is an explicit time-domain method using centered finite differences on the Yee space lattice [13]. As it is known that, in the total-field FDTD formulation, the incident wave is propagated through the grid, while in a scattered-field FDTD codes used here, the incident wave can be generated accurately via an exact analytical function at each location. The electric and magnetic field update equations in (1) and (2) are obtained by decomposing the field components into incident and scattered terms as

$$\boldsymbol{E}^t = \boldsymbol{E}^i + \boldsymbol{E}^s \tag{8a}$$

$$\boldsymbol{H}^t = \boldsymbol{H}^i + \boldsymbol{H}^s \tag{8b}$$

where subscripts i, s, and t stand for incident, scattered, and total fields, respectively. Then, the magnetohydrodynamic formulations can be discretized using leapfrog approximations. Without loss of generality, we consider the calculation of field component in x-direction, the difference scheme for Maxwell equations can be expressed as

$$H_x^s\Big|_{i,j+\frac12,k+\frac12}^{n+1/2} = H_x^s\Big|_{i,j+\frac12,k+\frac12}^{n-1/2} - \frac{\Delta t}{\mu_0}\left[\frac{E_z^s\Big|_{i,j+1,k+\frac12}^{n} - E_z^s\Big|_{i,j,k+\frac12}^{n}}{\Delta y} - \frac{E_y^s\Big|_{i,j+\frac12,k+1}^{n} - E_y^s\Big|_{i,j+\frac12,k}^{n}}{\Delta z} \right] \tag{9}$$

$$E_x^s\Big|_{i+\frac12,j,k}^{n+1} = E_x^s\Big|_{i+\frac12,j,k}^{n} + \frac{\Delta t}{\varepsilon_0}\left[\frac{H_z^s\Big|_{i+\frac12,j+\frac12,k}^{n+1/2} - H_z^s\Big|_{i+\frac12,j-\frac12,k}^{n+1/2}}{\Delta y} - \frac{H_y^s\Big|_{i+\frac12,j,k+\frac12}^{n+1/2} - H_y^s\Big|_{i+\frac12,j,k-\frac12}^{n+1/2}}{\Delta z} + e N_e\Big|_{i+\frac12,j,k}^{n} v_{ex}\Big|_{i+\frac12,j,k}^{n+1/2} \right] \tag{10}$$

Approximation for electron momentum equation (4) can be written as

$$v_{ex}\Big|_{i+\frac12,j,k}^{n+1/2} = -\frac{2e\Delta t}{m\left(2+v_m\Delta t\right)} E_x^t\Big|_{i+\frac12,j,k}^{n} + e^{-v_m\Delta t} v_{ex}\Big|_{i+\frac12,j,k}^{n-1/2} \tag{11}$$

The continuity equation (5) is solved using a simple analytical scheme as follows

$$N_e\Big|_{i,j,k}^{n+1} = N_e\Big|_{i,j,k}^{n} e^{v_{net}\Delta t} \tag{12}$$

We use the same grid spacing, noted as Δ in the x-, y- and z-directions, and it is chosen as $\Delta\leq \lambda/12$. Additionally, by considering the stability for hybrid equations, the time step Δt of the FDTD scheme should satisfy

$$\Delta t \leq \min\left\{ \frac{\Delta}{v_C\sqrt{3}},\ \frac{1}{10v_m},\ \frac{1}{v_C}\left(\frac{1}{\Delta^2}+\frac{e^2\mu_0 N_e}{4m_e}\right)^{-1/2},\ \frac{1}{10v_{net}} \right\} \tag{13}$$

where v_c is the velocity of light in vacuum.

Convolutional PML (CPML) is applied here as absorbing boundary condition (ABC) [14], which is based on a recursive-convolution technique.

III. Results and Discussion

In this section, we show the 3-D simulation results for ionization performance in atmosphere around altitude of 60km, with 300MHz incident HPM transmit vertically along the z-direction, the magnitude the sinusoidal wave is E_0, the pulse width is 1μs. The simulation domain is $1\lambda\times1\lambda\times30\lambda$ ($z\times y\times z$), the grid size is set to $\Delta= \lambda/20$, and the time step $\Delta t = 0.9\Delta/v_c/\sqrt{3}$, 10-cell CPML are used as ABCs, and 8 cells are defined between the ABCs and plasma domain.

Calculation with four different E_0 (5×10^4, 8×10^4, 9×10^4, 10^5 V/m) have been performed, the electron density is compared with the breakdown threshold ($N_e= 10^8N_{e0}$), from the results, the threshold electric field for breakdown is 9×10^4 V/m $<E_0<10^5$ V/m.

Fig. 1 shows the electron density when $E_0=10^5$ V/m after one pulse, the peak electron density has reached over 10^{14} cm^{-3}, which exceeds the breakdown threshold ($10^8N_{e0}\approx 10^{9.9}$ cm^{-3}). Fig. 2 shows the calculated plasma frequency, and the peak point reached about 15MHz. Fig. 3 depicts the power density of total field, where in the ionized region, for the effects of absorbing and reflection, the power density of waves has been disturbed obviously, it costs about 12.28 minutes for the calculation on a PC workstation.

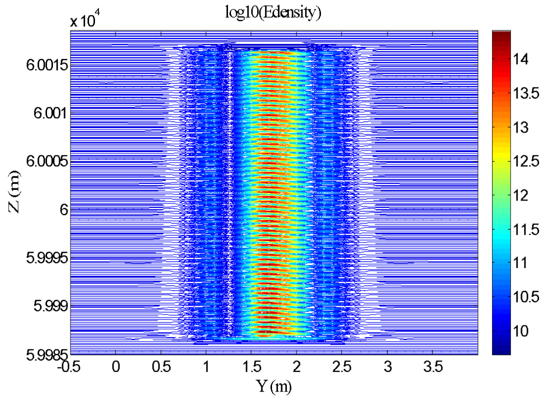

Figure 1. Evolution of electron density (cm^{-3}) in plane x=0.

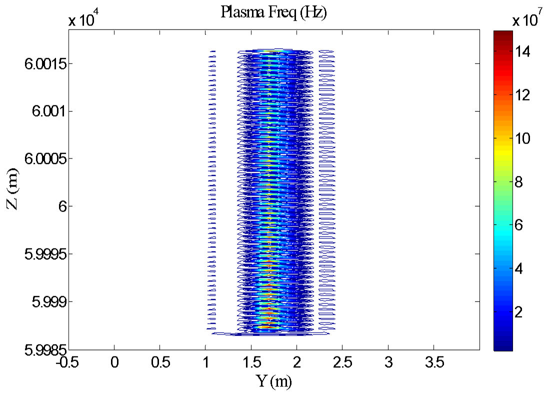

Figure 2. Calculated plasma frequency (Hz) in x=0 plane.

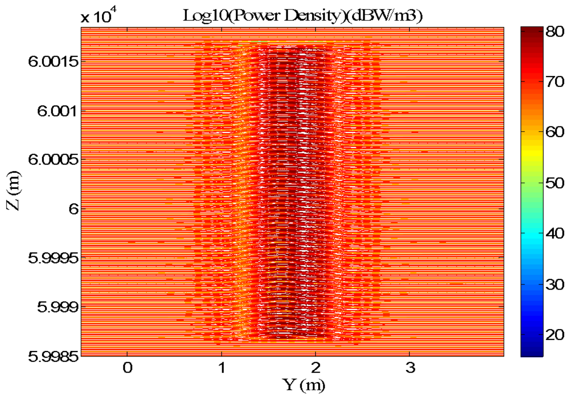

Figure 3. Calculated power density (dBW/m^3) in plane x=0.

It can be concluded from the results that, the threshold of power density for breakdown is 67.3 dBW/m^3 $<P_d<$68.2 dBW/m^3, while, by using the traditional ionization theory based on effective field approach and equivalent DC experiments [3][4], the air-breakdown-threshold is about 64.8 dBW/m^3 which is less than the 3-D FDTD results. However, there are many limitations to the effective field model, since diffusion is negligible and the dense coupling between wave and plasma can not be considered numerically, besides, the results in chamber were obtained under certain conditions, e.g. restricting the range of v_m/ω and E_e/P. So the results obtained by FDTD method are more accurate.

IV. Conclusion

In this paper, we use the magnetohydrodynamic formulations as the physical model to consider the ionization, in which, coupling Maxwell equations with plasma fluid equations are applied to describe the procedure. Then, in order to describe the formation of 3-D plasma patterns during HPM propagation in atmosphere, we develop a 3-D FDTD solver to simulate the time-vary procedure. An example is provided to demonstrate the efficiency of the method, where, a simulation domain with size $1\lambda\times1\lambda\times30\lambda$ near the attitude of 60km is defined, a sinusoidal plane wave is assumed to propagate along z-direction, CPML is set as the ABCs. By changing the input power density and observing the calculated parameters involving the electron density and plasma frequency, we indicate a more reasonable breakdown threshold, compared to the traditional results.

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