A hybrid FE-BI-DDM for electromagnetic scattering by multiple 3-D holes

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***Abstract*-A hybrid finite element-boundary integral-domain decomposition method (FE-BI-DDM) is introduced to efficiently solve the problem of electromagnetic scattering by multiple three dimensional (3-D) holes. Specifically, each hole is modeled by the edge-based finite element method. The holes are coupled to each other through the boundary integral equation based on Green's function. The resultant coupling system of equations is solved by an iterative domain decomposition method. Some numerical results are included to illustrate the validity and capability of the proposed method.**

I. INTRODUCTION

Conducting plates with multiple holes are widely used in engineering structures, e.g. missiles, aircraft, optic experiment platform, grating couplers etc. The accurate and efficient simulation of the scattering by such a structure is of vital importance in many areas of optical and electrical engineering, such as grating manufacture, surface defect detection, slot antenna design and radar cross section control, but is also a very challenging task.

For the problem of scattering by multiple two-dimensional (2-D) holes, there have been some numerical investigations. Dejoie *et al.* [1] applied an integral equation method to solve the problem of an elastic half-plane containing a large number of randomly distributed, non-overlapping, circular holes. Lee and Chen [2] adopted a semi-analytical approach that is based on null-field integral equation to solve the scattering problem of flexural waves in an infinite thin plate with multiple circular holes. Later in [3], they employed a multipole Trefftz method to analyze the scattering of flexural wave by multiple circular holes in an infinite thin plate. Recently, Alavikia and Ramahi [4] studied the problem of scattering from multiple 2-D holes in infinite metallic walls by means of a finite element method (FEM) that uses the surface integral equation with the free-space Green's function as the boundary constraint.

While some work has been done in the area of solving the problems of scattering by multiple 2-D holes, little work on the solution of three dimensional (3-D) problems have been reported in the previous literature. In the present work, we introduce a domain decomposition of the hybrid finite element -boundary integral (FE-BI) method described in [5] to solve the problem of electromagnetic scattering by multiple 3-D holes embedded in conducting plates. In the implementation of the method, the edge-based FEM is applied inside each hole to derive a linear system of equations associated with the unknown fields. The boundary integral equation (BIE) is then applied on the apertures of all the holes to truncate the computational domain and to connect the matrix subsystem generated from each hole. To reduce computational burdens, an iterative domain decomposition method (DDM) based on the substructuring method [6] and the multilevel fast multipole algorithm (MLFMA) [7] is developed to efficiently solve the coupling system of equations. In the following sections, the theoretical and implementation of the proposed method are firstly given. Some numerical results are then presented.

II. MATHEMATICAL FORMULATION

A. FE-BI Formulation for Scattering from a Single Hole

We first consider the problem of electromagnetic scattering by a single 3-D hole in a conducting plate, as shown in Fig. 1. The volume of the hole is denoted as V , and the planar surface area of the lower and upper aperture are denoted as S_1 and S_2 , respectively. Also, the hole is assumed to be filled with an inhomogeneous material having a relative permittivity ε_r and relative permeability μ_r , and the plane wave is assumed to be incident to the upper aperture. In accordance with the FE-BI method described in [5], the electric field inside the hole and at the apertures of the hole can be obtained by seeking the stationary point of the functional

$$\begin{aligned} F(\mathbf{E}) = & \iiint_V \left[\frac{1}{\mu_r} (\nabla\times\mathbf{E})\cdot(\nabla\times\mathbf{E}) - k_0^2 \varepsilon_r \mathbf{E}\cdot\mathbf{E} \right] dV \\ & - 2k_0^2 \iint_{S_1} \mathbf{M}_1(\mathbf{r}) \cdot \left[\iint_{S_1} \mathbf{M}_1(\mathbf{r}') \cdot \overline{\overline{G}}_0(\mathbf{r},\mathbf{r}') dS' \right] dS \\ & - 2k_0^2 \iint_{S_2} \mathbf{M}_2(\mathbf{r}) \cdot \left[\iint_{S_2} \mathbf{M}_2(\mathbf{r}') \cdot \overline{\overline{G}}_0(\mathbf{r},\mathbf{r}') dS' \right] dS \\ & + 2jk_0 Z_0 \iint_{S_2} \mathbf{M}_2(\mathbf{r}) \cdot \mathbf{H}^{inc}(\mathbf{r}) dS \end{aligned} \quad (1)$$

where $\mathbf{M}_1$ and $\mathbf{M}_2$ are the equivalent magnetic currents, which

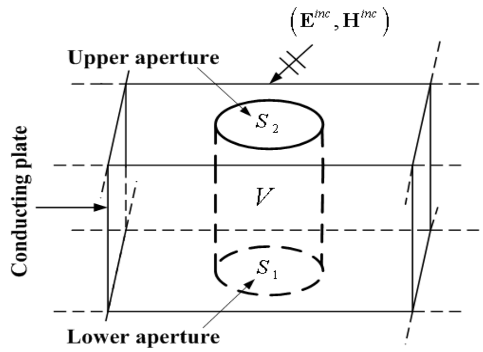

Fig. 1. Geometry of a 3-D hole in a conducting plate.

are related to the electric field on S_1 and S_2 by $\mathbf{M}_1 = \mathbf{E}_1 \times \hat{n}_1$ and $\mathbf{M}_2 = \mathbf{E}_2 \times \hat{n}_2$. $\hat{n}_1$ and $\hat{n}_2$ denote the outward unit vector normal to S_1 and S_2 , respectively. Also, $\mathbf{H}^{inc}$ is the incident magnetic field, k_0 and Z_0 are the free-space wave number and impedance, and $\overline{\overline{G}}_0$ is the free-space dyadic Green's function.

Using the edge-based FEM with tetrahedral elements [8], the functional can be converted into a matrix equation

$$\begin{bmatrix} K_{II} & K_{IL} & K_{IU} \\ K_{LI} & K_{LL}+P & 0 \\ K_{UI} & 0 & K_{UU}+Q \end{bmatrix} \begin{Bmatrix} E_I \\ E_L \\ E_U \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ b \end{Bmatrix} \quad (2)$$

where $\{E_I\}$ is a vector containing the discrete electric fields inside V , while $\{E_L\}$ and $\{E_U\}$ are the vectors containing the discrete electric fields on the lower and upper aperture of the hole. The matrix $[K]$ is contributed by the volume integral in (1), whereas matrices $[P]$ and $[Q]$ are contributed by the dual surface integrals. For the sake of clear description, we define two integral operators

$$L(\mathbf{g}_j) = -2k_0^2 \iint_{S_1} \mathbf{g}_j(\mathbf{r}') \cdot \overline{\overline{G}}_0(\mathbf{r},\mathbf{r}')\, dS' \quad (3)$$

$$U(\mathbf{g}_j) = -2k_0^2 \iint_{S_2} \mathbf{g}_j(\mathbf{r}') \cdot \overline{\overline{G}}_0(\mathbf{r},\mathbf{r}')\, dS' \quad (4)$$

where $\mathbf{g}_j$ are the RWG vector basis functions[9] defined on triangular elements. With the integral operator, the elements of matrices $[P]$ and $[Q]$ can be rewritten as

$$P_{ij} = \iint_{S_1} \mathbf{g}_i \cdot L(\mathbf{g}_j)\, dS \quad (5)$$

$$Q_{ij} = \iint_{S_2} \mathbf{g}_i \cdot U(\mathbf{g}_j)\, dS \quad (6)$$

where $\mathbf{g}_i$ and $\mathbf{g}_j$ denote the testing and expansion basis functions, respectively. In this notation, the testing and expansion locations are both on the apertures of the same hole.

B. *Extension to Multiple Holes*

To extend the method described above to case of multiple holes, we propose an expanded system of equations with individual $[P]$ and $[Q]$ sub-matrices [10]. The new $[P]$ and $[Q]$ sub-matrices take on the form

$$\left[P_{ij}\right]^{mn} = \iint_{S_L} \mathbf{g}_i^m \cdot L_{mn}(\mathbf{g}_j^n)\, dS \quad (7)$$

$$\left[Q_{ij}\right]^{mn} = \iint_{S_U} \mathbf{g}_i^m \cdot U_{mn}(\mathbf{g}_j^n)\, dS \quad (8)$$

The new integral operators L_{mn} and U_{mn} are defined as

$$L_{mn}(\mathbf{g}_j^n) = -2k_0^2 \iint_{S_L} \mathbf{g}_j^n(\mathbf{r}') \cdot \overline{\overline{G}}_0(\mathbf{r},\mathbf{r}')\, dS' \quad (9)$$

$$U_{mn}(\mathbf{g}_j^n) = -2k_0^2 \iint_{S_U} \mathbf{g}_j^n(\mathbf{r}') \cdot \overline{\overline{G}}_0(\mathbf{r},\mathbf{r}')\, dS' \quad (10)$$

where the subscript " S_L " represents the surface integration in (7) and (9) is performed over the lower apertures of all the holes. Similarly, the subscript " S_U " represents the surface integration in (8) and (10) is performed over the upper apertures of all the holes. Also, the indices $[m,n]$ represent the testing and expansion holes, respectively.

According to the principle of edge-based FEM, there is no direct coupling between the unknowns of any two independent computational domains. Therefore, the FEM matrix $[K]$ is only defined when $m = n$. In other words, the holes are coupled to each other only through the BIE based on Green's function. As a result, the coupling system of equations for m holes in a conducting plate would be expressed in the matrix notation

$$\begin{bmatrix} \widetilde{K}_{II} & \widetilde{K}_{IL} & \widetilde{K}_{IU} \\ \widetilde{K}_{LI} & \widetilde{K}_{LL}+\widetilde{P} & 0 \\ \widetilde{K}_{UI} & 0 & \widetilde{K}_{UU}+\widetilde{Q} \end{bmatrix} \begin{Bmatrix} \widetilde{E}_I \\ \widetilde{E}_L \\ \widetilde{E}_U \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \widetilde{b} \end{Bmatrix} \quad (11)$$

where

$$\left[\widetilde{P}\right] = \begin{bmatrix} P^{11} & P^{12} & \cdots & P^{1m} \\ P^{21} & P^{22} & \cdots & P^{2m} \\ \vdots & \vdots & \ddots & \vdots \\ P^{m1} & P^{m2} & \cdots & P^{mm} \end{bmatrix} \quad (12)$$

$$\left[\widetilde{Q}\right] = \begin{bmatrix} Q^{11} & Q^{12} & \cdots & Q^{1m} \\ Q^{21} & Q^{22} & \cdots & Q^{2m} \\ \vdots & \vdots & \ddots & \vdots \\ Q^{m1} & Q^{m2} & \cdots & Q^{mm} \end{bmatrix} \quad (13)$$

With the aid of the following notation

$$diag.\sum_{i=1}^{m}[K_i] = \begin{bmatrix} K_1 & & & \\ & K_1 & & \\ & & \ddots & \\ & & & K_m \end{bmatrix} \quad (14)$$

The FEM associated matrix sub-blocks are of the form

$$\begin{gathered} \left[\widetilde{K}_{II}\right] = diag.\sum_{i=1}^{m}\left[K_{II}^i\right], \left[\widetilde{K}_{LL}\right] = diag.\sum_{i=1}^{m}\left[K_{LL}^i\right], \\ \left[\widetilde{K}_{IL}\right] = diag.\sum_{i=1}^{m}\left[K_{IL}^i\right], \left[\widetilde{K}_{LI}\right] = diag.\sum_{i=1}^{m}\left[K_{LI}^i\right], \\ \left[\widetilde{K}_{IU}\right] = diag.\sum_{i=1}^{m}\left[K_{IU}^i\right], \left[\widetilde{K}_{UI}\right] = diag.\sum_{i=1}^{m}\left[K_{UI}^i\right], \\ \left[\widetilde{K}_{UU}\right] = diag.\sum_{i=1}^{m}\left[K_{UU}^i\right] \end{gathered} \quad (15)$$

C. *Iterative Domain Decomposition Solver*

To efficiently solve the coupling system of equations (11), we develop an iterative domain decomposition solver based on the substructuring method [6] and MLFMA [7]. Specifically, we first eliminate the unknowns in the interior of each hole according to the following equation

$$\{E_I^i\} = -[K_{II}^i]^{-1}[K_{IL}^i]\{E_L^i\} - [K_{II}^i]^{-1}[K_{IU}^i]\{E_U^i\} \quad (16)$$

where $i = 1, 2, \cdots, m$. Once all the interior unknowns are eliminated, the original coupling system of equations can be reduced to a small one which only includes the unknowns on the apertures of all the holes, as follows

$$\begin{bmatrix} A+\widetilde{P} & B \\ C & D+\widetilde{Q} \end{bmatrix} \begin{Bmatrix} \widetilde{E}_L \\ \widetilde{E}_U \end{Bmatrix} = \begin{Bmatrix} 0 \\ \tilde{b} \end{Bmatrix} \quad (17)$$

in which

$$\begin{aligned} [A] &= diag.\sum_{i=1}^{m}[A^i], [B] = diag.\sum_{i=1}^{m}[B^i], \\ [C] &= diag.\sum_{i=1}^{m}[C^i], [D] = diag.\sum_{i=1}^{m}[D^i] \end{aligned} \quad (18)$$

where

$$[A^i] = [K_{LL}^i] - [K_{LI}^i][K_{II}^i]^{-1}[K_{IL}^i] \quad (19)$$

$$[B^i] = -[K_{LI}^i][K_{II}^i]^{-1}[K_{IU}^i] \quad (20)$$

$$[C^i] = -[K_{UI}^i][K_{II}^i]^{-1}[K_{IL}^i] \quad (21)$$

$$[D^i] = [K_{UU}^i] - [K_{UI}^i][K_{II}^i]^{-1}[K_{IU}^i] \quad (22)$$

The solution to the reduced system (17) can be obtained by an iterative solver such as the generalized minimum residual (GMRES) method. However, traditional GMRES iteration method incurs very high computational cost and memory requirements with the increasing of the unknowns on the apertures of all the holes. In addition, the conventional approaches to computing the BIE matrix elements consume a considerable portion of the total solution time, and this, in turn, can place an inordinately heavy burden on the CPU regarding memory and time. To overcome these difficulties, we employ the MLFMA to the BIE matrix to significantly reduce the memory requirement and computational complexity. The detailed description of the MLFMA is given in [7] and is not repeated here.

III. NUMERICAL RESULTS

To illustrate the validity of the proposed method for scattering by multiple holes, we consider two identical circular holes in a conducting plate. Both holes have a diameter of 1.0λ and depth of 0.5λ, and are separated by 1.0λ in x-direction, λ being the operating wavelength. For numerical solution, each hole is discretized independently into 2855 tetrahedral elements. As a result, a total of 6644 FEM unknowns and 514 BIE unknowns are generated. By virtue of the iterative domain decomposition solver described above, only one half of the FEM unknowns need to be deal with. Since the number of BIE unknowns is not so much, the reduced system can be easily solved. The monostatic RCS computed by the proposed FE-BI-DDM are plotted in Fig. 2, and compared with those from the method of moments (MOM)

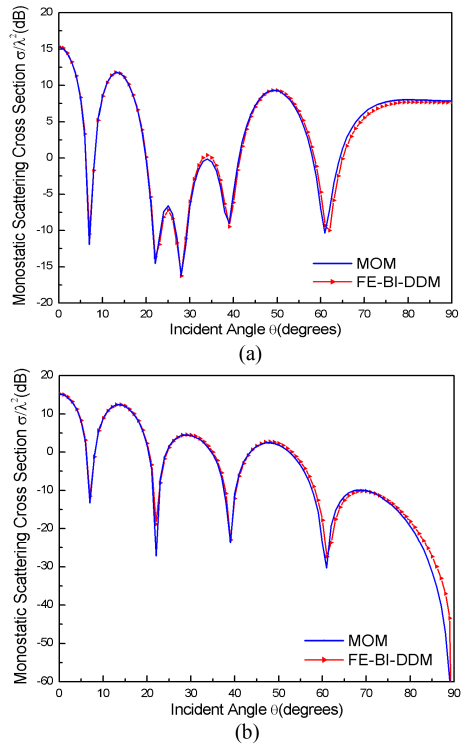

Fig. 2. Comparison of the monostatic scattering cross sections for two identical circular holes in a conducting plate obtained from FE-BI-DDM and MOM. (a) $\theta\theta$ polarization; (b) $\varphi\varphi$ polarization.

based on the surface boundary integral equation. As is evident from the figure, good agreement is obtained between the two methods.

Then, we investigate the scattering behavior of four identical circular holes in a conducting plate. Each hole has a diameter of 2.0λ and depth of 3.0λ. These holes are arranged in three ways and are separated by distances of 0.5λ. In the first way, these holes are placed along y-axis. In the second way, these holes are placed along x-axis. In the last way, these holes are arranged periodically in both the x and y directions, i.e. a 2×2 array. Fig.3 depicts the computed scattering cross section as a function of the angle of incidence in the xoz plane. As can be seen, when these holes are placed along y-axis, the scattering cross section curve in the xoz plane is relatively smooth, whereas the curves corresponding to the other two ways have several peaks. This indicates that the coupling effects among these holes are stronger in the last two ways than that in the first one.

Finally, to illustrate the capability of the proposed method to solve large problems, we consider a 10×10 array of circular holes in a conducting plate. Each hole of the array has a diameter of 1.0λ and depth of 2.0λ. The periodicity is 2.0λ in both x and y directions. A plane wave with incidence angles $\theta^{inc} = 45^\circ$, $\varphi^{inc} = 0^\circ$ and polarization angle $\alpha = 0^\circ$ is obliquely incident to the he upper apertures of the array. We calculate the bistatic scattering cross section of the array, and the results are shown in Fig.4. In this problem, a total of 1142000 tetrahedral elements are used to discretize the array holes,

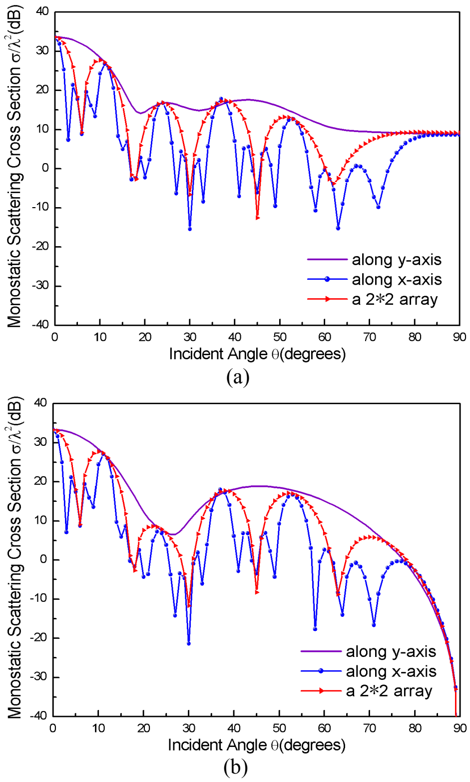

Fig. 3. Monostatic scattering cross section of four identical circular holes in a conducting plate: (a) $\theta\theta$ polarization; (b) $\varphi\varphi$ polarization.

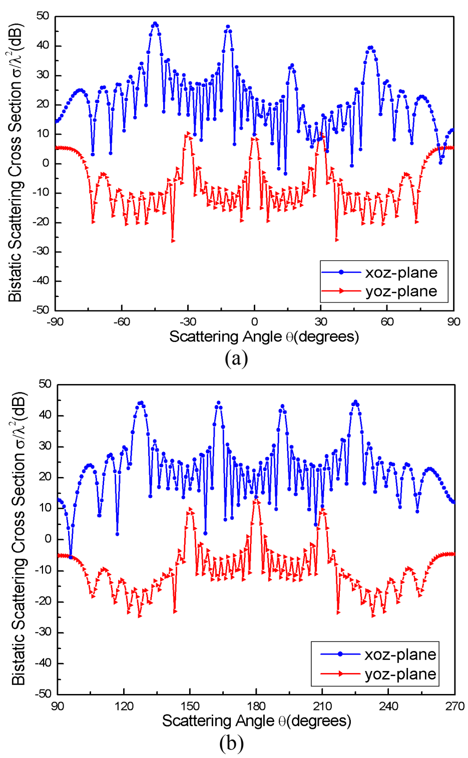

Fig. 4. Bistatic scattering cross section for a 10×10 array of circular holes in a conducting plate: (a) above the plate; (b) below the plate.

which result in 1200300 FEM unknowns and 51400 BIE unknowns. Although the number of FEM unknowns is very large, only one percent of those need to be dealt with. Furthermore, the reduced full system can be converted into a sparse one by virtue of the MLFMA. In this problem, 5-level MLFMA is used, and the memory required is about 630Mb.

IV. CONCLUSION

This paper presents a domain decomposition of the FE-BI method for the analysis of electromagnetic scattering from multiple 3-D holes in conducting plates. In the implementation of the method, the edge-based FEM is used to obtain the solution of vector wave equation inside each hole. The BIE using the free-space Green's function is applied on the apertures of the holes as a global boundary condition. To reduce the computational burdens of the method, an iterative substructuring method was employed to solve the coupling system of equations. As a result, the original problem was reduced to a small one which only includes the unknowns on the apertures. Furthermore, the reduced system was solved by the CGM, where MLFMA is employed to reduce the memory requirement and computational complexity. Some numerical results are presented to demonstrate the validity and capability of the proposed method.

ACKNOWLEDGMENT

This work was supported by the he Fundamental Research Funds for the Central Universities (Grant No. k5051307003).

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