FREQUENCY CHARACTERIZATION
OF A THIN WIRE ANTENNA USING DIAKOPTIC THEORY

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ABSTRACT

A systematic approach using diakoptic antenna theory is presented which yields frequency dependent input impedance expression for a linear antenna. In diakoptic theory the antenna is modelled by an equivalent network which eliminates the need for an integral equation formulation. A frequency dependent impedance matrix results after expanding the free space Green's function into a power series in the wavenumber k. The unknown current vector in the impedance representation is assumed to be expanded in a power series in k. Grouping coefficients of the same power in k leads to a numerically efficient algorithm which is used to determine the input admittance as a function of frequency. Numerical results for the input impedance are compared with data obtained from a conventional integral equation solution.

INTRODUCTION

In modern communication systems the frequency characterization of radiating structures is needed for broadband operation. A system designer could make use of a frequency dependent expression for the antenna input impedance. Diakoptic theory [1] was used to obtain such an impedance function [2]. Here, the theory is extended to obtain an equivalent modified impedance expression via numerically efficient algorithm. Current is expanded into an M-th order polynomial function and the antenna is diakopted into N segments.

FORMULATION

A thin linear antenna is diakopted into N electrically short segments (Fig.1), where each segment is treated as a two port net-
work. 

\[ V' = Z' I' \]  

(1)

Here \( Z' \) is the impedance matrix which takes into account coupling between segments, \( V' \) is the voltage excitation vector and \( I' \) is the unknown current vector. The free space Green's function is expanded into the power series

\[ e^{-jkr} \sum_{m=0}^{\infty} \frac{(-jkr)^m}{m!} \]

(2)

Retaining \( M \) terms in (2) leads to a power series expansion in \( k \) for the impedance matrix

\[ Z' = k^{-1}C'_{-1} + kC'_{1} + k^2C'_{2} + \ldots + k^M C'_{M} \]

(3)

where \( k \) is an unspecified wavenumber and constant coefficient matrices \( C'_{m} \) are determined from assumed linear current distributions along the segments. The unknown vector \( I' \) in (1) is found by the following systematic procedure.

(a) The impedance matrix \( Z' \) in (3) is expanded around a particular value of \( k = k_{a} + \Delta k \). Terms are grouped according to like powers of \( \Delta k \).

(b) The current vector \( I' \) is assumed to be expanded in terms of \( \Delta k \)

\[ I' = I^{(0)} + (\Delta k)I^{(1)} + (\Delta k)^2I^{(2)} + \ldots + (\Delta k)^M I^{(M)} \]  

(4)
This assumption allows the solution to converge around the expansion point $k_a$.

(c) Results of (a) and (b) are substituted back into (1) and terms are ordered in like power of $(\delta k)$.

(d) Equating coefficients of like power of $(\delta k)$ results in a direct solution for the unknown vectors $I^{(0)}, I^{(1)}, \ldots, I^{(M)}$ in (4).

\[
(\delta k)^0: \quad I^{(0)}(\xi_1) = (\xi_1^{-1})^{-1} k_a v,
\]

\[
(\delta k): \quad I^{(1)}(\xi_1) = (\xi_1^{-1})^{-1} [v' - \xi_1^{-1} I^{(0)}(\xi_1)]
\]

\[
(\delta k)^2: \quad I^{(2)}(\xi_1) = - (\xi_1^{-1})^{-1} [\xi_1^{-1} I^{(1)}(\xi_1) + \xi_1^{-1} I^{(0)}(\xi_1)]
\]

\[
(\delta k)^M: \quad I^{(M)}(\xi_1) = - (\xi_1^{-1})^{-1} [\xi_1^{-1} I^{(M-1)}(\xi_1) + \xi_1^{-1} I^{(M-2)}(\xi_1) + \cdots + \xi_1^{-1} I^{(0)}(\xi_1)]
\]

where the first two of the modified coefficients are

\[
\xi_1^{-1} = \xi_1^{-1} + k_a^2 \xi_2^{-1} + k_a^3 \xi_3^{-1} + \cdots
\]

\[
\xi_0^{-1} = 2k_a \xi_1^{-1} + 3k_a^2 \xi_2^{-1} + 4k_a^3 \xi_3^{-1} + \cdots
\]

**NUMERICAL RESULTS**

A computer code was developed to consider up to $M=7$ terms in the expansion for a diakopted antenna consisting of up to $N = 8$ segments. Numerical results for an impedance function $Z(k) = R + jX$ are presented in Fig. 2, where the an integral equation solution [3] is used as a reference. Good agreement was observed in two lower frequency regions where the solution is expanded around $k_a L = 0$ and $k_a L = 0.95$. However, the expansion around $k_a L = 0.95$ diverged near $k_a L = 1.5$, the resonance region where $X$ approaches zero. The expansion around resonance $k_a L = 1.5$ has a narrow region of validity. For higher frequencies beyond resonance the expansion $k_a L = 2.25$ becomes less accurate because more segments have to be considered in the method. Currently, we are investigating
poles of such impedance functions. Lower order poles for a linear antenna seem to be in a good agreement with those obtained from the singularity expansion method.

Fig. 2. Input impedance $Z(k) = R + jX$ of a thin linear antenna of length $L = 0.5$ meters with a radius $a$ defined by $2 \ln(2L/a) = 10$. Curves represent expansions around $a = 0.95, 1.5$ and $2.25$.

REFERENCES

