NUMERICAL ANALYSIS OF $TE_0$ DIELECTRIC ROD RESONATORS
PLACED BETWEEN TWO PARALLEL CONDUCTOR PLATES

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Introduction

Recently S. A. Long, et al. [1] have shown the possibility of using dielectric resonators as compact antenna elements in microwave and millimeter wave region. Theoretical studies of such resonators, which are usually placed in a free space or on a conducting ground plane, have been reported by many authors [2]-[6].

In this paper, for dielectric rod resonators placed between two parallel conductor plates, resonant frequencies and Q-factors of the $TE_0$ modes are numerically analyzed by introducing complex frequency into characteristic equations derived on the basis of the mode matching technique. Influence of a distance between the conductor plates on the resonant frequencies and the Q-factors is discussed from the computed results. Validity of the theory is verified by experiments.

Analysis

Dielectric rod resonators of complex relative permittivity $\varepsilon_r(1-j \tan \delta)$ and diameter D are placed between two parallel, infinitely large lossless conductor plates, as shown in Fig. 1. A time factor $e^{j\omega t}$ is assumed, where $\omega = \omega_1 + j\omega_2$ ($f_1 = \omega_1 / 2\pi$, $Q_r = \omega_1 / 2\omega_2$) is the complex angular frequency. The $TE_0$ modes only are treated. From the structural symmetry in Fig. 1(b) the resonant modes can be classified into those for which T-plane ($r\theta$-plane at $z=0$) is an electric wall and the others for which it is a magnetic wall. The former case also corresponds to Fig. 1(a). The resonator is divided into homogeneous subregion I, II, and III as in Fig. 1(b). The electromagnetic fields in each subregion are expanded in eigen modes which satisfy the boundary conditions on the conductor surface and the T-plane. Then imposing the boundary condition at the interfaces of the subregions and applying the orthogonality of the eigen modes, we get the homogeneous equations for the expansion coefficients. The resonant frequencies are determined by the condition that the determinant of the coefficient matrix vanishes [7]; that is,

$$\det H(\tilde{\omega}; \varepsilon_r, D, L, M) = 0$$  \hspace{1cm} (1)

where the matrix element $H_{qp}$ ($q, p=1, 2, \ldots, N$) is given by
\[
\begin{align*}
\eta_p &= \frac{\dot{x}_p \cot \dot{x}_p - z_q \cot z_q}{\dot{x}_p \tan \dot{x}_p - z_q \tan z_q} \left[ J_1(\dot{u}_p) - \frac{H_1(\dot{v}_q)}{\dot{u}_p J_0(\dot{u}_p)} - \frac{\dot{v}_q H_0(\dot{v}_q)}{\dot{u}_p J_0(\dot{u}_p)} \right] \\
&\quad \times \frac{1}{[\dot{x}_p^2 - z_q^2] \left[ (\dot{x}_p / M)^2 - (z_q / L)^2 \right]} \quad \text{(2)}
\end{align*}
\]

\[
\begin{align*}
\dot{u}_p &= \sqrt{\left( \frac{\omega R}{c} \right)^2 \frac{x}{r} - \left( \frac{R}{L} \right)^2}, \quad \dot{v}_q = \pm \sqrt{\left( \frac{\omega R}{c} \right)^2 - \left( \frac{R}{L} q \right)^2}, \quad \text{(3)}
\end{align*}
\]

\[
Z_q = \{ q \pi L / h, (2q - 1) \pi L / 2h \} \quad \text{(4)}
\]

In the above, the upper and lower (or the first and second) expressions in \{ \} correspond to the electric and magnetic T-plane modes, respectively. Also \( c \) is the light velocity in a vacuum, \( J_n(x) \) the Bessel function of the first kind, and \( H_n(x) \) the Hankel function of the second kind. Furthermore, \( (\dot{x}_p, \dot{v}_q) \) is given as the \( p' \)th solution of the following simultaneous equations:

\[
\begin{align*}
-\dot{x}_p \cot \dot{x}_p &= \frac{L}{M} \dot{v}_q \cot \dot{v}_q, \\
\dot{x}_p^2 - \dot{v}_q^2 &= \left( \frac{\omega R}{c} \right)^2 (\epsilon - 1) \quad \text{(5)}
\end{align*}
\]

Then, putting \( \dot{v}_q = v_{q1} + jv_{q2} \), we get

\[
(\omega_1 \omega_2 / c^2) - (v_{q1} v_{q2} / R^2) = 0 \quad \text{(6)}
\]

from the imaginary part of the second of (3). Since \( \omega_1 \) and \( \omega_2 \) are both positive for a damped free-oscillation, \( v_{q1} \) and \( v_{q2} \) are both positive or both negative, as seen from (6); that is, the \( v_q \) values exist in either the first or the third quadrant of a complex \( v \) plane. Then the fields behave as

\[
\begin{align*}
\dot{e}_q &= e^{j(\omega_1 t - v_{q1} \tau)} e^{-\omega_2 t} v_{q2} \dot{r} \quad \text{and} \quad \dot{r} = \frac{R}{\omega_2} \quad \text{(7)}
\end{align*}
\]

Some discussions of (7) yield the following result: the field having \( v_q \) in the first quadrant is in a leaky state; a part of energy \( q \) leaks away from the resonator in the radial direction, while one having \( v_q \) in the third quadrant is in a trapped state; the energy is trapped in and near the rod [8]. Thus, for \( v_q \) in (3), we choose the positive sign when \( (h / q, 2h / (2q - 1)) > \lambda_0 / 2 \) and the negative sign when \( (h / q, 2h / (2q - 1)) < \lambda_0 / 2 \), where \( \lambda_0 \) is the resonant wavelength.

When \( (h, 2h) < \lambda_0 / 2 \), \( v_q \) for any \( q \) is always in the third quadrant; the resonant mode is \( q \) in the trapped state. Particularly, the case of \( (h, 2h) = \lambda_0 / 2 \) represents a cutoff of the trapped state. On the other hand, when \( (h, 2h) > \lambda_0 / 2 \), the resonant mode is in the leaky state [7], because at least one of the \( v_q \) values is in the first quadrant.

**Computation and Experiments**

The computations and experiments were performed using a (Zr,Sn)TiO ceramic rod (MURATA MFG. CO., LTD.) with \( \varepsilon_r = 37.43 \) and \( \tan \delta = (0.255 + 0.170 f_{GHz}) \times 10^{-4} \) measured. For the TE01 (1+\( \delta \))/2 mode of the image type resonator in Fig. 1(a), the complex frequencies versus the distance \( M \) were computed from (1). Fig. 2
shows locations of $v'$ in the complex $v$ plane. At $M=10\ mm$, all of $v'$s are in the trapped state, and the number of $v'$ in the leaky state increase with $M$. Fig. 3 shows the convergence of the solutions versus the number $N$ of determinant. The solution of three figures is obtained if $N=15$. These computed results are shown in Fig. 4 by solid lines. Similarly, Fig. 5 shows one for the $TE_{016}$ mode of the open type resonator in Fig. 1(b). Broken lines in the figures show the cutoffs. The left-hand side of the cutoff is the trapped state region while the right-hand side is the leaky state region. It is seen in Fig. 4 that a gentle peak of $Q_f$ appears near the cutoff of $v^2$.

In addition the measured values of the resonant frequency $f_0$ and the unloaded $Q$, $Q_u$ by the swept-frequency method are indicated by dots in the figures. In both cases, the $f_1$ curves agree very well with the $f_0$ values. The $Q_f$ curves in the trapped state, are greater than the $Q_u$ values since the conductor loss is not considered in the analysis of $Q_f$, while the $Q_f$ curves in the leaky state agree well to the $Q_u$ values since the radiation loss is predominant.

**Conclusion**

Validity of this analysis was verified by good agreement between the computed and experimental results. The $Q_f$ values by this method are somewhat lower than ones presented by Tsuji, et al [4],[5]. If a proper technique exciting the $TE_{016}$ mode is used, it is possible to apply this resonator as a compact, omnidirectional antenna with a horizontally polarized wave.

**References**


Fig. 1. Dielectric rod resonators placed between two parallel conductor plates: (a) image type, (b) open type.
Fig. 2. Locations of $\nu$, as $M$ is varied for a image type resonator in Fig. 4.

Fig. 3. Convergence of complex frequency ($f_1, Q_e$) for a image type resonator in Fig. 4.

Fig. 4. Computed results of complex frequency ($f_1, Q_e$) and measured values ($f_0, Q_i$) for the TE01 (1+6)/2 mode of the image type resonator.

Fig. 5. Computed results of complex frequency ($f_1, Q_e$) and measured values ($f_0, Q_i$) for the TE016 mode of the open type resonator.