A Compressed Best Uniform Approximation for Fast Computation of RCS over Wide Angular-band

Zhiwei Liu[1,2], Shan He[1], Yueyuan Zhang[1], Xiaoyan Zhang[1,2], Yingting Liu[1], Jun Xu[1]
1 East China Jiaotong University, Nanchang, 330013, China
2 The State Key Laboratory of Millimeter Wave, Nanjing, 210096, China

***Abstract*–Using the best uniform approximation hybridized with Singular Value Decomposition (BUA-SVD) is proposed to reduce the time requirement for computation of monostatic radar cross-section (RCS). In contrast to our previous work, the best uniform approximation technique is applied to compute the key excitation vectors instead of electric current vectors. Reduction of the number of multiple excitation vectors can lead to significantly reduced computation time. Moreover, with low-rank property, the excitation vectors may be further compressed by SVD, resulting in a more efficient method. SVD can lead to reduce the computational complexity further with the best uniform approximation. Numerical results demonstrate that BUA-SVD is efficient for monostatic RCS calculation with high accuracy.**

I. INTRODUCTION

Electromagnetic (EM) wave scattering problems address the physical issue of detecting the diffraction pattern of the EM radiation scattered from a large and complex body when illuminated by an incident incoming wave. A good understanding of these phenomena is crucial to RCS calculation, antenna design, EM compatibility, and so on. All these simulations are very demanding in terms of computer resources, and require efficient numerical methods to compute an approximate solution of Maxwell's equations. Using the equivalence principle, Maxwell's equations can be recast in the form of integral equations that relate the electric and magnetic fields to the equivalent electric and magnetic currents on the surface of the object. To solve integral equations by applying traditional method of moment in [1], the computation complexity for the iterative solver is $O(kN^2)$ and the memory requirement is $O(N^2)$, where N refers to the number of unknowns and k refers to the number of iterative steps. Obviously, it is impractical to use personal computer to solve equations with more than 10,000 unknowns. This difficulty can be overcome by using the multi-level fast multi-pole algorithm (MLFMA) [2, 3]. The use of MLFMA reduces the memory requirement to *O(NlogN)* and the computation complexity to *O(kNlogN)*.

Although MLFMA results in an efficient solution of the integral equation, it is still time-consuming for the calculation of monostatic RCS since it requires repeated solution of the integral equation at each incident angle. As is well known, many interpolation methods have been proposed to circumvent this difficulty. Conventional interpolation methods, such as asymptotic waveform evaluation (AWE) and the cubic-spline (CS) interpolation method, can easily approximate the monostatic RCS. AWE which is introduced in [4] and [5] is a kind of classical method which is widely used in computational EM. It utilizes the high-order derivatives of the incident current vector at the interval center to extrapolate the value of nearby points, and is considered to be the AWE extrapolation method in [6] has been introduced by Wei. In the AWE technique which is introduced in [6] and [7], the induced current is expanded in the Taylor series around an angle, and the Padé approximation is used to improve the accuracy. As a result, the induced current vector is expected to be accurate near this sample, but imprecise when the incident angle moves away from the angle.

As an alternative technique, the best uniform approximation in [8] has been introduced by Chebyshev. This approximation is important in approximation theory because the roots of the Chebyshev polynomials of the first kind, which are also called Chebyshev nodes, are used as nodes in polynomial interpolation. The resulting interpolation polynomial minimizes the problem of Runge's phenomenon and provides an approximation that is close to the polynomial of best approximation to a continuous function under the maximum norm. This approximation leads directly to the method of Clenshaw-Curtis quadrature. Therefore, the best uniform approximation can improve efficiency and save much time.

It is noteworthy that selection of sampling points is crucial for interpolation and extrapolation methods. The non-uniform sampling method is more flexible and efficient than the uniform sampling method. As a result, optimally selecting those angles that would be most informative will reduce the number of repeated solutions which one must consider for monostatic scattering computations. In [8], the best uniform approximation is proposed to optimally select the most informative angles in monostatic RCS curve, resulting in an efficient computation of monostatic scattering. In [9], it is reported that multiple excitation vectors or right hand side vectors can be compressed by use of the low-rank property. Inspired by [10], the multiple right hand sides can be approximately described by a low-rank form. In linear algebra, the singular value decomposition (SVD) is a factorization of a real or complex matrix, with many useful applications in signal processing and statistics. Applications which employ the SVD include computing the pseudoinverse, least squares fitting of data, matrix approximation, and determining the rank, range and null space of a matrix. In this paper, the

combination of the best uniform approximation and singular value decomposition (BUA-SVD) is applied to efficient computation of monostatic RCS. The numerical simulations demonstrate that this framework can reduce the computation time significantly.

II. THEORY OF BUA-SVD

A. the Best Uniform Approximation (BUA)

The best uniform approximation in [8] has been introduced by Chebyshev as an interpolation technique. In order to get the RCS of the target more quickly, the algorithm must be applied multilevel fast multipole method (MLFMA) point by point calculation within a given frequency band. To combine it with the best uniform approximation, the specific process is as follows:

For a given frequency band $f \in [f_m, f_n]$, corresponding to the wave number $k \in [k_m, k_n]$, do the coordinate transformation first. Let

$$\tilde{k} = \frac{2k - (k_m + k_n)}{k_n - k_m}, \tag{4}$$

so The surface current can be written as:

$$I(k) = I\left(\frac{\tilde{k}(k_n - k_m) + (k_m + k_n)}{2}\right), \tag{5}$$

where $\tilde{k}_i \in [-1,1]$.

Assume that $T_l(\tilde{k})$ (l=1,2,…,n) as the l-order Chebyshev polynomials, and it is defined as :

$$T_0(\tilde{k}) = 1,\ T_1(\tilde{k}) = \tilde{k},\ T_{l+1} = 2\tilde{k}T_l(\tilde{k}) - T_{l-1}(\tilde{k}),\ 2 \le l \le n. \tag{6}$$

So the Chebyshev Approximation of $I(k)$ can be expressed as:

$$I(k) = I\left(\frac{\tilde{k}(k_n - k_m) + (k_m + k_n)}{2}\right) \approx \sum_{l=0}^{n-1} c_l T_l(\tilde{k}) - \frac{c_0}{2}. \tag{7}$$

We suggest $\tilde{k}_i$ (i=1,2,…,n) as the n-th zreo point of $T_n(\tilde{k})$ ($\tilde{k} \in [-1,1]$), so

$$\tilde{k}_i = \cos\left(\frac{i - 0.5}{n}\pi\right), i = 1, 2, \cdots, n \tag{8}$$

$$c_l = \frac{2}{n}\sum_{i=1}^{n} I(k_i) T_l(\tilde{k}_i). \tag{9}$$

Here, k_i is called the Chebyshev nodes in $[k_m, k_n]$. Where

$$k_i = \frac{\tilde{k}_i(k_n - k_m) + (k_m + k_n)}{2} (i = 1, 2, \cdots, n) \tag{10}$$

Above all, we calculate the approximate current throughout the whole frequency band to analyze the EM scattering characteristics of the target quickly.

B. Singular Value Decomposition (SVD)

Theoretically, the combination of MoM and MLFMA is able to accurately analyze the scattering of any geometry. Improved by the best uniform approximation, the computation of a monostatic RCS can be accelerated greatedly. However, in some cases, the number of coefficients of the interpolation polynomials is so large as to compromise the efficient calculation of monostatic scattering. This process can be computationally prohibitive for electrically large objects. In order to alleviate this difficulty, a singular value decomposition based method is proposed and discussed in this section.

Firstly, a brief review of compression of right hand sides is given. The computation of monostatic RCS can be considered as linear equations with multiple right hand sides

$$\mathbf{A}\cdot\mathbf{X} = \mathbf{B} \tag{11}$$

where $\mathbf{A}$ is the impedance matrix, $\mathbf{X}$ is the multiple complex coefficient vector of RWG basis and $\mathbf{B}$ is the multiple right hand side generated by the incident wave. In addition

$$\mathbf{X} = [\mathbf{x}(\theta_1), \mathbf{x}(\theta_2), \ldots, \mathbf{x}(\theta_n)] \tag{12}$$

$$\mathbf{B} = [\mathbf{b}(\theta_1), \mathbf{b}(\theta_2), \ldots, \mathbf{b}(\theta_n)] \tag{13}$$

where θ_i is the i^{th} incident angle. Using traditional singular value decomposition, the matrix $\mathbf{B}$ can be described in the form of an eigenvalue and eigenvector.

$$\mathbf{B} = \mathbf{U}\cdot\mathbf{\Sigma}\cdot\mathbf{V}^{\mathrm{H}} \tag{14}$$

The superscript 'H' denotes the conjugate transpose. If the dimension of $\mathbf{B}$ is $N \times M$, the dimension of matrices $\mathbf{U}$、$\mathbf{\Sigma}$ and $\mathbf{V}$ are $N \times M$, $M \times M$, $M \times M$, respectively. N is the number of unknowns. $\mathbf{\Sigma}$ is a diagonal matrix including all the eigenvalues of $\mathbf{B}$ while $\mathbf{U}$ and $\mathbf{V}$ contain all the eigenvectors of $\mathbf{B}$. When $\mathbf{B}$ is the multiple right hand sides in the linear system connecting with the SIE used for monostatic RCS, the matrix $\mathbf{B}$ is low-rank and can be approximately described as a low-rank SVD form.

$$\mathbf{B} = \mathbf{U}_k\cdot\mathbf{\Sigma}_k\cdot\mathbf{V}_k^{\mathrm{H}} \tag{15}$$

where the dimension of matrices $\mathbf{U}_k$、$\mathbf{\Sigma}_k$ and $\mathbf{V}_k$ are $N \times k$, $k \times k$, $M \times k$, respectively. Only the k largest eigenvalues and corresponding eigenvectors are reserved. Substituting (15) into (11), the linear equations can be rewritten as

$$\mathbf{X} \approx (\mathbf{A}^{-1}\cdot\mathbf{U}_k)\cdot\mathbf{\Sigma}_k\cdot\mathbf{V}_k^{\mathrm{H}} \tag{16}$$

Here, $\mathbf{A}^{-1}\cdot\mathbf{U}_k$ can be computed by any iterative solver. If using a direct solver to compute the inversion of matrix $\mathbf{A}$, the proposed method will become useless. Therefore, the number of repeated solutions of $\mathbf{Ax} = \mathbf{b}$ is k for SVD method. Using traditional method, the number is M. Generally, k is much smaller than M which leads to an efficient method for computation of monostatic RCS over a wide angular band.

Using the best uniform approximation strategy, we can write the induced current into the sum of the samples shown in (15). We rewrite this formulation in matrix form

$$\mathbf{X} = \mathbf{A}^{-1}\cdot\mathbf{B}_s\cdot\mathbf{C} \tag{17}$$

where $\mathbf{X}$ is a matrix contains all induced currents over the whole angular band. $\mathbf{C}$ is the coefficient matrix of the non-uniform interpolation method with the dimension of $s \times M$. $\mathbf{B}_s$ contains all the key samples of the excitation vectors. According to formulation (17), the required number of repeated solution of $\mathbf{Ax} = \mathbf{b}$ is s. Using singular value decomposition for matrix $\mathbf{B}_s$

$$\mathbf{B}_s = \mathbf{U}_{sk}\cdot\mathbf{\Sigma}_{sk}\cdot\mathbf{V}_{sk}^{\mathrm{H}} \tag{18}$$

Then

$$\mathbf{X} = (\mathbf{A}^{-1}\cdot\mathbf{U}_{sk})\cdot\mathbf{\Sigma}_{sk}\cdot\mathbf{V}_{sk}^{H}\cdot\mathbf{C} \quad (19)$$

As a result, the required number of repeated solution is reduced to *sk*.

C. Combination of BUA with SVD

The key problem for SVD is to obtain the decomposition form of multiple right hand sides. The traditional SVD method is a good analytical solution for this problem. However, SVD requires the computation of the matrix including all right hand sides and the complexity of the computation time of SVD is $O(nm^2 + mn^2)$, where *m* and *n* are with respect to the number of rows and columns. When the number of unknowns or right hand vectors is large, this analytical solution is not practical. In order to alleviate this difficulty, the best uniform approximation algorithm may be applied and this performs more efficiently than the traditional SVD method.

III. Results and Discussion

In this section, a number of numerical results are presented to demonstrate the accuracy and efficiency of the proposed method for fast calculation of monostatic RCS over a wide angular band. All experiments are conducted on a Quad-Core AMD Opteron (tm) with 4.00 GB local memory and run at 2.31 GHz in single precision. Two geometries are applied to illustrate the performance of our proposed method. They consist of an Almond with 1210 unknowns and a cube with 49260 unknowns. In our numerical experiments, the two geometries are illuminated by a plane wave with the incident pitch angles range from 0 to 180 deg. The frequency is 3.0 GHz for the Almond and 300 MHz for Cube. For all cases the azimuth angle is 0 deg.

The algorithm produces a sequence of decompositions of a matrix into a sum of low-rank matrix and error matrix. Neither the original matrix nor the error matrix will be computed completely. Choosing the Convergence Error (CE) is the most important consideration in SVD. In order to avoid the numerical error, CE is required to be sufficiently small. However, large CE is needed for efficiency. In Fig.1, the results of the monostatic RCS of Almond by three different CE are compared with the reference result (the Direct Method). The reference result is the RCS curve computed with repeated solution at each angle. Other curves are computed by our proposed method. We select a part of the curve where the difference is much bigger with the incident pitch angles range from 20 to 100 deg. From this figure, when CE is set to be 0.1, the RCS curve is not accurate enough. When CE is set to be 0.01, the proposed method will perform a good result. A larger number of vectors leads to larger computation time. If using the proposed method without compression (CE=0.0), it would Consume more time because of the high-rank. As a result, despite of the number of excitation vectors, the value of CE is set to be 0.01 in this paper to keep the RCS curve accurate enough.

Since the number of right hand vectors is small for 1-D angular sweep in this paper, it is feasible to apply the proposed

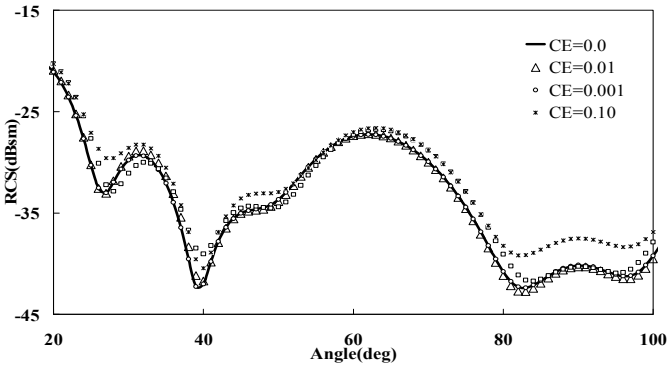

Fig.1 Comparison of the results by Proposed Method with different CE

method for computing the eigenspace of multiple right hand sides. For monostatic RCS simultaneous theta and phi sweep, the number of right hand sides is 721×721(4 point per deg). As is shown in Fig.2-3, the monostatic RCS curve of Almond and Cube which computed by the proposed method is compared with the curve computed by the reference result repeatedly. It is obvious that the proposed method is accurate since there is no significant difference between the RCS result obtained by the reference result and the proposed method. As is shown in TABLE I, when compared with the reference result, the proposed method provides little advantage on total computation time since the number of right hand side is small.

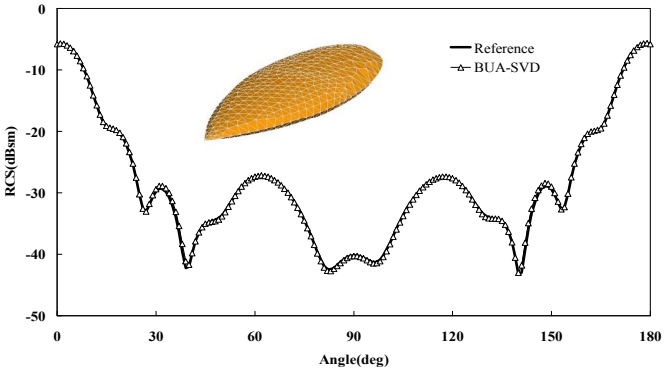

Fig.2 The monostatic RCS results for Almond

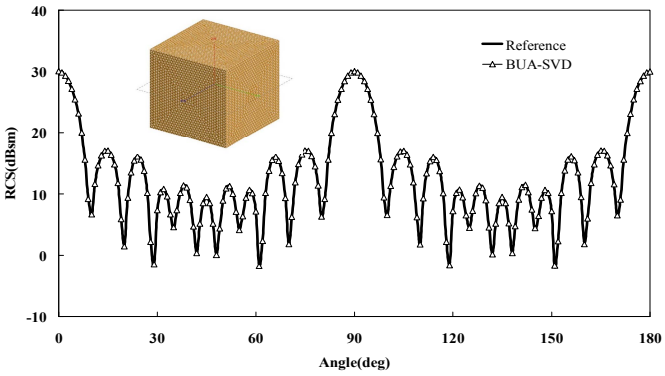

Fig.3 The monostatic RCS results for Cube

TABLE I
COMPUTATION TIME OF MONOSTATIC RCS WITH DIFFERENT METHODS

Geometry	Unknown	f(GHz)	Computation Time (s)	
			Repeated Solution	Proposed Method
Almond	1210	3.0	3520.578	231.9844
Cube	49260	0.3	53497.78	1672.125

IV. CONCLUSION

In this paper, using the best uniform approximation with Singular Value Decomposition (BUA-SVD) has been proposed for efficient analysis of monostatic scattering. Unlike interpolation of the electric current, the best uniform approximation algorithm is used to approximate the multiple right hand sides on a set of non-uniform sampling angles and SVD is employed to reduce the consumption time automatically. The most informative angles may be selected by this procedure. Moreover, applying SVD to compute the eigenvectors of the selected vectors leads to reduced times for the iterative solutions of linear systems. Numerical experiments demonstrate that the proposed method is more efficient when compared with the traditional direct method.

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