A TWO-DIMENSIONAL SYNTHETIC APERTURE RADAR IMAGING SIMULATION
OF MOVING OCEAN WAVES

Koichiro Wakasugi, Naru-fumi Kishi, and Masaru Matsue

Department of Electrical Engineering
Kyoto Institute of Technology
Matsugasaki, Kyoto 606, Japan

ABSTRACT

An analytic model of the ocean surface SAR images with a more realistic three-dimensional framework is presented under the assumption that a trochoidal swell propagates through a uniform field of Bragg-type distributed scatterers. Two-dimensional SAR images are calculated for the interpretation and prediction of actual SAR images of the ocean surface as a function of ocean wave amplitude, wave frequency, propagation direction and radar frequency, off-nadir angle of the antenna, and spatial resolutions.

1. INTRODUCTION

The spaceborne synthetic aperture radar (SAR) has opened up a new dimension in our capability to study the ocean because it provides synoptic radar images with high resolution in short time [Iomiyasu, 1978; Beal et al., 1981]. However, it is controversial as to what physical mechanism is causing the formation on SAR imaging of the ocean surface [Elachi and Brown, 1977; Valenzuela, 1980; Harger, 1983]. The radar cross section modulation by the long ocean waves is responsible for the image formation, especially for a real aperture radar. Since the SAR uses the Doppler history of echoes to achieve its high resolution, the orbital motion of water particles can also play a dominant role in SAR image formation [Swift and Wilson, 1979; Alpers and Rufench, 1979].

This paper discusses a two-dimensional SAR imaging based on the treatment of Swift and Wilson [1979]; the calculations assume that a trochoidal ocean gravity wave propagates a uniform field of Bragg-type distributed scatterers. Since three dimensions are considered in our model, two-dimensional SAR images can be obtained as a function of ocean wave amplitude, wave frequency, direction of wave propagation and radar frequency, off-nadir angle of the antenna, and spatial resolutions in azimuth and range. The analytic model with a more realistic three-dimensional framework will be a help for the interpretation and prediction of actual SAR images of the ocean surface.

2. THEORY

Let us consider a line source in circular motion, as shown in Fig.1. The line source is moving in a circle of radius \( \alpha \) and with angular velocity \( \omega \), and the axis of rotation is perpendicular to the wave propagation direction. The motion of a point on the line whose center is at \((x_c, y_c, 0)\) is given by

\[
\begin{align*}
  x(t) &= x_c - \alpha \cos \theta \sin(\phi - \omega \tau - t) \\
  y(t) &= y_c - \alpha \sin \theta \sin(\phi - \omega \tau - t) \\
  z(t) &= \alpha \cos(\phi - \omega \tau - t)
\end{align*}
\]
where $\phi$ and $\theta$ represent the initial location of the line source and the angle of the wave propagation relative to the x axis. The SAR is moving along the x axis with velocity $v$, altitude $h$. If $R(x_c, y_c, t)$ is the distance between the SAR and the point, it is then approximated as

$$R(x_c, y_c, t) \sim R_c + \sqrt{[(x_c - vt)^2 - 2\alpha_x h \cos \phi - \omega_0 t] + 2\alpha_y y_c \sin \theta \sin(\phi - \omega_0 t)}$$

$$2R_c$$

where $\alpha_x = \alpha_y = \alpha$: the subscripts $x$ and $y$ are used for discriminating the motion components associated with the altitude and the range directions, respectively. We assume $(x_c - vt)/R_c << 1$ and $\alpha/R_c << 1$ in deriving Eq.2. and $R_c$ is the distance between the SAR and the center of the antenna footprint (i.e., $R_c \approx h^2 + y_c^2$).

The trochoidal wave train is assumed to propagate through a field of scatterers with a distributed cross section $\sigma(x_c, y_c)$ [LeBlond and Hysak, 1980]. Since the phase history is given by $\exp[2iKR(x_c, y_c, t)]$ ($K$: radar wavenumber), the SAR signal processing gives a two-dimensional image intensity $I(x, y)$ as follows:

$$I(x, y) = J |F_x(y, y_c)|^2 \cdot J \sigma(x_c, y_c) |F_y(x, x_c)|^2 \, dx_c \, dy_c$$

where $F_x$ and $F_y$ are the azimuthal and range response functions of a moving point centered at $(x_c, y_c, 0)$. These functions depend on ocean wave parameters such as wave amplitude $\alpha$, wave frequency $\omega_0$, propagation direction $\theta$, and on radar system parameters such as radar wavenumber $K$, off-nadir angle of the antenna $\delta_c$, and azimuthal and range resolutions $\epsilon_x$ and $\epsilon_y$. Since we neglect the cross section modulation, $\sigma(x_c, y_c)$ is assumed to be unity for the following discussions.

Fig.2 shows an example of SAR images representing a region of $\pm 1.0$ km in azimuth and range directions. For the calculations, the SEASAT SAR parameters are chosen [Jordan, 1980]: the resolutions $\epsilon_x$ and $\epsilon_y$ are fixed at 25 m and the radar wavelength is chosen as 0.25 m (L-band). The SAR moves at an altitude of 800 km with a speed of 8 km s$^{-1}$. The ocean wave is assumed to be a swell of 250 m in wavelength $L_c$ ($L_c = 2\pi g/\omega_0^2$). The figure shows a formation of wave-like patterns even for a uniform radar scattering cross section. This is due to the velocity bunching as was noted by several authors [Elachi and Brown, 1977; Alpers and Rufench, 1979; Swift and Wilson, 1979]. Parametric calculations are conducted as a function of wave propagation direction $\theta$, off-nadir angle $\delta_c$, and a normalized wave amplitude $\zeta = \alpha/L_c$ in the following section.

3. DISCUSSIONS AND CONCLUDING REMARKS

Since three dimensions are considered in our model, we can examine the effects of ocean and radar parameters on SAR images in a more realistic framework.

Fig.3 illustrates the maximum azimuthal shift of scatterers in image plane as a function of the radar off-nadir angle $\delta_c$ and of the ocean wave direction $\theta$. The shift is normalized by $\lambda_c \cos \theta$ an equivalent azimuthal wavelength of the ocean wave. The shift increases
Fig. 1. Geometry of synthetic aperture radar imagery.

Fig. 2. Examples of SAR image calculated for different amplitude of the ocean wave.

Fig. 3. Dependence of the maximum azimuthal image shifts on (a) the radar off-nadir angle $\delta_0$ and (b) the wave direction $\theta$.

Fig. 4. Dependence of SAR image contrast on the amplitude $\zeta$ and the wave direction $\theta$. 

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as the off-nadir angle increases, as shown in Fig. 3(a). This is mostly
due to the y-component of motions, and the z-component produces a
constant image shift which is determined only by the normalized wave
amplitude \( \zeta \). The dependence on the wave direction \( \theta \) is illustrated in
Fig. 3(b). Although the actual shift by the y-component increases with
\( \theta \), the equivalent wavelength \( L(\theta) \) also increases and the normalized
shift becoming a maximum at \( \theta = 45^\circ \). Fig. 3 shows that the amount of
shifts is generally within a half of the equivalent wavelength \( L(\theta) \),
but it is large enough compared with the spatial resolutions of the SAR
considered.

Fig. 4 shows the dependence of SAR image intensity on the amplitude
and direction of the ocean wave. The ordinate is a contrast ratio which is
defined as an intensity ratio of the minimum to the maximum of the
calculated SAR images. While the azimuthal shifts are proportional to
the wave amplitude \( \zeta \), the image contrast is determined by the degree of
scatterer's bunching. Therefore, the figure shows a strong contrast by
\( \sim 10 \) dB when the wave amplitude \( \zeta \) is relatively small or when the
direction of ocean wave \( \theta \) approaches the right angle.

In the present work, we have emphasized the effects of trochoidal
wave motions on SAR imaging formation with a more realistic three-
dimensional framework. SAR images are calculated as a function of
actual ocean and radar parameters. The image contrast is estimated to
be several to 10 dB, which is comparable to the value expected by a
cross section modulation due to the long waves [Elachi and Brown, 1977].
This modulation effect dominates for ocean waves propagating
perpendicular to the SAR flight direction. However the azimuthal shifts
are several times larger than the spatial resolutions of the SAR as was
shown in Fig. 3. it is relatively simple to take into account the
modulation effect through the radar cross section \( \sigma(x, y, z) \) for the
present analytic framework.

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