Evaluation of Dielectric Sample with Spacer in Rectangular Waveguide

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1. Introduction

Broadband microwave wave applications require an accurate data about material characteristics. Many techniques have been reported to evaluate the electromagnetic properties of material with loss using waveguides, coaxial lines and cavities.

Recently, we proposed the broadband measurement method of complex permittivity \( \varepsilon_r = \varepsilon_r' - j\varepsilon_r'' \), where the sample may occupy only the part of the waveguide cross section to get the adequate signal level of S parameters [1]. The method enables to evaluate the high dielectric constant material that may have the low or somewhat high loss. To keep a constant spacing between the sample and waveguide wall that is important to get the reproducible data, the well-defined spacer is attached to support the sample.

In this paper, we will extend the method of [1] to evaluate the sample with spacer in rectangular waveguide. The method is based on the hybrid electromagnetic theory, the extended spectral domain approach combined with the mode-matching method (ESDMM) [2].

2. Theoretical Basis of Analysis

Fig. 1 shows a dielectric sample with the dielectric spacer keeping the fixed spacing between the sample and the side wall of rectangular waveguide. The complex permittivity of the spacer \( \varepsilon_{sp} \) is known beforehand. The dominant TE\(_{10}\) mode is incident on the sample, and the TE\(_{m0}\) modes will be generated as scattered fields. The scattered fields are analyzed by the above mentioned accurate hybrid analytical method, ESDMM. The whole analysis region of the present problem is divided into five regions (in Fig. 2) by introducing the aperture fields \( e_a(x) \) at \( z = 0 \); \( e_b(x) \) at \( z = s \); \( e_c(x) \) at \( z = t \) and \( e_d(x) \) at \( z = u \). After division, each region can be treated independently by applying the equivalence theorem [1]. The EM fields in the homogeneous (input and output) regions (region I and V) can be expressed in terms of simple eigen functions, i.e., the sinusoidal functions which satisfy the boundary
conditions on the waveguide walls. On the contrary, the fields in inhomogeneous regions (region II, III, IV) cannot be expressed by the simple functions. The eigen functions in these regions (subregion A, B, C) can be obtained by using the mode-matching procedure [2]. They are determined by the equations resulting from the continuity condition at interface between the air, the sample and spacer. These eigen functions $\Phi_m(x)$ satisfy the following biorthogonal relation totally in inhomogeneous air, sample and spacer region.

$$\int_{S_A} \Phi_m(x) \Phi_n(x) \, dx + \int_{S_B} \Phi_m(x) \Phi_n(x) \, dx + \int_{S_C} \Phi_m(x) \Phi_n(x) \, dx = \delta_{mn}$$

(1)

where $S_A$, $S_B$, $S_C$ indicate the air, the sample and the spacer subregion, respectively and $\delta_{mn}$ is Kronecker’s delta. By utilizing the biorthogonal relation (1), the fields in the inhomogeneous region can be transformed to the spectral domain as in the homogeneous regions. In the transformed domain, the Green’s functions can be obtained easily and fields can be related to aperture fields. By applying the continuity conditions of the magnetic fields, we obtain the integral equations with respect to the aperture field. Then, Galerkin’s procedure is applied to these integral equations to determine the aperture fields. The S parameters can be obtained by taking the inner products of the dominant waveguide mode (input signal) with the aperture field at the end of sample.

![Diagram](image_url)

Fig. 1 (a) Dielectric sample supported by the dielectric spacer in rectangular waveguide
(b) Dimensions and coordinates

![Diagram](image_url)

Fig. 2 (a) Introduction of aperture field, (b) Separation into subregion.

3. Numerical Procedure and Results

First, the unknown aperture fields are expressed in terms of the appropriate basis functions $\xi_k(x)$ as
\[ e_p(x) = \sum_{k=1}^{N} b_k \xi_k(x) \]  

where \( b_k \) are unknown expansion coefficients. We choose the flexible subsectional functions \( \xi_k(x) \) [2] as basis functions. The matrix size in this method is the order of the total number of basis function. Then the Galerkin’s method is applied to solve these matrix.

The preliminary numerical computations were performed to confirm the validity of the method. Fig. 3 shows the frequency dependency of scattering characteristics of the sample with dielectric spacer. The complex scattering parameters are calculated by the ESDMM and FEM. Both results are in good agreement over the operating frequency of WR - 90.

The estimation of the unknown permittivity of the sample is the typical inverse problem, that is, the iterative computations are performed by changing the permittivity to reach to the minimum difference between the measured and calculated scattering parameters. The virtual experiment is performed to investigate the accuracy of the estimation by the inverse problem. At first, the permittivity of the sample material permittivity is set up as \( \hat{\varepsilon}_p \) (preassigned). The complex scattering parameters with the sample loaded \( S_{21}^{L} \) and unloaded \( S_{21}^{U} \) are evaluated by FEM using the preassigned \( \hat{\varepsilon}_p \). These complex scattering parameters are treated as the virtual measured values, and the inverse estimation is performed to obtain the unknown permittivity \( \hat{\varepsilon}_r \) by using ESDMM. Table 1 shows the preassigned \( \hat{\varepsilon}_p \) and the inverse estimated \( \hat{\varepsilon}_r \). The dielectric constant is well recurred.

<table>
<thead>
<tr>
<th>Preassigned ( \varepsilon_p )</th>
<th>Estimated ( \varepsilon_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.00 - j 1.00</td>
<td>1.99 - j 0.99</td>
</tr>
<tr>
<td>5.00 - j 1.50</td>
<td>4.98 - j 1.49</td>
</tr>
<tr>
<td>10.00 - j 3.00</td>
<td>9.97 - j 2.99</td>
</tr>
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</table>

Table 1 the virtual experiments at 10 GHz

4. Measurement and Results

Fig. 5 shows waveguide measurement setup. X-band waveguides are used and the operating frequency range is 8.2 - 12.4 GHz. Waveguides are connected to VNA (Agilent 8719ET Vector Network Analyzer) though the waveguide adapter and the coaxial cable. Only \( S_{21} \) is measured and \( S_{11} \) is not required in this evaluation method. To omit the calibration at boundary of coaxial lines and
waveguide, the complex scattering parameter of sample unloaded are subtracted from those of sample loaded. Fig. 6 shows the frequency dependent permittivity of material. Commercially available microwave absorber is used as the sample. The permittivity of the PTFE spacer is known as $\varepsilon_{rs} = 2.27 - j0.04$. Good agreements are observed in measured frequency range.

Fig. 5 Waveguide measurement setup    Fig. 6 Measured frequency dependent permittivity of material, $\varepsilon_{rs} = 2.27 - j0.04$ (the spacer)

5. Conclusion

We proposed the evaluation method for the dielectric sample with the dielectric spacer. This method is based on the hybrid electromagnetic theory, the extended spectral domain approach combined with the mode-matching method (ESDMM) that requires only transmission characteristics $S_{21}$. The validity of the method was demonstrated by the virtual experiments and confirmed by the actual measurements. Good agreements with published data are observed in measured frequency range. The present method can be extended to evaluate the characteristics of compound and layered aterials.

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References