Production of Bessel-Gauss Beams at THz by Use of UPA

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***Abstract-* Applying the principle of antenna pattern synthesis, a uniform planar array (UPA) is designed to generate an arbitrary-order Bessel-Gauss beam (including zero-order and high-order) at Terahertz (THz) range. Numerical results show that the designed Bessel-Gauss beams are in excellent agreement with the desired ones and the project of creating Bessel-Gauss beam is practicable. The generated beams can be applied in THz systems.**

***Index Terms*–Bessel-Gauss beam; uniform planar array (UPA); Terahertz (THz)**

I. INTRODUCTION

Physical generation of an ideal diffraction-free beam or Bessel beam can not be made, as it is unbounded in a transverse plane and would require an infinite amount of energy. In order to conquer this difficulty, a Bessel-Gauss beam was thus proposed [1]. It may be considered as a Bessel beam modulated by a Gaussian function so that its boundary does not extend infinitely and its field carries a limit of energy [2]. Therefore, the production of this beam becomes much easier practically. Lots of approaches [3, 4] have been suggested to create a Bessel-Gauss beam of zero order or high order. In fact, these approaches can be divided into two kinds. One is known as a passive way, which implies a Bessel-Gauss beam generated by transforming an incident Gaussian beam using an optical element, such as computer-generated holograms (CGHs) [5], axicons [6], and diffractive phase elements (DPEs) [7]; the other is an active project in which a resonator should be constructed to shape a Bessel-Gauss beam and output directly from it. These schemes have been reported in articles [8, 9]. In the present paper, a novel way is proposed to generate a Bessel-Gauss beam of arbitrary order. It is known that the pattern of array antenna can be synthesized by adjusting the weight of each element. We can get inspired from the technique of pattern synthesis of array antenna. Accordingly a uniform planar array is employed to form a Bessel-Gauss beam by optimally selecting its weights. The numerical results demonstrate that Bessel-Gauss beams with arbitrary orders can be produced effectively.

II. OPTIMAL DESIGN OF UPA

A. Bessel-Gauss Beam

In the cylindrical coordinates system, the complex amplitude distribution of the *nth* -order Bessel-Gauss beam is given by

$$E_n(\rho,\varphi,z=0)=E_0 J_n(k_\perp \rho)\exp(in\varphi)\exp(-\rho^2/w_0^2) \qquad (1)$$

where E_0 is a constant, J_n represents the *nth* -order Bessel function of the first kind, $\rho^2=x^2+y^2$, $k_\perp{}^2+k_z{}^2=k^2=(2\pi/\lambda)^2$, $k_\perp$ and k_z denote respectively the transverse and longitudinal wave numbers, λ is a wavelength in free space, the waist of Gaussian beam is denoted by w_0 . Therefore, the amplitude distribution of the *nth* -order Bessel-Gauss beam can be written as

$$U_n(\rho,\varphi,z=0)=U_0 J_n(k_\perp \rho)\exp(-\rho/w_0^2) \qquad (2)$$

It can be seen from Eq. (2) that the Bessel-Gauss beam of order n may be considered as a Gaussian modulation had been added on the Bessel beam. Unlike the ideal Bessel beam, the extension of its lateral oscillation is localized and it thus carries a limited energy. Thereupon, the creation of this beam becomes much easier in practice when compared with the Bessel beam. Fig. 1 displays the radial profiles of the zero-order Bessel-Gauss beam and ideal Bessel beam, respectively. And the high-order distributions of them are illustrated in Fig. 2.

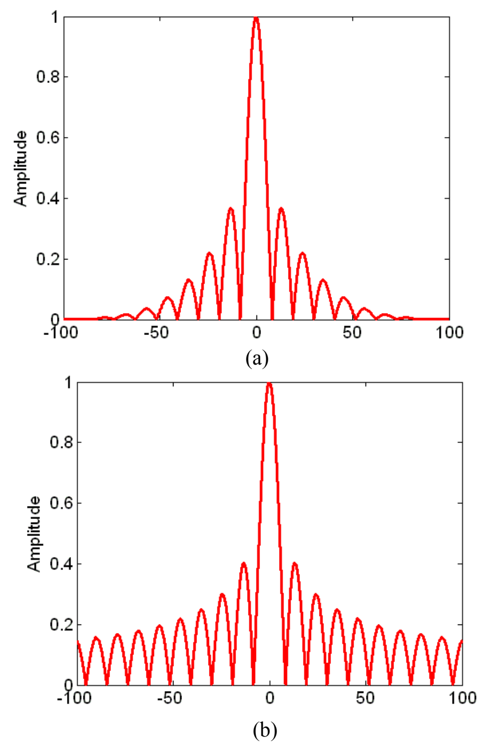

Fig. 1. The amplitude distributions of the zero-order Bessel-Gauss beam (a) and Bessel beam (b).

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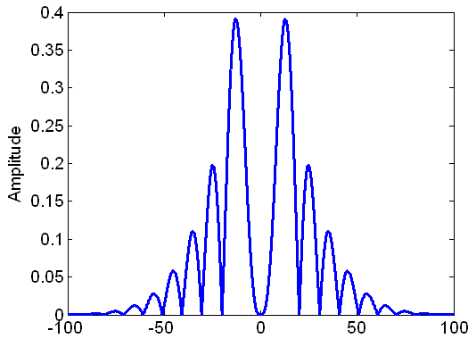
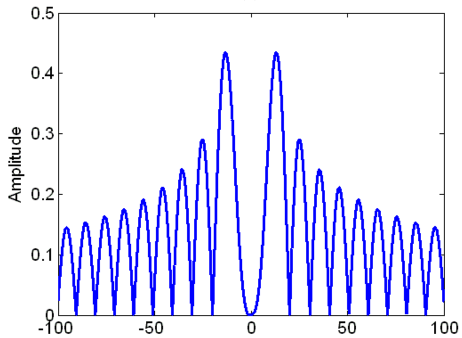


(a)



(b)

Fig. 2. The amplitude profiles of the high-order Bessel-Gauss beam (a) and Bessel beam (b).

B. *Uniform Planar Array (UPA)*

A UPA is formed by placing elements along a rectangular grid, as illustrated in Fig. 3. It can be employed to control and shape the radiation pattern of array antenna. It have been demonstrated that it is more versatile and can provide more symmetrical pattern with lower side lobes [10]. Provided that $M \times N$ elements are positioned along the x-axis and y-axis, and the element in the top left corner of array is set as the reference point, and the azimuth and pitch angels of the incident signal are represented by φ and θ (see Fig. 3), its radiation pattern can thus be expressed as [10, 11]

$$\begin{aligned} G_0 &= \sum_{m=1}^{M} I_{mx} \exp(j\beta_{mx}) \exp[-j(m-1)(kd\sin\theta\cos\varphi)] \\ &\times \sum_{n=1}^{N} I_{my} \exp(j\beta_{my}) \exp[-j(n-1)(kd\sin\theta\sin\varphi)] \\ &= G_x \times G_y \end{aligned} \quad (3)$$

Eq. (3) indicates that the total eradiation pattern G_0 of a UPA is the product of the pattern of the arrays in the x and y directions. The amplitude and phase of initial excitation current along the x-axis and y-axis are denoted by I_{mx}, I_{my}, β_{mx}, β_{my}, respectively. Therefore the desired pattern of the UPA can be shaped by choosing optimally the weight factor of I_{mx}, I_{my}, β_{mx}, β_{my}. To radiate the Bessel-Gauss beams, a genetic algorithm is used in this paper to select the optimal coefficients of each element. The flow of executing genetic algorithm program is depicted detailedly in ref. [12], and consequently will not be covered herein for avoiding repetition.

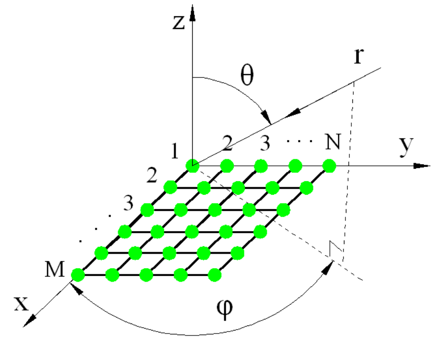
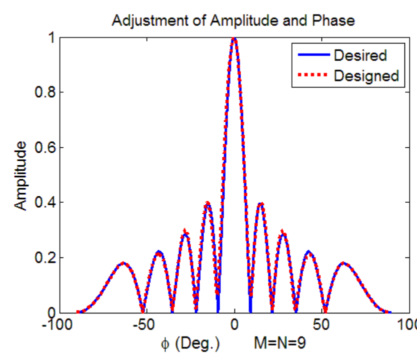


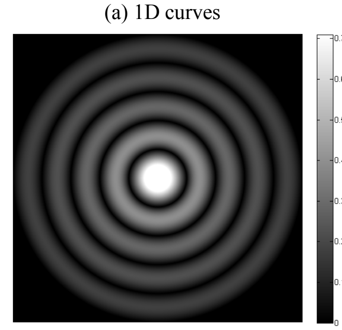
Fig. 3. The geometry of a UPA

III. Simulation Results

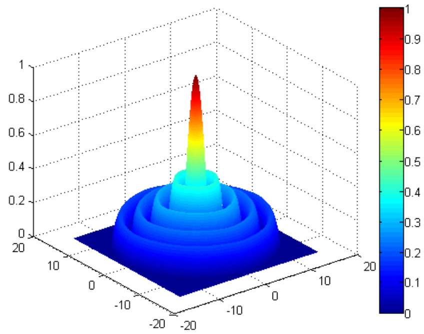
Using the optimal algorithm introduced above, we have successfully synthesized several UPAs to radiate Bessel-Gauss beams. In the first instance, the 9×9 -element UPA, with $d = 0.9\lambda$ and $\lambda = 1.0mm$, is designed to form the zero-order Bessel-Gauss beam by adjusting the amplitude and phase of exciting current of each element, as illustrated in Fig. 4. At the visible scope, the generated beam, with $k_\perp = 3318mm^{-1}$ and w_0=0.009mm, has 10 null points and 9 maximums. We can readily see from Fig. 4(a) that the desired beam exhibits an excellent agreement with the designed one. For the sake of understanding the properties of the constructed Bessel-Gauss beam clearly, the transverse amplitude distributions in two- and three-dimensional figures are depicted in Figs. 4(b) and 4(c), respectively. The normalized amplitude and phase of the exciting current of each element are listed in Table 1.



(a) 1D curves



(b) 2D pattern



(d) 3D profile

Fig. 4. Zero-order Bessel-Gauss pattern shaped by 9×9 -element UPA

Table 1. Amplitudes and phases of the exciting current for 9×9 -element UPA

No.	1	2	3	4	5	6	7	8	9
I_{mx}	0.89	0.26	0.26	0.72	0.58	0.07	0.41	0.79	0.29
I_{my}	0.56	0.31	0.43	0.45	0.62	0.55	0.46	0.37	0.81
β_{mx}	0.23	3.1	2.07	3.14	0.31	2.80	0.22	3.01	0
β_{my}	3.13	3.0	3.04	0.13	3.1	3.05	0.12	3.06	3.13

Fig. 5 shows another zero-order Bessel-Gauss pattern having $k_\perp = 10529mm^{-1}$ and w_0=0.0022mm, which is generated by the 8×8 -element UPA through selecting the phase-only components of exciting current. The related parameters utilized in Fig. 5 are : $\lambda = 0.32mm$, $d = 0.5\lambda$. Likewise, as can be observed distinctly from Fig. 5, the designed beam almost overlaps with the desired one. The optimal results of the phase of the current are given in Table 2.

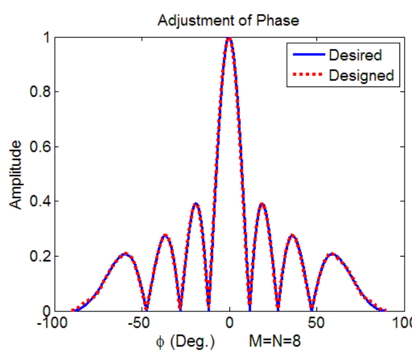


Fig. 5. Bessel-Gauss beam of zero order radiated by 8×8 -element UPA

Table 2. Phase components of the current for 8×8 -element UPA

No.	1	2	3	4	5	6	7	8
β_{mx}	2.82	2.60	1.49	3.14	0.01	2.20	1.24	3.01
β_{my}	1.50	2.22	0.48	2.06	2.06	0.47	2.22	1.51

The Bessel-Gauss beam of zero order can also be shaped by altering the amplitude components, as illustrated in Fig. 6. The relevant parameters used in this example are: $M = N = 5$, $\lambda = 0.30mm$, $d = 0.5\lambda$, $k_\perp = 11539mm^{-1}$, w_0=0.0015mm. From Fig. 6 one can see that two curves display a good accord. Table 3 lists the optimal amplitude components.

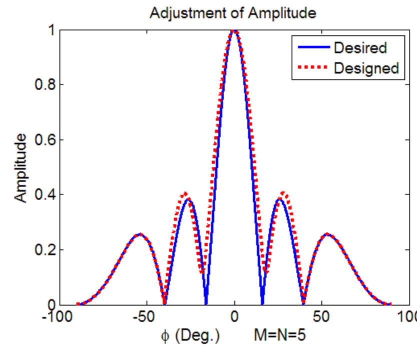
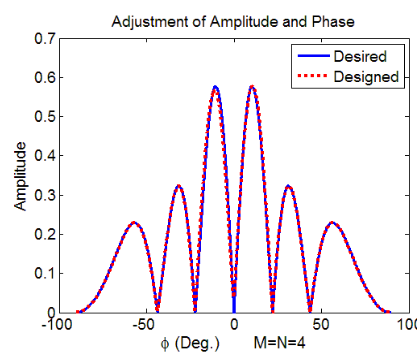


Fig. 6. Zero-order Bessel-Gauss beam eradiated by 5×5 -element UPA

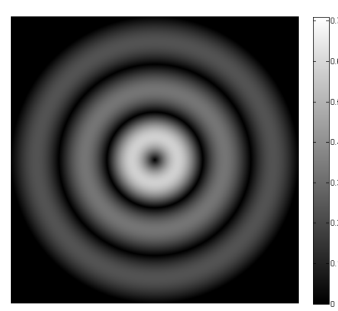
Table 3. Amplitude components of the current for 5×5 -element UPA

No.	1	2	3	4	5
I_{mx}	0.7557	0	0	0.1746	0.8951
I_{my}	0.9966	0.2561	0.0207	0.9999	0.718

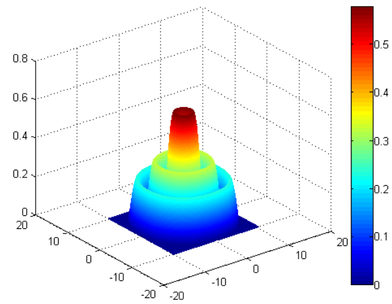
In the last instance, the 4×4 -element UPA is synthesized, by the way of tuning simultaneously the amplitude and phase, to eradiate the first-order Bessel-Gauss beam, whose amplitude at the origin is zero. Figs. 7 (a), (b) and (c) give the 1D, 2D and 3D distributions of the first-order Bessel-Gauss beam, respectively. The other parameters used in Fig. 7 are : $\lambda = 0.30mm$, $d = 0.6\lambda$, $k_\perp = 11303mm^{-1}$, w_0=0.0018mm. We can observe obviously that the created beam is almost identical with the expected one.



(a) 1D distributions



(b) 2D distribution



(d) 3D distribution

Fig. 7. First-order Bessel-Gauss beam produced by 4×4 -element UPA

Table 4. Amplitudes and phases for 4×4 -element UPA

No.	1	2	3	4
I_{mx}	0.8926	0.6979	0.5171	0.483
I_{my}	0.4626	0.2847	0.2551	0.4754
β_{mx}	0	0.188	3.1416	3.1416
β_{my}	0	3.1345	0.0173	3.1416

IV. CONCLUSION

In this paper, a new approach to generate efficiently Bessel-Gauss beam of arbitrary order at THz range is suggested. It employs the UPA to shape a desired beam by adjusting the amplitude-only, phase-only, amplitude and phase components of the exciting current of each element. Simulation results demonstrate that the proposed method is a feasible scheme. The produced beam can find its applications in communication, measuring, power transmission, and imaging system at THz spectrum range.

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