FINITE ELEMENT ANALYSIS OF EFFECTS OF OBSTACLES ON CUTOFF FREQUENCY AND FIELD IN ARCHED TUNNEL AS WAVEGUIDE

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Abstract: Effects of trains in an arched tunnel on the cutoff frequency and field were determined at the range of VHF, UHF, and SHF bands by the finite-element method. According to this study, the tunnel is a transmission channel of high-pass type waveguide. The tunnel and the trains were assumed to be infinitely long and fully conductive. Generally speaking, the trains in the tunnel lowered TE_{nm} wave cutoff frequencies and raised TM_{nm} cutoff frequencies.

The field is represented by contour lines. Thus, its change is clearly shown by a change in the distribution of the lines caused by the train in the tunnel. Although the train changed field distribution for both TE_{nm} and TM_{nm} mode, greater changes were usually observed in higher order mode fields.

I. INTRODUCTION

Secure communications are a prerequisite in tunnels, underground markets, and mines, including coal mines. Recently, an international conference on communications in tunnels [1] was held to enhance public safety and prevent disasters. In underground environments, communicability is vital in emergency cases such as fires, as well as in routine management operations [2]. This study aims at establishing a basis for solving such problems.

Everyone has experienced that radio broadcasting cannot be heard in the tunnel. However, when a certain relationship exists between the cross section of the tunnel and the free-space wavelength of the electromagnetic wave, the wave propagates through the tunnel, thus permitting radio communications that utilize the tunnel as a waveguide.

II. THEORY

Fig. 1 shows a model for trains in a tunnel. The tunnel and the trains are both assumed to be straight and infinitely long, and the tunnel wall and the trains are assumed to be perfect conductors. The cross section of the tunnel is taken as the X-Y plane and the direction of radio wave transmission as the Z axis.

The internal region of the tunnel is expressed as G and the boundary of G (the walls of the tunnel and the trains) as Γ. With this
notation, the $\text{TEnm}$ modes and the $\text{TMnm}$ modes are expressed by the following Helmholtz equation:

i) $\text{TE}_{nm}$ modes

$$\nabla^2 H_z + k^2 H_z = 0, \quad \text{in } G$$
$$\frac{\partial H_z}{\partial n} = 0, \quad \text{on } \Gamma \quad (1)$$

ii) $\text{TM}_{nm}$ modes

$$\nabla^2 E_z + k^2 E_z = 0, \quad \text{in } G$$
$$E_z = 0, \quad \text{on } \Gamma \quad (2)$$

where $H_z$ is the $z$ component of the magnetic field, $E_z$ is the $z$ component of the electric field, $k$ is the cutoff wave number, and $\partial / \partial n$ is the partial differentiation along a normal line against the boundary $\Gamma$.

The functional $I[\phi]$ corresponding to the above Helmholtz equation is expressed as follows:

$$I[\phi] = \iint_G \left\{ \left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 \right\} \, dx \, dy - k^2 \iint_G \phi^2 \, dx \, dy \quad (3)$$

where $\phi = H_z(\text{or } E_z)$.

This function is solved for each set of boundary conditions by applying the finite-element method [3],[4].

The elements used are triangular ones on which $\phi$ is approximated as quadratic function.

If each eigenvalue $(k)^2$ is determined, the cutoff frequency $f_c$ is obtained from the following formula:

$$f_c = k/2\sqrt{\varepsilon_0 \mu_0} .$$

$\phi(H_z \text{ or } E_z)$ is determined as an eigenvector of each eigenvalue.
III. NUMERICAL RESULTS

In this paper, the inside of the tunnel is divided into over 176 elements. Therefore, the relative errors for the TE_{11} mode and for up to the TM_{11} mode are less than 0.05 percent and 0.3 percent, respectively.

Fig. 2 shows cutoff frequency change by train height for TE_{11} mode and for TM_{01} mode. Figs. 3, 4, 5, 6 and 7 give the \( \phi \) distribution of the individual modes.

In these figures, the maximum value of \( \phi \) is specified as 1, and represents \( H_z \) for the TE_{11} mode and \( E_z \) for the TM_{01}, TM_{11} modes. The field distribution in the tunnel depends on the train, as shown in Figs. 3 - 7.

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**Fig. 2.** Cutoff frequency of the tunnel.

**Fig. 3.** The field pattern of the TE_{11} mode.

**Fig. 4.** The field pattern of the TE_{11} mode.
IV. CONCLUSION

This study analyzes the effects of trains in tunnels on the basis of certain assumptions, and elucidates part of the basic characteristics. The results have revealed that cutoff frequencies for ordinary empty tunnels are in the lower region of the VHF band. The details of other higher modes are shown colored slides. As a general rule, the trains in the tunnel lower the cutoff frequency for the TE_{nm} modes and raise that for the TM_{nm} modes.

Although the train changed field distribution for both TE_{nm} and TM_{nm} mode, greater changes were usually observed in higher order mode fields.

V. REFERENCES


