An Efficient Modal Series Representation of Green's Function of Planar Layered Media for All Ranges of Distances from Source Using CGF-PML-RFFM

A. Torabi, A. A. Shishegar
Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran

Abstract- **A closed-form series representation for spatial Green's function of planar layered media for all distances from source, is presented. By terminating the structure by perfectly matched layer (PML) that is backed by perfect electric conductor (PEC) in semi-infinite layer at the top and/or bottom, the discreet set of surface wave (SW) poles is complemented by eigenmodes of the closed structure by PML which construct the continuous spectrum contribution of the original structure. Then applying characteristic Green's function (CGF) technique, a closed-form representation of spatial Green's function is derived. Very close to the source, where the large number of modes must be considered, the method is become inefficient. By combining CGF technique and rational function fitting method (RFFM), Green's function of very near field would be efficiently constructed with few number of poles extracted in modified VECTFIT algorithm in similar form of CGF-PML result. In this way, an efficient modal series representation is derived by using CGF-PML and CGF-RFFM for far from and close to the source respectively. The main advantage of this representation is that for desired accuracy the number of required modes is controllable. Excellent agreements with direct numerical integration of the spectral integral are shown in several examples.**

I. INTRODUCTION

One of the most extensively studied topics in electromagnetics is the analysis of printed circuits embedded in planar layered media. To have excellent accuracy and also fast computation, integral equation based techniques could be used. In these methods such as electric field integral equation (EFIE) potential form of the Green's functions are required. The spatial domain Green's function of vector and scalar potential are represented as oscillatory Sommerfeld integrals and generally do not have analytical solution. Due to highly oscillating integrands and slow decaying, numerical integration is expensive.

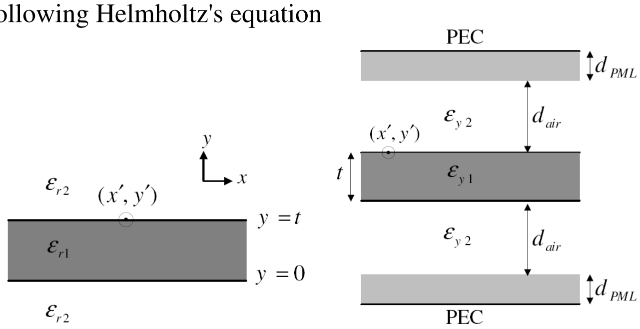
In the literature, different forms of series representation have been proposed for layered media. It is showed that by combining discrete complex images technique (DCIT) with characteristic Green's function (CGF) method and using well-known Weyl's identity, a closed form expression for spatial Green's function can be obtained [1], [2]. CGF-CI is fast since it needs no numerical integration attempt. In DCIT [3], the most important and cumbersome steps is the analytic extraction of the surface wave poles and also quasi-static part of the spectrum before the complex exponential approximation via the generalized pencil of function (GPOF) method [4]. Although by choosing the sampling path attentively, extraction of surface waves part can be removed but for long distances from the source, extraction of surface wave poles is usually necessary for high precision. Numerical technique like finite difference (FD) [5] can be convenient to implement but it has not sufficient accuracy specially for near field regions. Another interesting technique to evaluate the Sommerfeld integrals resulting in series presented in [6] which is based on a reduced order modeling of differential equation of spectral domain. By applying VECTFIT algorithm for spectral Green's function in rational function fitting method (RFFM), a uniform series expression for spatial Green's function can be obtained [7], [8]. The main challenge of RFFM is that the generated complex poles behave exponentially increasing in semi-infinite layer at the top and/or bottom like leaky wave poles. Perfectly matched layer (PML) was introduced by Berenger has been used to analyze open waveguide problems involving microstrip substrate. In this technique, by adding a metal backed PML to an open layered media, the structure becomes closed without changing its electromagnetic behavior. Then, an efficient continuous spectrum contribution in terms of a set of discreet modes of the closed waveguide is possible [8].

In this paper, combination of characteristic Green's function method and PML technique is used for series representation of spatial domain Green's function of infinite planar dielectric substrate. The main advantage of CGF-PML result is the analytically knowledge of source and observation points dependencies. Very close to the source, to have sufficient accuracy, a large number of PML modes must be considered in derived series expression which makes the method inefficient due to slowly convergent. For very near field regions combination of CGF method and RFFM is utilized. In this way field distribution could be expressed with few numbers of poles whereas the final form of CGF-RFFM is similar to CGF-PML. Therefore, with finite number of poles, a uniform series representation of spatial Green's function is derived for all distances from the source.

This paper is organized as follows. CGF-PML is briefly described in section II. CGF-RFFM is studied in section III. Numerical results and validation of the method are presented in section IV. Eventually, conclusion is provided in section V. Meanwhile $e^{j\omega t}$ time dependency is used.

II. CGF-PML FORMULATION

For Green's function of magnetic vector potential, A_z , the following Helmholtz's equation



(a) (b)

Figure 1. (a) A line source on a infinite dielectric substrate, (b) A line source on an infinite dielectric substrate closed structure by PML.

$$[\nabla^2 + k_0^2 \varepsilon_r(x, y)] A_z(x, y; x', y') = -\delta(x - x')\delta(y - y'), \quad (1)$$

should be solved along with proper boundary conditions. In the CGF formulation [1], [2], the studied structure is separated into two 1-D layered media (called N_x (read normal to x) and N_y (read normal to y)) i.e.

$$\varepsilon_r(x, y) = \varepsilon_x(x) + \varepsilon_y(y). \quad (2)$$

In this way, G_x and G_y are solutions of the 1-D Helmholtz's equation in the N_x and N_y layered media respectively. That is

$$\frac{d^2 G_\gamma}{d\gamma^2} + \left(\varepsilon_\gamma(\gamma) k_0^2 + \lambda_x\right) G_\gamma = -\delta(\gamma - \gamma'), \quad (3)$$

where $\gamma = x$, y and $\lambda_x + \lambda_y = 0$. For an infinite dielectric substrate shown in Fig. 1(a), it can be shown that this separation is rigorously possible [1], [2]. One choice for the N_x and N_y structures' parameters is: $\varepsilon_{x1} = \varepsilon_{x2} = 0$, $\varepsilon_{y1} = \varepsilon_{r1}$ and $\varepsilon_{y2} = \varepsilon_{r2}$. Therefore, using CGF technique formulation, A_z of the original 2-D Helmholtz's equation is represented exactly in terms of its Characteristic Green's function G_x and G_y as [1]

$$A_z(x, y; x', y') = \left(\frac{-1}{2\pi j}\right) \oint_{C_{\lambda y}} G_x(x, x', -\lambda_y) G_y(y, y', \lambda_y) d\lambda_y, \quad (4)$$

$$G_y(y, y', \lambda_y) = \frac{\left(1 + R_y e^{-j2\beta_{y1} y_<}\right)\left(1 + R_y e^{-j2\beta_{y1}(t - y_>)}\right) e^{-j\beta_{y1}(y_> - y_<)}}{2j\beta_{y1}\left(1 - R_y^2 e^{-j2\beta_{y1} t}\right)}, \quad (5)$$

$$R_y = \frac{\beta_{y1} - \beta_{y2}}{\beta_{y1} + \beta_{y2}}, \ \beta_{y1,2} = \sqrt{\varepsilon_{y1,2} k_0^2 + \lambda_y},$$

$$G_x(x, x', \lambda_x) = \frac{e^{-j\sqrt{\lambda_x}|x - x'|}}{2j\sqrt{\lambda_x}}, \quad (6)$$

and integration contour $C_{\lambda y}$ should enclose only the singularities of G_y characteristic Green's functions, in a counterclockwise sense as shown in Fig. 2(a) [1]. Numerical integration of the integral in (4) is very expensive and time-consuming. To circumvent this integration we can use PML technique. Consider again an infinite substrate like N_y structure which is shown in Fig. 1(a) with thickness of t . Then, above and below the substrate, an air regions are considered with thickness of d_{air} terminated by a PMLs that are backed by PEC shown in Fig. 1(b). For PMLs, thickness of d_{PML} and material parameters κ_0 and σ_0 are considered. It is shown that by stretching the coordinates, the air region can be combined with the PML to form a single layer with complex thickness $\tilde{d}_{air} = d_{air} + d_{PML}(\kappa_0 - j\sigma_0 / \omega\varepsilon_0)$ [10]. In this way modal analysis of the closed structure would be relatively simple.

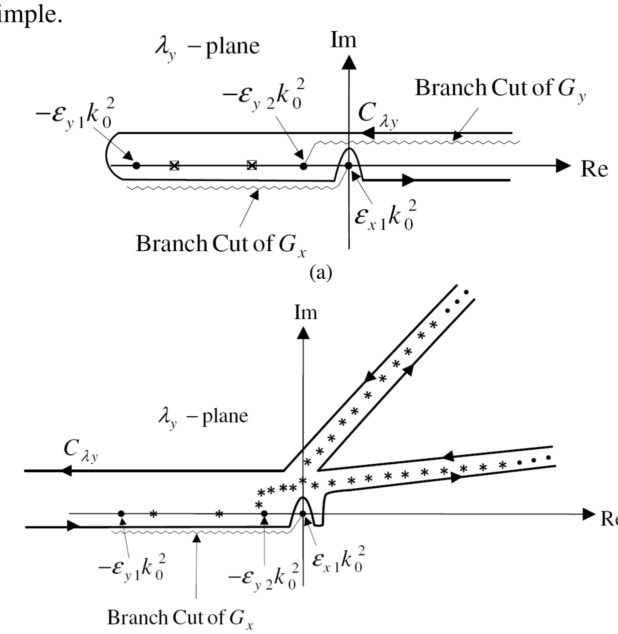


Figure 2. Integration path in λ_y plane, (a) ⊠: SW poles of G_y (b) ∗: SW poles of $G_y^{(c)}$.

If PMLs properly work, then CGF G_y of open structure in Fig. 1(a) would be the same as the CGF of the closed structure of Fig. 1(b) which is named by $G_y^{(c)}$ ((c) denotes 'closed') which can be achieved simply by using usual spectral techniques. CGF $G_y^{(c)}$ has no branch cut and branch point singularities. Whereas, $G_y^{(c)}$ will have infinite discreet poles singularities which can be computed by argument principle method (APM) [11]. The poles of $G_y^{(c)}$ are the eigenmodes of the closed structure and can be categorized into odd and even TE modes. Then, closed path $C_{\lambda y}$ must include infinite number of discreet poles of $G_y^{(c)}$, which is demonstrated in Fig. 2(b). Now by using well-known Cauchy-Riemann theorem for closed path $C_{\lambda y}$ in Fig. 2(b), a series representation for integral of (4) can be obtained as (7) which can be truncated by definite number of PML poles efficiently. Except real poles, two types of eigenmodes can be regarded which are called *Berenger* and *quasi-leaky* wave poles. It must be noted that independent of source and observation location, extraction of PML poles must be done for once and therefore for multiple $\mathbf{r}$ and $\mathbf{r}'$, A_z is simply computed by (7).

$$A_z(x, y; x', y') = \sum_{p=1}^{\infty} -\mathrm{Res}_p \frac{e^{-j\beta_{xp}|x - x'|}}{2j\beta_{xp}}, \quad (7)$$

where $\beta_{xp} = \sqrt{-\lambda_{yp}}$ and $\text{Res}_p = \lim_{\lambda_y \to \lambda_{yp}} \left(\lambda_y - \lambda_{yp}\right) G_y^{(c)}$.

III. CGF-RFFM FORMULATION

For large distances from the source (large $|x - x'|$), series of equation (7) may be truncated by finite number of poles N_{PML} poles) with good accuracy. This is because of that the higher order modes have large absolute value of imaginary part. But for very close to the source considerable number of modes must be considered. In order to have efficient series representation CGF-RFFM is utilized for small $|x - x'|$. Using modified VECTFIT (vector fitting) algorithm [8] for CGF G_y ,(5) will give

$$G_y(y, y', \lambda_y) \approx \sum_{p=1}^{N_{RFFM}} \frac{\text{Res}_p}{\beta_x^2 - \beta_{xp}^2} = \sum_{p=1}^{N_{RFFM}} \frac{-\text{Res}_p}{\lambda_y - \lambda_{yp}}. \quad (8)$$

Res_p is the p*th* pole and residue respectively and N_{RFFM} is the total number of poles (including SW poles) for approximating G_y . It is assumed that the poles are extracted on the proper Riemann sheet. For (8) uniformly sampled points are used i.e. $\beta_x \in [-t_{max} k_0 - j\delta, t_{max} k_0 + j\delta]$, where δ is a small number or even zero and t_{max} must be greater than $\sqrt{\varepsilon_{r,\max}}$ ($\varepsilon_{r,\max}$ is the maximum value of relative dielectric constant), in order to ensure that all the SW poles are included. To obtain more accurate approximation larger t_{max} with more sample points can be used. Modified VECTFIT algorithm is so fast and will converge almost with arbitrary initial guesses for poles and residues. By substituting (9) in (4) for integration path C_{λ_y} shown in Fig. 3, we will have

$$A_z(x, y; x', y') \approx \sum_{p=1}^{N_{RFFM}} -\text{Res}_p \frac{e^{-j\beta_{xp}|x-x'|}}{2j\beta_{xp}}. \quad (9)$$

Except surface wave poles of the original structure, some other complex poles which are extracted in VECTFIT algorithm in rational function fitting are responsible to construct the field comes from the continuous spectrum contribution. In fact in the CGF-RFFM against CGF-PML, the obtained poles are dependent to source and observation point location. Therefore, for specific source and observation point location, the best complex poles which result in exact A_z are produced in CGF-RFFM. This is the main disadvantage of CGF-RFFM which makes this method inefficient in large scale problem. But for small distances from the source, to have result similar to CGF-PML, it is useful to utilized CGF-RFFM. The main reason is that by few number of complex poles, very near field can be expressed in a series form, although for each y and y' configuration, a separate VECTFIT algorithm must be run. VECTFIT algorithm is fast enough and is more inexpensive than CGF-PML series computation with large number of poles for small $|x - x'|$.

IV. NUMERICAL RESULTS

In this section some numerical examples are shown to prove the efficiency and versatility of the proposed method. The codes have been carried out on a 2.26 GHz personal computer.

Let us first define a parameter d_s which denotes the distance from the source where for $|x - x'| < d_s$ and $|x - x'| > d_s$, CGF-RFFM and CGF-PML are used respectively.

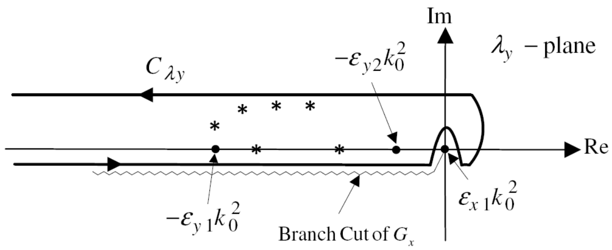


Figure 3. Integration path in λ_y − plane, ∗: poles extracted by VECTFIT.

The final result can be written as

$$\begin{aligned} A_z(x, y; x', y') &= \prod_{d_s}(x, x') \sum_{q=1}^{N_{RFFM}} -\text{Res}_q^r \frac{e^{-j\beta_{xq}^r|x-x'|}}{2j\beta_{xq}^r} \\ &+ \left(1 - \prod_{d_s}(x, x')\right) \sum_{q=1}^{N_{PML}} -\text{Res}_q^p \frac{e^{-j\beta_{xq}^p|x-x'|}}{2j\beta_{xq}^p} \end{aligned} \quad (10)$$

where $\prod_{d_s}(x, x')$ is 1 for $|x - x'| < d_s$ and 0 for $|x - x'| > d_s$. N_{RFFM} is the number of poles (with β_{xq}^r , Res_q^r) extracted for y and y' at hand in CGF-RFFM and N_{PML} is the number of poles (with β_{xq}^p , Res_q^p) in CGF-PML method. d_s can control the N_{PML} in the way that for smaller d_s , bigger N_{PML} must be considered to CGF-PML's result be accurate and vice versa. The important point is that the extracted poles in VECTFIT algorithm in CGF-RFFM are not essentially orthogonal as PML modes of close structure in CGF-PML method. Therefore, to have efficient usage of Green's function in mode-matching type problems, small d_s s may be regarded. To validate the accuracy of the proposed method numerical integration of (4) along the path $C_{\lambda y}$ is evaluated.

In Fig. 4 the result of A_z for dielectric slab shown in Fig. 1(a) with $t = 0.2\lambda$, $\varepsilon_{r1} = 10$ and $\varepsilon_{r2} = 1$ is shown. The excitation source is located at $x' = 0$ and $y' = t$ and distribution of field is considered on the upper surface of the slab, $y = t$. For $d_s = 0.2\lambda$, CGF-RFFM is used with $N_{RFFM} = 8$ poles and CGF-PML with 50 poles. By considering the smaller d_s ($d_s = 0.1\lambda$) with the same CGF-RFFM results, 70 PML poles must be taken to have excellent match with exact numerical integration. For CGF-PML, $d_{air} = 0.1\lambda$ and also we choose $d_{PML} = 0.05\lambda$ and $\kappa_0 - j\sigma_0 / \omega\varepsilon_0 = 6 - j6$ for PMLs. To search the efficiency of CGF-RFFM-PML, let us consider a dielectric substrate with $t = 0.2\lambda$, $\varepsilon_{r1} = 15$ and $\varepsilon_{r2} = 1$. A_z for near field is shown in Fig. 5. For $d_s = 0.5\lambda$, CGF-RFFM-PML is implemented with 12 poles for CGF-RFFM and 30 poles for CGF-PML. It can be seen that accurate modal series representation by 42 poles is obtained in comparison with the numerical integration. By decreasing d_s to 0.2λ, 62 poles are required due to more 20 poles considered in CGF-PML method. To compare the rapidity of the method, let us have a comparison shown in Fig. 6 for dielectric slab with $t = 0.3\lambda$, $\varepsilon_{r1} = 20$ and $\varepsilon_{r2} = 1$ for line source located at $x' = 2\lambda$ and $y' = 0$. CGF-PML result which was also developed in, is illustrated for $d_s = 0.1\lambda$ (without using CGF-RFFM). With 50 PML poles some deviations can be seen near the $x' + d_s$ which is due to insufficient number of PML modes. A_z for $d_s = 0.02\lambda$ in CGF-PML with 350 poles is also shown which it's required time is at least 9 minutes. This time is mainly related to poles extraction and computation of truncated series in (7). Even so, by using just 40 poles for $d_s = 0.2\lambda$ in CGF-PML and 12 poles in CGF-RFFM, efficient series expression of A_z can be obtained with 52 poles in less than 1 minute.

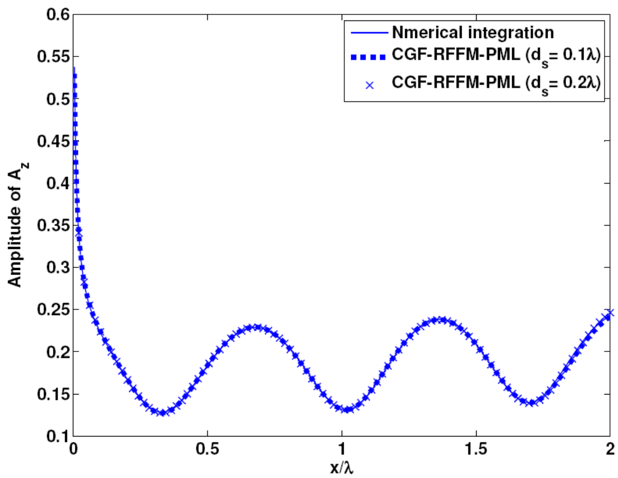


Figure 4. Amplitude of A_z for numerical integration and the CGF-RFFM-PML for dielectric slab with $t = 0.2\lambda$, $\varepsilon_{r1} = 10$ and $\varepsilon_{r2} = 1$, where $y = y' = t$ and $x' = 0$ for $d_s = 0.2\lambda, 0.1\lambda$, $N_{RFFM} = 8$ and $N_{PML} = 50, 70$.

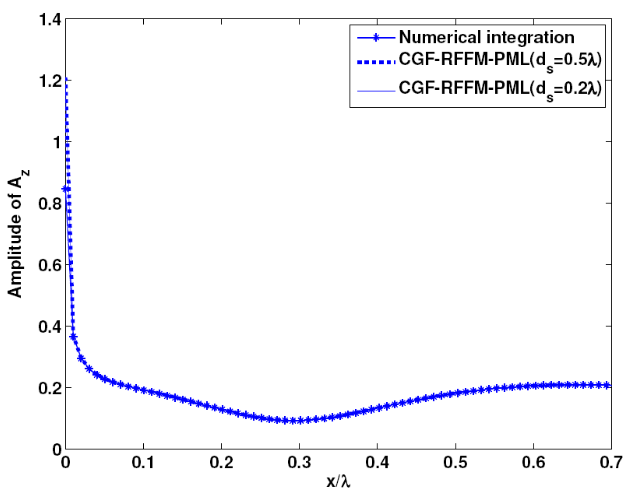


Figure 5. Amplitude of A_z for numerical integration and the CGF-RFFM-PML in the near field for dielectric slab with $t = 0.2\lambda$, $\varepsilon_{r1} = 15$ and $\varepsilon_{r2} = 1$, where $y = y' = t$ and $x' = 0$ for $d_s = 0.5\lambda, 0.2\lambda$, $N_{RFFM} = 12$ and $N_{PML} = 30, 50$.

V. Conclusion

A closed-form and efficient series representation for spatial Green's function of a dielectric slab for all distances from source is presented by using perfectly matched layer method and CGF technique. By combining CGF technique and rational function fitting method, efficient modal series expression of Green's function is achieved in very near field, with few poles extracted in modified VECTFIT algorithm. The main advantage of this representation is that the number of required modes is controllable.

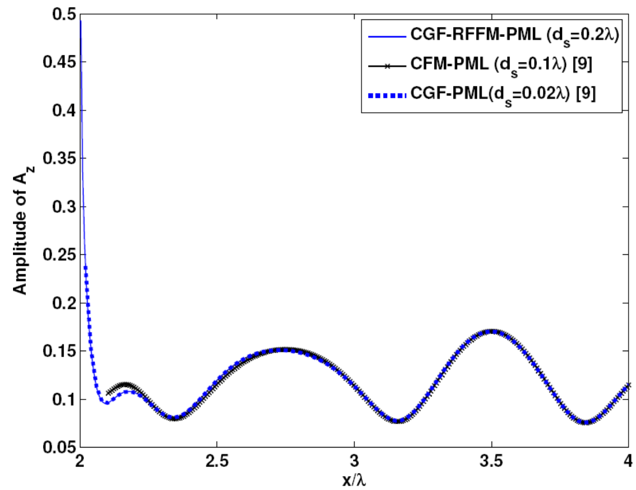


Figure 6. Amplitude of A_z for CGF-PML and the CGF-RFFM-PML for dielectric slab with $t = 0.3\lambda$, $\varepsilon_{r1} = 20$ and $\varepsilon_{r2} = 1$, where $y = y' = t$ and $x' = 2\lambda$ for different d_s and N_{PML} .

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