THE MMP-METHOD APPLIED TO 3-D SCATTERING PROBLEMS

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ABSTRACT

In this paper the MMP-method [1,2,3] is applied to three-dimensional scattering problems. Some simple rules are presented, which allow to choose an appropriate MMP-expansion for a given problem, i.e. to set the poles. An example shows the merits of this technique.

INTRODUCTION

Many scattering problems consist of different domains with linear, homogeneous and isotropic materials. If one supposes a time-harmonic incident field, one has to solve the homogeneous, complex vector Helmholtz equation to get the scattered field of each domain.

The solution of the Helmholtz equation leads to the well known multipole expansion. The MMP-method is an extension of this multipole expansion. In spherical coordinates the MMP-expansion for each domain has the form

\[ \hat{f}_{\text{MMP}} = \sum_{J=1}^{J} \sum_{N=1}^{N} \sum_{M=0}^{M} \hat{B}_{n}^{(J)}(kr) \hat{B}_{n}^{(J)}(kr) \}

\[ \cos \phi_{j} \cdot \sin \phi_{j} \]

\[ a_{nmj} \cdot \hat{p}_{n}^{m}(\cos \theta_{j}) \]

with

- \( \hat{B}_{n}^{(J)} \): Riccati-Bessel function of the first, second or third kind
- \( \hat{p}_{n}^{m} \): associated Legendre function of the first kind
- \( k \): wave constant

For further details and the derivation of equation (1) see [3,4]. The common multipole expansion has the same form as equation (1), but with \( J=1 \).

To show the advantage of the MMP-method, in Fig. 1 the error in the matching-points of a finite cylinder is shown for different MMP-expansions. The total number of coefficients is always the same, i.e. the calculation effort is the same. It is easy to see that the MMP-expansion allows an adaption to the shape of the body.
Fig. 1a, J=1  Fig. 1b, J=3  Fig. 1c, J=7

Fig. 1: Error on the boundary of a finite cylinder for various MMP-expansions

CHOICE OF POLES

A good choice of the locations of the poles still needs some experience, but there are some rules which allow to set the multipoles straightforward. A computer program which uses these strategies is in development. The rules are based on the behaviour of poles. One multipole expansion in the centre of a sphere is optimal for this sphere. This gives the simple idea to build up a more complicated body by spheres, as depicted in Fig. 2.
If the poles are too close together, one can get numerical problems, i.e., the poles become dependent. The highest order of each expansion is related to the density of the matching points. A more detailed consideration of the dependencies between different poles [3] leads to the rule:

A body is built up by spheres. The center of each sphere must lie outside the other spheres. The highest orders $N_j$ and $M_j$, resp. must be smaller than $\alpha_{\text{max}}$. Where $\alpha_{\text{max}}$ is the largest view angle.

**RESULTS**

As an example of 3-D scattering the Poynting vectors near two finite cylinders are shown. The conductivity of the two cylinders is supposed to be infinite. Fig. 3 shows the configuration and Fig. 4 the results of the computation. One sees that there is an interaction between the two cylinders.

![Diagram of cylinders and Poynting vectors](image)

- h = 180 cm
- d = 25 cm
- a = 100 cm
- $\theta = 60^\circ$
- $f = 150 \text{ MHz}$

Fig. 3: Configuration
Fig. 4: Real part of the Poynting vector near two finite cylinders. E-plane.

REFERENCES

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