Equivalent Radius Analytic Formulas of Substrate Integrated Cylindrical Cavity

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Abstract—**Equivalent radius of a Substrate Integrated Cylindrical Cavity (SICC) is an important parameter for calculating the resonant frequency and designing a SICC device. In this paper, based on two sets of equivalent width analytic formulas of Substrate Integrated Waveguide (SIW), the equivalent radius analytic formulas of SICC were derived by the conformal transformation method for the first time. In order to verify the validity of these formulas, using the formulas given in this paper calculated the equivalent radii and the corresponding resonant frequencies of TM_{010} mode SICCs , and the resonant frequencies of TM_{010} mode SICCs were also calculated by the method introduced in [12] and simulated by electromagnetic simulation software. The results show that the formulas given in this paper have higher precision, and are convenient, suitable to more application fields, so they will play important role in the analysis and design of SICC devices.**

Index Terms—substrate integrated cylindrical cavity; resonant frequency; conformal transformation method; equivalent radius.

I. INTRODUCTION

In the last years, the Substrate Integrated Waveguide (SIW) was studied widely. SIW is a type of dielectric-filled rectangular waveguide that is synthesized in a planar substrate with two linear arrays of metallic vias. By comparison with conventional metallic rectangular waveguide, SIW has the characteristics of compact bulk, planar structure, easy to fabricate, etc., is very suitable to integrated circuits. Presently, a large number of SIW-based devices have been implemented, and many of them, like filters, oscillator and power divider, are based on the substrate integrated cavities [1]-[8] . In conventional metallic waveguide cavity, resonant frequency can be determined in closed form for canonical shape. In [1]-[2], the cavities are made of substrate integrated rectangular waveguides, their resonant frequencies can be determined by using the similar methods of conventional metallic rectangular cavities, because the equivalent width analytic formulas of SIW have been given [9]-[10] . In [3]-[8], the oscillator, filters and power divider are made on the substrate integrated cylindrical cavities. Substrate Integrated Cylindrical Cavity (SICC) is a cylindrical cavity with sidewall of metallic vias instead of solid metallic sidewall connecting the solid metallic top and bottom plates, as shown in Fig.1. For SICC, the analytic calculation of the resonant frequency can be cumbersome, because the equivalent radius analytic formula of SICC has not been given at present. In [11], nonlinear eigenvalues method was adopted to analyze arbitrarily shaped substrate integrated waveguide resonators. In [12], the resonant frequency determination methods of TM_{010} mode and TM_{110} mode circular substrate integrated resonators are given. In this paper, based on two sets of equivalent width analytic formulas of SIW given in [9] and [10], two sets of analytic formulas for calculating the equivalent radius of SICC are derived by using the conformal transformation method, respectively. Using the equivalent radius analytic formulas given in this paper, the resonant frequencies of SICC can be determined easily. As an example, the equivalent radii and the corresponding resonant frequencies of TM_{010} mode SICCs are calculated using the formulas given in this paper, and the resonant frequencies are compared with the results calculated by the method given in [12] and the simulated results of electromagnetic simulation software.

II. ANALYTIC FORMULAS FOR CALCULATING THE EQUIVALENT RADIUS OF SUBSTRATE INTEGRATED CYLINDRICAL CAVITY

The geometry of SICC is shown in Fig.1. Similar to SIW can be equivalent to solid metallic sidewall rectangular waveguide filled with dielectric, the SICC can be equivalent to a solid metallic sidewall cylindrical cavity if the radius R of SICC is replaced by the equivalent radius R_{eq} .Conformal transformation method was adopted to derive the equivalent radius analytic formula of SICC in this paper. Fig.2 shows the transformation process in detail.

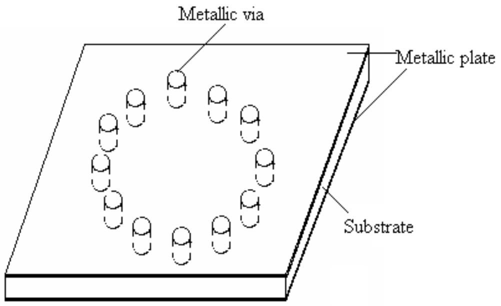

Fig. 1 Geometry of the substrate integrated cylindrical cavity

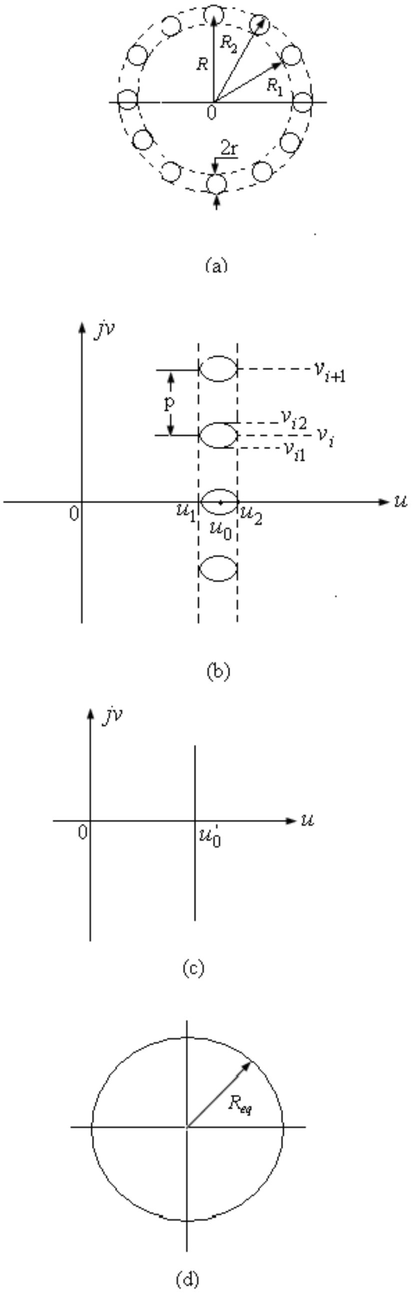

Fig.2 Transformation process of substrate integrated cylindrical cavity

In Fig.2 (a), the number of metallic vias is *n,* the radius of metallic via is r，the radius of SICC is R , so the inner circle radius $R_1 = R - r$, and the exterior circle radius $R_2 = R + r$. Suppose Fig.2 (a) is in *Z*-plane, and $Z = |Z|e^{j\varphi}$, then the transformation $w = \ln(Z) = \ln|Z| + j\varphi = u + jv$ transforms the *Z*-plane into *w*-plane， as show in Fig.2（b）. In the *w*-plane ， $u = \ln|Z|, v = \varphi$. So the circles which radii are R_1 and R_2 in *Z*-plane are transformed into beelines $u_1 = \ln R_1$ and $u_2 = \ln R_2$ in *w*-plane, respectively. The circular array of metallic vias in *Z*-plane is transformed into linear array between u_1 and u_2 in *w*-plane.

In Fig.2（b）

$$\begin{aligned} u_0 &= (u_1 + u_2)/2 = \ln(R_1R_2)/2 \\ &= \ln[(R-r)(R+r)]/2 \end{aligned} \tag{1}$$

$$v_i = \varphi_i, \quad v_{i1} = \varphi_{i1}, \quad v_{i2} = \varphi_{i2} \tag{2}$$

Where, $\varphi_i = i\frac{2\pi}{n}(i = 1,2,\cdots,n)$, $\varphi_{i1} = \varphi_i - \theta$, $\varphi_{i2} = \varphi_i + \theta$, and

$$\theta = \sin^{-1}(r/R) \tag{3}$$

The meanings of $\varphi_i, \varphi_{i1}, \varphi_{i2}$ *and* θ are shown in Fig.3.

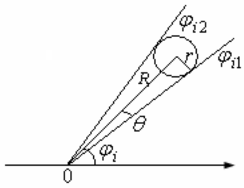

Fig.3 Geometry parameters of metallic via

In Fig.2 (b), the separation between adjacent metallic vias is

$$p = v_{i+1} - v_i = \varphi_{i+1} - \varphi_i = \frac{2\pi}{n} \tag{4}$$

There are three equivalent methods for metallic via equivalent radius, that is

$$r_{eq} = \frac{1}{2}(u_2 - u_1) = \frac{1}{2}(\ln R_2 - \ln R_1) = \frac{1}{2}\ln(\frac{R+r}{R-r}) \tag{5a}$$

$$r_{eq} = \frac{1}{2}(v_{i2} - v_{i1}) = \theta = \sin^{-1}(\frac{r}{R}) \tag{5b}$$

$$r_{eq} = \frac{1}{2}[\frac{1}{2}\ln(\frac{R+r}{R-r}) + \sin^{-1}(\frac{r}{R})] \tag{5c}$$

It can be proven that the results calculated from the three formulas are nearly equal. It indicates that metallic via is also cylindrical in *w*-plane.

The metallic vias array in Fig.2 (b) can be equivalent into solid metallic plane, as shown in Fig.2(c). In this paper, two sets of equivalent width analytic formulas of SIW given in [9] and [10] are used to derive the position of the equivalent solid metallic plane, respectively.

The inner area of circular metallic vias array in Fig.2 (a) is transformed into the area of $u < u_1$ in Fig.2 (b). Suppose another similar metallic vias array is at $u = -u_0$, then the two metallic vias arrays compose a SIW. So in Fig.2 (c), according to [9], the equivalent solid metallic plane position u_0' can be determined by the following formulas

$$u_0' = a_{eq}/2 \tag{6}$$

Where a_{eq} is the equivalent width of the corresponding SIW, it can be determined by

$$a_{eq} = 2u_0 \cdot \bar{a} = \ln[(R-r)\cdot(R+r)]\cdot\bar{a} \tag{7}$$

In which, $\bar{a}$ is the normalized equivalent width, it is determined by[9]

$$\bar{a} = \xi_1 + \frac{\xi_2}{\frac{p}{2r_{eq}} + \frac{\xi_1+\xi_2-\xi_3}{\xi_3-\xi_1}} = \xi_1 + \frac{\xi_2}{\frac{2\pi}{n\ln(\frac{R+r}{R-r})} + \frac{\xi_1+\xi_2-\xi_3}{\xi_3-\xi_1}} \tag{8}$$

Where

$$\xi_1 = 1.0198 + \frac{0.3465}{\frac{2u_0}{p} - 1.0684}$$
$$\xi_2 = -0.1183 - \frac{1.2729}{\frac{2u_0}{p} - 1.201} \tag{9a}$$
$$\xi_3 = 1.0082 - \frac{0.9163}{\frac{2u_0}{p} + 0.2152}$$

That is

$$\xi_1 = 1.0198 + \frac{0.3465}{\frac{n\ln[(R-r)\cdot(R+r)]}{2\pi} - 1.0684}$$
$$\xi_2 = -0.1183 - \frac{1.2729}{\frac{n\ln[(R-r)\cdot(R+r)]}{2\pi} - 1.201} \tag{9b}$$
$$\xi_3 = 1.0082 - \frac{0.9163}{\frac{n\ln[(R-r)\cdot(R+r)]}{2\pi} + 0.2152}$$

According to [10], the equivalent solid metallic plane position $u_0^{'}$ can also be determined by

$$u_0^{'} \approx u_0 + \frac{p}{4}\ln(\frac{p}{4r_{eq}}) = \ln\{\sqrt{(R-r)\cdot(R+r)}\cdot[\frac{\pi}{n\ln(\frac{R+r}{R-r})}]^{\frac{\pi}{2n}}\} \tag{10}$$

Through the transformation $Z^{'} = e^{w}$, then the solid metallic plane $w = u_0^{'}$ in Fig.2(c) is transformed into the solid metallic cylindrical sidewall, its radius (that is the equivalent radius of SICC) is

$$R_{eq} = e^{u_0^{'}} \tag{11}$$

As shown in Fig.2 (d).

The equivalent radius of SICC can be calculated from (6)~(9) and (11), these formulas are derived from [9].

From (10) and (11), we can get another formula of the equivalent radius of SICC derived from [10], that is

$$R_{eq} = \sqrt{(R-r)\cdot(R+r)}\cdot[\frac{\pi}{n\ln(\frac{R+r}{R-r})}]^{\frac{\pi}{2n}} \tag{12}$$

In the design of SIW, the via diameter is chosen to be equal or smaller than a tenth of the wavelength of the maximum operation frequency and the separation between adjacent vias is equal to or smaller than twice the diameter of the metallic via, so in the design of SICC, if corresponding to Fig.2(b), then

$$2r_{eq} = \ln(\frac{R+r}{R-r}) \le \frac{\lambda_{\min}}{10}$$
$$p = \frac{2\pi}{n} \le 4r_{eq} = 2\ln(\frac{R+r}{R-r})$$

Where $\lambda_{\min}$ is the minimum wavelength corresponding to the maximum operation frequency.

If corresponding to Fig.2 (a), then

$$r \le \frac{\lambda_{\min}}{20}$$
$$R\cdot\frac{2\pi}{n} \le 4r$$

That is

$$\begin{cases} \frac{\pi}{n} \le \ln(\frac{R+r}{R-r}) \le \frac{\lambda_{\min}}{10} \\ \frac{\pi R}{2n} \le r \le \frac{\lambda_{\min}}{20} \end{cases} \tag{13}$$

III. CALCULATION OF SUBSTRATE INTEGRATED CYLINDRICAL CAVITY RESONANT FREQUENCY AND EQUIVALENT RADIUS

Conventional solid metallic sidewall cylindrical cavities usually work at TE_{11p}, TM_{01p} and TE_{01p} modes. TM_{010} mode SICC can be realized in substrate easily, and can be excited by SIW, so in this paper, the TM_{010} mode SICC is studied.

In the design process of cavity, the resonant frequency is a fundamental parameter needed to be determined accurately. The resonant frequency f_0 of TM_{010} mode solid metallic sidewall cylindrical cavity can be determined by the cavity radius *R*, that is

$$f_0 = \frac{c}{2.62R\sqrt{\varepsilon_r}} \tag{14}$$

Where, c is the light velocity in air, ε_r is the relative permittivity constant of dielectric substrate. Formula (14) can also be used to calculate the resonant frequency of TM_{010} mode SICC if the cavity radius *R* is replaced by the equivalent radius R_{eq} of SICC. R_{eq} can be calculated by the formulas (6)~(9), (11) or (12) derived in the above section.

In order to comparison, with the help of electromagnetic simulation software HFSS, the resonant frequencies of TM_{010} mode SICC are determined by the calculation of eigenmode. Otherwise, the resonant frequencies are also calculated by the method given in [12] and by the formula (14) with the actual radius *R* of SICC, as shown in Fig.2(a).

Fig.4 shows the curves of resonant frequency f_0 versus the via radius *r* of SICC when the SICC radius *R*=8mm. In which, "**" denote the simulated results by HFSS; "---"denote the calculated results from the formulas (6)~(9), (11) and (14) ; "oo" denote the calculated results from the formulas (12) and (14) ; "—" denote the calculated results using the method given in [12]; "++" denote the calculated results from the formula (14) and the actual radius *R* of SICC.

It can be seen from Fig.4 that the resonant frequencies calculated from the formulas (6)~(9), (11) , (14) and calculated by the method given in [12] are close to the simulated resonant frequencies, they are relatively accurate, and they have almost the coequal precision ; the resonant frequencies calculated from the formula (14) and the actual SICC radius *R* have the maximal errors compared to the simulated results, are inaccurate, so the actual radius *R* of SICC cannot be simply used to calculate the resonant frequencies of SICC, should be replaced by the equivalent radius R_{eq} of SICC.

Fig.5 shows the equivalent radius curves versus actual radius of SICC. The equivalent radii are calculated from formulas (6~9) and (11). It can be seen from Fig.5 that the equivalent radius increases with the decrease of via radius. This can be indicated that when the via radius decreases, the gap between adjacent vias increases when the number of vias and radius of SICC are unchangeable, then the electromagnetic field distribution space in SICC is extended, so the equivalent radius increases.

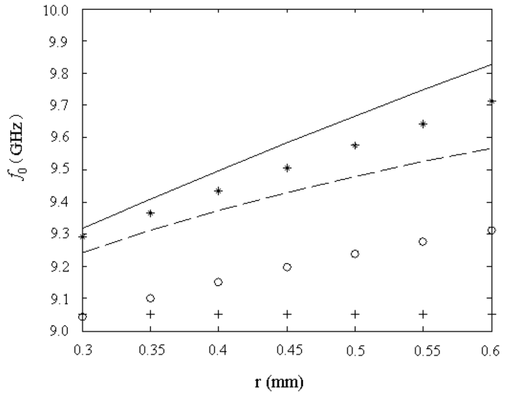

Fig. 4 Resonant frequency curve of SICC versus via radius

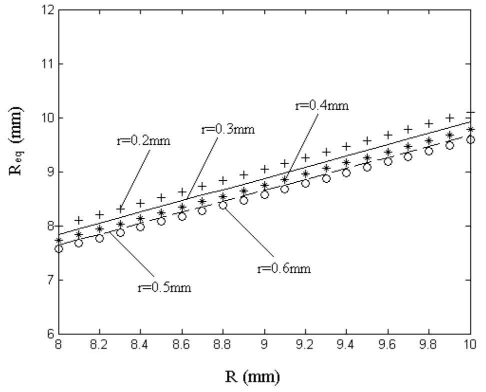

Fig. 5 Equivalent radius curve versus actual radius of SICC

IV. CONCLUSION

In this paper, two sets of analytic formulas for calculating the equivalent radius of SICC are derived by the conformal transformation method based on two sets of equivalent width analytic formulas of SIW, respectively. In order to comparison, the resonant frequencies of TM_{010} mode SICCs are calculated from the two sets equivalent radius , actual radius and by the method given in [12], these results are compared with the simulated results obtained from electromagnetic simulation software HFSS. The results show that the formulas (6)~(9) , (11) given in this paper and (14) are more precision, similar to the method given in [12]. However, the formulas given in [12] are only for calculating the resonant frequencies of TM_{010} mode and TM_{110} mode SICCs. The formulas given in this paper are for calculating the equivalent radius of SICC. Equivalent radius is a more basic parameter compared with resonant frequency. Resonant frequency of any mode SICC can be calculated from the equivalent radius of SICC, and the other parameters of SICC can also be calculated from its equivalent radius, so the formulas given in this paper are suitable to more application fields, and they are handy, so they will play important role in the analysis and design of SICC devices.

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