

# A Time Domain analysis for a hyperbolic reflector antenna based on a mathematic continuation of ellipsoidal surface curvatures

Shih-Chung Tuan<sup>1</sup> and Hsi-Tseng Chou<sup>2</sup>

1. Dept. of Communication. Eng., Oriental Institute of Technology, Taiwan

2. Dept. of Communication Eng., Yuan Ze University, Taiwan

**Abstract-** A transient analysis of a hyperbolic reflector antenna is performed based on a mathematic continuation of surface curvatures of an ellipsoidal reflector antenna with the analysis previously performed in [1]. This work makes the time-domain (TD) analysis useful for the design of reflector antennas since both ellipsoidal and hyperbolic reflectors are widely used as sub-reflectors for dual-reflector antenna system. In particular, this work interprets the scattering phenomena in terms of reflected and diffracted fields with respect to the frameworks of geometrical theory of diffractions, and allows one to analyze the reflector based on the local scattering mechanism.

## I. INTRODUCTION

This paper extends previous works on the transient analysis of electromagnetic (EM) fields scattering from a perfectly conducting ellipsoidal reflector in [1], and presents the corresponding analysis for the case of a hyperbolic reflector. This work is motivated by the fact that both ellipsoidal and hyperbolic reflectors are widely used as sub-reflectors in the dual-reflector antenna systems [2]. The transient analysis presented in this paper makes the time-domain (TD) analysis of sub-reflector antennas more complete for practical applications. In particular, the analysis is performed based on a mathematic continuation of surface curvatures of an ellipsoidal reflector [1], and as a result, the analysis of such ellipsoidal reflectors can be applied straightforwardly. Furthermore, the analysis successfully decomposes the scattering fields into components of reflection and edge diffractions, and allows one to interpret the scattering phenomena within the frameworks of geometrical theory of diffraction (GTD) [3] and its wave propagation mechanisms. Examples are presented to demonstrate the wave scattering phenomena.

## II. ANALYSIS DESCRIPTION

The hyperbolic reflector, illustrated in Figure 1(a), is described by a part of the following surface

$$\frac{(z+c)^2}{a^2} - \frac{x^2 + y^2}{b^2} = 1, \quad (1)$$

and truncated at  $z = z_a$ , where  $a$  and  $b$  ( $a > b$ ) are related to the radii of its two principal axes with  $c = \sqrt{a^2 + b^2}$ . The two foci,  $F_1$  and  $F_2$ , are located at  $z = -c$  and  $z = c$ , where  $F_2$  was selected as the origin of a global coordinate system I the

following analysis while the feed is located at  $F_1$  for a focus feeding. Given an arbitrary point,  $Q_s$ , on the hyperbolic surface, it is straightforward to show that

$$|F_1 Q_s - Q_s F_2| = 2a. \quad (2)$$

The feed's radiation is a  $\hat{x}_f$ -polarized spherical wave with a cosine-taper and a transient step function, and is described by (2)-(3) in [1]. Here  $(r_f, \theta_f, \phi_f)$  is defined in the feed's spherical coordinate system with  $\hat{z}_f = +\hat{z}$  and  $\hat{x}_f = \hat{x}$  as illustrated in Figure 1(a).

The analysis employs a TD aperture integration (TD-AI) technique, where the scattering fields are found by the radiation from a set of equivalent currents,  $(\vec{J}_a, \vec{M}_a)$ , defined on an aperture,  $S_a$ , in the front of the reflector. They are found from the aperture fields,  $(\vec{E}_a, \vec{H}_a)$ , obtained by geometrical optics (GO) from the incident fields. The aperture,  $S_a$ , is selected to be the projection of the reflector's surface onto a sphere's surface,  $S_r$ , which is centered at  $F_2$  with a radius  $r_1$ , and makes  $(\vec{E}_a, \vec{H}_a)$  have a uniform distribution of propagation time delay. Figure 1 (a) illustrate the equivalent aperture configurations of an ellipsoidal reflector described in [1], and the hyperbolic reflector of interest, respectively, where in both cases the shapes of equivalent apertures are identical and may share the similar procedure in the TD scattering analysis. As a result, the radiation integral of scattering field in TD can be expressed in (6) of [1].

## III. ANALYTIC DEVELOPMENT FOR SCATTERED FIELDS

The scattering field is found by using the continuation of surface curvature from an ellipsoidal surface to hyperbolic reflectors in [1].

### (A) The Continuation of Surface Curvature

The hyperbolic surface in (1) can be analogously described as

$$\frac{(z+c)^2}{a^2} + \frac{x^2 + y^2}{b_{eq}^2} = 1 \quad (3)$$

where  $b_{eq} = jb$  is a complex radii of surface curvature, and makes the surface an ellipsoidal one. The trigonometric

representation of an arbitrary position vector,  $\bar{r}' = (x', y', z')$  is described by

$$\bar{r}' = (b_{eq} \sin \theta_{eq} \cos \phi, b_{eq} \sin \theta_{eq} \sin \phi, -c + a \cos \theta_{eq}) \quad (4)$$

where  $\theta_{eq} = j\theta$  with  $(\theta, \phi)$  being illustrated in Figure 1. Based on this curvature continuation into a complex space, the solution in [1] can be extended for the analysis of hyperbolic reflector.

### (B) Contribution Contours

The scattering field at  $t$  is contributed from the equivalent currents on an equal time-delay contour,  $C_\delta$ , formed by the intersection of  $S_a$  and  $S_R$  with  $S_R$  being a sphere centered at the field point,  $\bar{r}_o$ , with a radius,  $R \geq 0$ . The condition of this contour is  $R + r_1 - 2a = vt$ , where  $r_1$  is selected to make  $S_a$  close to the hyperbolic reflector as shown in Figure 1. In particular,  $C_\delta$  shares the properties as that in [1], and will not be repeated for brevity. However, it is circular with its center,  $Q_c$ , located on the straight line going through  $F_2$  and  $\bar{r}_o$ , the reflection path of feed's radiation to arrive  $\bar{r}_o$ . The scattering field vanishes as  $C_\delta$  vanishes. It is noted that when  $C_\delta$  is a closed circle, then the scattering field corresponds to the reflected fields. Otherwise, it is a partial circle with ends at the boundary of  $S_a$ , and will consist of reflection and edge diffractions. The mathematic expression of  $C_\delta$  is identical to (9) in [1] by replacing  $R = vt - (2a - r_1)$  in [1] with  $R = vt - r_1 + 2a$  here.

### (C) Resulting Solution

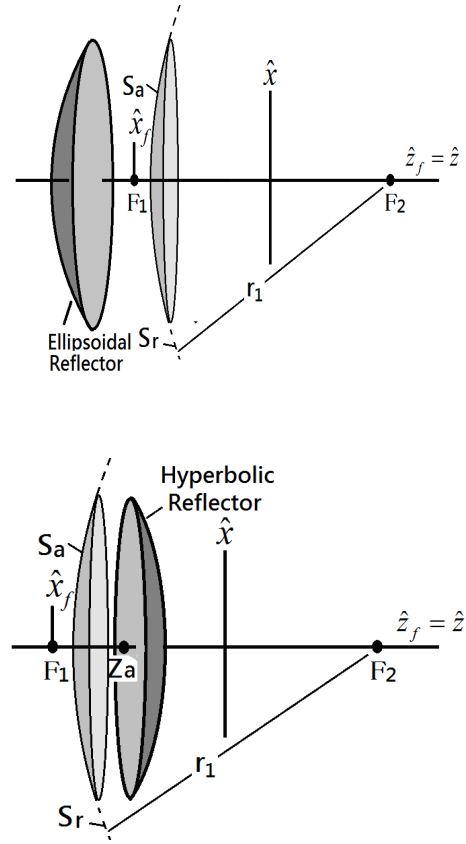
The scattering field at  $\bar{r}_o$  due to the illumination of a TD step-function feed radiation can be expressed in a closed-form as

$$\bar{E}_s(\bar{r}_o, t) = \left( \bar{E}_r(\bar{r}_o, t) \cdot T(\phi_e - \phi_b) + \bar{E}_c(\bar{r}_o, t) \Big|_{\phi_e=\phi_e} - \bar{E}_c(\bar{r}_o, t) \Big|_{\phi_e=\phi_b} \right) \Pi(t) \quad (5)$$

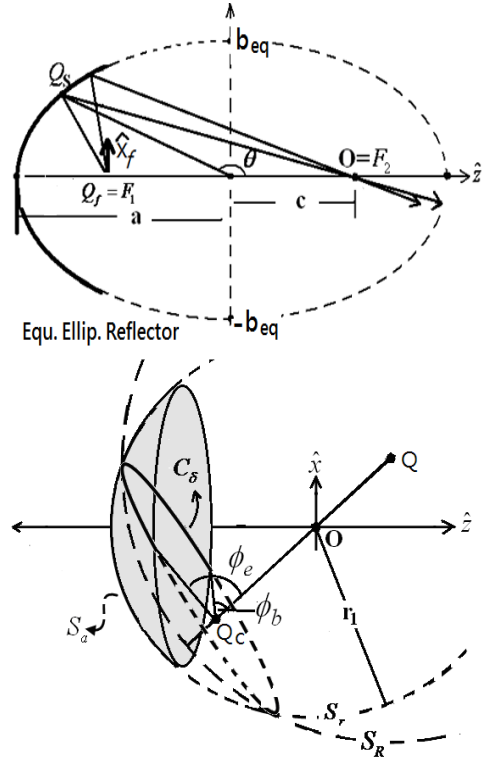
where  $T(\varphi) = \varphi / (2\pi)$  and  $\Pi(t) \equiv u(t - t_1) - u(t - t_2)$  with  $u(t)$  is a Heaviside step function. In (5),  $t_1$  and  $t_2$  are the starting and ending time that the contributing contour,  $C_\delta$ , overlaps with the reflecting surface. Also  $\phi_{b,e}$  are the two angular intersection points between  $C_\delta$  and the edge of reflecting surface, spanned from the center of  $C_\delta$ . When the entire  $C_\delta$  overlaps on the reflecting surface,  $\phi_e = \phi_b + 2\pi$ . The reflected and edge diffracted components are given by

$$\bar{E}_{r,c}(\bar{r}_o, t) = \frac{r_1 V_o}{4\pi r_p} \left\{ \begin{aligned} & (1 + \hat{R}(t) \cdot \hat{r}_i) \bar{\Omega}_1^{r,c} - \hat{z}_p \left( \frac{z_p^2}{R^2(t)} + \frac{(R(t) - r_1) z_p z_c}{R^2(t) r_1} \right) (\hat{z}_p \cdot \bar{\Omega}_1^{r,c}) \\ & + \frac{(R(t) - r_1) z_p \rho_t}{R^2(t) r_1} [(\hat{z}_p \cdot \bar{\Omega}_1^{r,c}) \hat{x}_p + (\hat{z}_p \cdot \bar{\Omega}_1^{r,c}) \hat{y}_p] \end{aligned} \right\} \quad (6)$$

The sign change due to the difference of surface's unit normal direction has been considered in (6). The parameters not defined in this letter are identical to these in [1] with the new parameter mapping in (3) and (4) used.



(a) Reflectors and their apertures

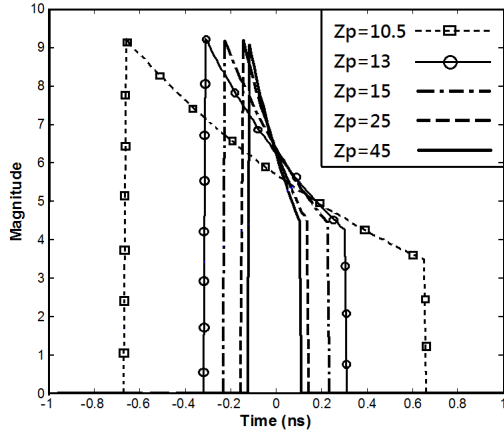


(b) Geometrical parameters

Fig. 1: A rotationally symmetric hyperbolic and a near-field focused reflector which is taken from a part of hyperbolic.

## IV. NUMERICAL EXAMPLES

The examples consider a reflector size with the parameters in (1) given by  $a = 4\text{m}$ ,  $b = 3\text{m}$  and  $c = 5\text{m}$ . The reflector in Figure 1 is rotationally symmetric with a radius  $r_a = 1\text{m}$ . Figure 2 (a) shows the TD response on the z-axis with  $z_p$  being the distance measured from  $F_2$  toward the  $-z$  direction. In contrast to the radiation of an elliptical reflector, this reflector radiates defocusing fields. The time duration becomes shorter at the observation of farther distance. Figure 2(b) shows the TD response at  $r_o = 15\text{m}$  with various observation angles. In this case, the field at the axis corresponds to the main beam while that at the far away angles are the sidelobes. Figure 3 (a) and (b) show the cross-polarization and curvature effects in comparison with a parabolic reflector antenna with a focal length, 1m, where the observation is at a same distance to the reflector surface. In this case the parameters of hyperbolic reflector is same to those in Figure 2(b) with the observation at  $r_o = 15\text{m}$  on the  $-z$ -axis.



(a) Observation on the axis

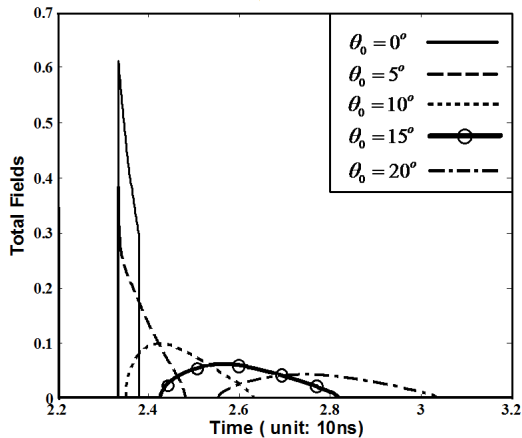
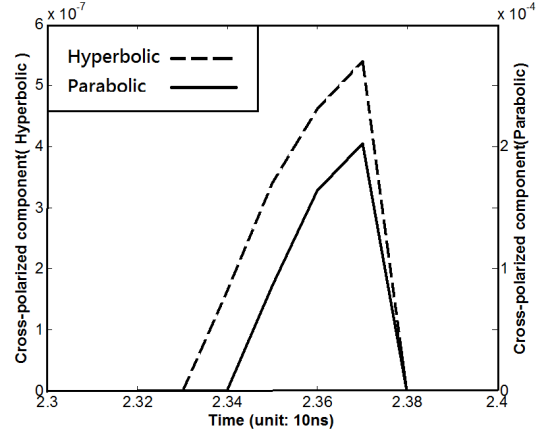
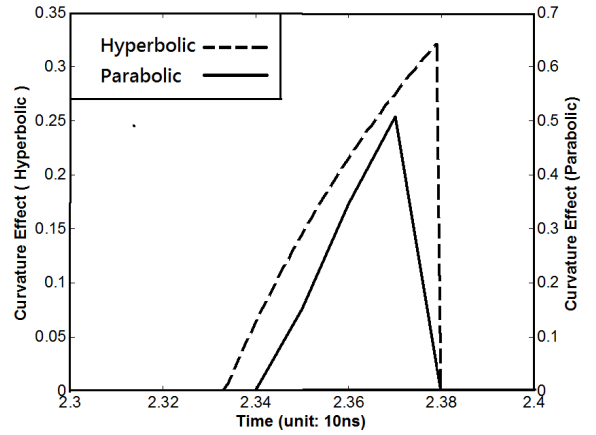
(b) Observation at  $r_o = 15\text{m}$ 

Fig. 2: TD field response of scattering fields.



(a) Cross-polarization Effects



(b) Curvature effects

Fig. 3: Cross-polarization and curvature effects in comparison with a parabolic reflector.

## V. CONCLUSIONS

This paper presents the TD analysis of a hyperbolic reflector based on the mathematic continuation of an ellipsoidal reflector antenna. The formulation has been developed with numerical results presented to validate the results.

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