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Characteristic Analysis on Double Side Spiral Resonator's Thickness Effect on Transmission Efficiency for Wireless Power Transmission

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Abstract Wireless power transmission using magnetic resonant coupling through the utilization of helical and spiral inductors offers the capability to potentially eliminate the "last wire" in power supply applications. In this paper, we will focus on double-side spiral resonators and especially on how their spacing affects the transfer efficiency between the two resonators. We fabricated double-side spiral resonators with different thicknesses and conducted measurement of the transmission efficiency-distance feature with two resonators connected to different ports of vector network analyzer, located in coaxial positions. We observed the measurement results that the thickness of double side spiral resonators affects the over coupled peak transmission efficiency, critical coupled transmission efficiency as well as the critical coupled distance between two resonators. We also conducted equivalent circuit analysis on the explanation of the thickness effect of double side resonator's transmission efficiency at different distance ranges.

Key words wireless power transmission, double side spiral resonator, thickness effect, equivalent circuit analysis

1. Introduction

Wireless power transmission using resonant coupling has been studied and successfully achieved theoretical transmission efficiency of 95%. The efficiency at a distance of eight times the resonator radius is 45% [1], [2]. Both helical and spiral resonators are adopted in wireless power transmission using resonant coupling. In literature [1], the phenomenon of electromagnetic resonant coupling is explained in detail. In a wireless power transmission system using electromagnetic resonant coupling, with the transmission distance increasing in over coupled range, where frequency splitting happened, the peak transmission can be maintained independent of distance if the correct frequency is chosen [3]. Two resonant frequencies which are corresponding to the peak efficiency can be observed. Two resonators are critically coupled when the two resonant frequencies combines into one, with the corresponding distance called critical coupled distance. The critical transmission efficiency would be almost the same with the peak efficiency of over coupled range. The under coupled distance range is defined as the distance range which is beyond the critical coupled distance, where a single resonant frequency corresponding to peak efficiency can be observed. In under coupled distance range, the peak transmission efficiency shows an apparently decreasing trend as the distance between two resonators increases [4], [5]. As mathematical analysis and simulation of resonators' coupling, it has become widely accepted to use a series resonating circuit as an equivalent circuit of a resonator to conduct analysis around the resonant frequency, including double side spiral resonators [4], [6], [7].

Recently, double-side spiral resonator has been focused on due to that it is feasible to fabricate spiral resonators with extremely thin thickness in millimeter range, making embedding resonators into electrical devices such as laptop, mobile phone more applicable than helical resonators. With this consideration, we also determined the dimension (size) of our resonators. In this paper, we exploit double side spiral resonators with different thicknesses, concentrating on how the thickness of double side spiral resonators affect the transmission efficiency at different distance ranges, during which both inductive coupling and electrical coupling have effect on power transmission procedure. Inductive coupling stands for the fact that two resonators are coupled by electromagnetic induction, by means of mutual inductance, while electrical coupling means that energy is transmitted by means of mutual capacitance between two resonators.

Our contributions in this paper are summarized as follows.

First, we show measurement results of peak transmission efficiency of double-side spiral resonators which were fabricated with different thicknesses. Second, we clarify how the thickness of spiral resonators influences the transmission efficiency in both short and long distance ranges, focusing on the thickness effect on over coupled peak transmission efficiency, critical coupled transmission efficiency and critical coupled distance. Third, we include a discussion on explanation of the thickness effect on peak transmission efficiency-distance feature by conducting equivalent circuit analysis.

2. Measurement Setup

In this section, we introduce the structure of resonators we fabricated and the experimental system including resonator deployment.

We have fabricated double-side spiral resonators as shown in Fig.1. The resonator is formed by two pieces of spiral coil with styrene foam of certain thickness t between each other. SMA port was used to connect two coils' largest turns. The pitch between adjacent turns of spiral coil is 5 mm, which is also the distance between the largest turn and the edge of foam. The copper wire with a cross section's diameter of 1.12 mm was used. Following the above fashion, we fabricated three prototypes of the double-side resonators with the only difference of the spacing (thickness) t between the two coils. The thickness t of one pattern is 10 mm, while the other two's thicknesses are 25 mm and 50 mm. The resonators' diameter of all three patterns is the same, which is $d = 200$ mm. For simplicity, we call the pattern with t of 10 mm as Low pattern, the pattern with t of 25 mm as Middle pattern and another one as High pattern. We connected single resonator to vector network analyzer (VAN) and adjusted the length of copper wire on both sides, while we were observing the smith chart of S_{11} by VNA. By doing this, we tuned the resonant frequency of all three patterns to 13.56 MHz.

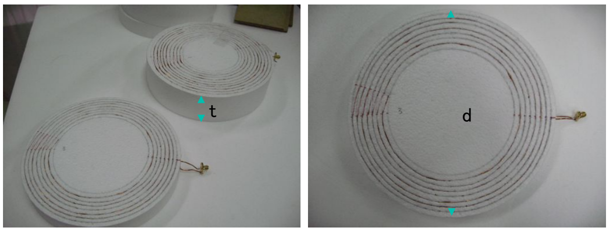

Figure 1 Double-side spiral resonator's structure

We measured the parasitic resistance (R), self inductance (L) and self capacitance (C) of each pattern with the RLC meter function of vector network analyzer (VNA). According to the measurement results, we also calculated the quality factor (Q) of each pattern according to Equation 1. Table 1 shows the average results.

$$Q = \frac{2\pi f_0 L}{R} \tag{1}$$

Table 1 Electromagnetic parameters of each resonator pattern

Pattern	R (Ohm)	L (μH)	C (pF)	f_0 (MHz)	Q
Low	1.20	11.51	11.97	13.56	816.91
Middle	1.80	20.38	6.79	13.56	960.72
High	2.76	33.06	4.17	13.56	1020.62

Figure 2 The experimental system architecture.

We conducted three sets of measurements (Low-Low [L-L], Middle-Middle [M-M] and High-High [H-H]) utilizing the one double-side coil as the power transmitter and the other coil as the power receiver modifying the spacing of the two coils in a setup as shown in Fig.2. In each set of measurement, both the transmitting resonator and receiving resonator were of the same pattern.

3. Measurement Results

In this section, we present the peak transmission efficiency-distance distance performance of the three geometries, the "Low (L)" pattern, the "Middle (M)" patter and the "High (H)" pattern.

As shown in Fig.3, we conducted measurement at six distance ranges while connecting two resonators of the same pattern to vector network analyzer (VNA). Fig.3 shows the S_{21} with sweeping frequency from 0.1 MHz to 30 MHz. The thinner resonators sets are critically coupled at the distance where the thicker resonator sets are still over coupled with two different resonant frequencies. We extract the peak transmission efficiency of each set at different distances and show the results in Fig.4.

As we can see from Fig.3 and Fig.4, we summarize the thickness effect of double side spiral resonators on transmission efficiency as follows. There are totally three findings about the thickness effect. First, the critical transmission efficiency of thinner resonators is higher than that of thicker ones. Second, the peak transmission efficiency of thinner resonators in over coupled range is higher than that of thicker resonators. Third, the critical coupled distance of thicker resonators is farther than that of thinner ones.

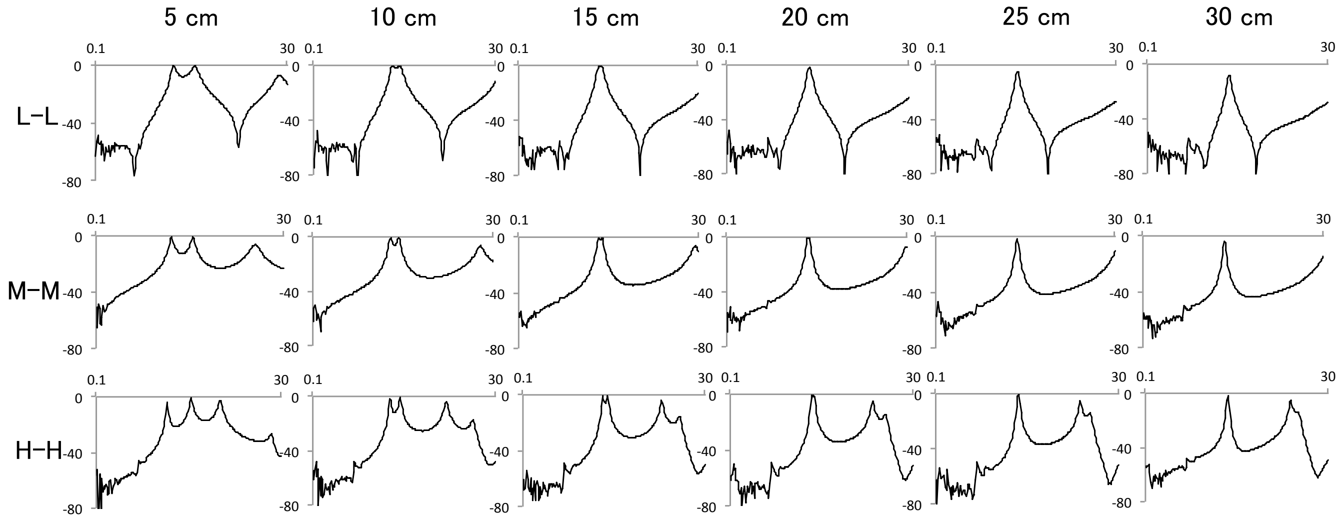

Figure 3 Transmission efficiency-distance features of three sets of measurement.

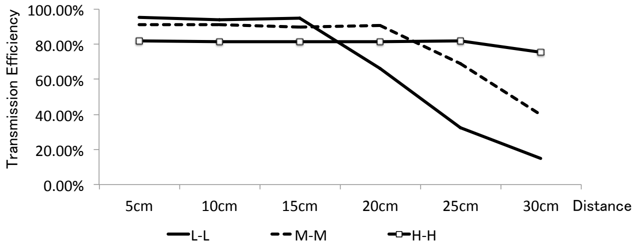

Figure 4 The peak transmission efficiency at different distance ranges.

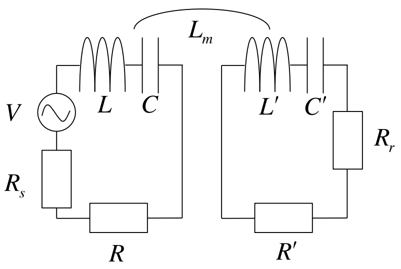

Figure 5 Equivalent circuit of two double side spiral resonators.

4. Equivalent Circuit Analysis

In this section, we conduct equivalent circuit analysis of double side spiral resonators with different thicknesses. By doing this, we include discussion on the explanation for the thickness effect which we summarized in previous section.

As we have mentioned in previous section, we utilized a series resonating circuit as the equivalent circuit of single double side spiral resonators. Thus we can conclude the circuit as shown in Fig.5 as the equivalent circuit of two coupled resonators.

In our analysis, the relationship between parameters of two resonators are shown in Equation 2 - 6. Since we utilized VNA to conduct the measurement, the R_s and R_r are the same, 50 Ω as shown in Equation 2. In each set of measurement, we utilized two resonators of the same pattern, resulting that the parasitic resistance, self inductance and self capacitance of the two resonators are the same, as shown in Equation 3, 4 and 5. The k in Equation 6 is coupling coefficient between two resonators.

$$R_s = R_r = 50\Omega \tag{2}$$

$$L = L' \tag{3}$$

$$C = C' \tag{4}$$

$$R = R' \tag{5}$$

$$L_M = k\sqrt{LL'} = kL \tag{6}$$

4.1 Analysis on critical transmission efficiency

From the equivalent circuit model in Fig.5, S_{21} can be derived as shown in Equation 7 [3]. By using the relationship of R_s and R_r in Equation 2, the transmission equation 8 can be derived, which can only be used to conduct analysis around the critical coupled resonant frequency, which was 13.56 MHz in our experiments.

$$S_{21} = 2\frac{V_r}{V}(\frac{R_s}{R_r})^{1/2} \tag{7}$$

The Z_0 in Equation 8 stands for R_s and R_r, which are both 50 Ω. Equation 9 shows the S_{21} at the resonant frequency ω_0, which was 13.56 MHz in our experiment. The derivative as shown in Equation 10 is taken with the respect to L_m. By solving Equation 10, the mutual inductance L_m under critical coupling condition could be calculated as shown in Equation 11. The L_m is only determined by the distance between two resonators in our experiment. The transmission distance corresponding to $L_{m_{critical}}$ and $k_{critical}$ is the critical coupled distance of two resonators, from which we could derive critical coupling coefficient $k_{critical}$ in Equation 12.

$$S_{21}(\omega) = \frac{2Z_0 j\omega L_m}{(Z_0 + R + j\omega L - \frac{j}{\omega C})^2 + \omega^2 {L_m}^2} \tag{8}$$

$$S_{21}(\omega_0) = \frac{2Z_0 j\omega_0 L_m}{(Z_0 + R)^2 + {\omega_0}^2 {L_m}^2} \tag{9}$$

$$\frac{\partial |S_{21}(\omega_0)|}{\partial L_m} = 0 \tag{10}$$

$$L_{m_{critical}} = \frac{Z_0 + R}{\omega_0} \tag{11}$$

$$k_{critical} = \frac{Z_0 + R}{\omega_0 L} \tag{12}$$

By Equation 9 and 11, we could calculate the critical coupling S_{21} as shown in Equation 13. And the transmission efficiency $\eta_{critical}$ can be calculated in Equation 13. Since the parasitic resistance R of thicker resonators is larger than that of thinner ones as shown in Table 1, the critical transmission efficiency of thicker ones is lower than that of thinner ones according to Equation 13. Now we have conducted the equivalent circuit analysis and explanation on measurement result that the critical transmission efficiency of thinner resonators is higher than that of thicker ones, which is the first finding of the thickness effect in previous section.

$$S_{21_{critical}} = \frac{jZ_0}{Z_0 + R} \tag{13}$$

$$\eta_{critical} = |S_{21_{critical}}|^2 \times 100\% = |\frac{jZ_0}{Z_0 + R}|^2 \times 100\% \tag{14}$$

4. 2 Analysis on over coupled peak efficiency

In-equation 15 shows the condition when the resonators are over coupled. Under this condition, the derivative with the respect to ω can be conducted as shown in Equation 16. We acquired the two frequencies, ω_1 and ω_2, corresponding to two splitting peaks of transmission efficiency in Equation 16, 17 [8]. As well, we measured the coupling coefficient k of three sets resonators at different distance ranges and show the results in Fig.6. We can then calculate L_m of each set at different distances by Equation 6. By adopting Equation 6, 17, 18 and the coupling coefficient k in Fig.6, we calculated the peak efficiency of three sets in over coupled distance range using Equation 8. We also calculated the under coupled efficiency of three sets by using Equation 6 and 9 as illustrated in Fig.7. This explains the measurement result that the peak transmission efficiency of thinner resonators in over coupled range is higher than that of thicker resonators, which is our second finding of the thickness effect in previous section.

$$L_m > L_{m_{critical}} \tag{15}$$

$$\frac{\partial |S_{21}(\omega)|}{\partial \omega} = 0 \tag{16}$$

$$\omega_1 = \sqrt{\frac{2L - CZ_0^2 - \sqrt{4L_m^2 + Z_0^4 C^2 - 4LCZ_0^2}}{2(L^2 - L_m^2)C}} \tag{17}$$

$$\omega_2 = \sqrt{\frac{2L - CZ_0^2 + \sqrt{4L_m^2 + Z_0^4 C^2 - 4LCZ_0^2}}{2(L^2 - L_m^2)C}} \tag{18}$$

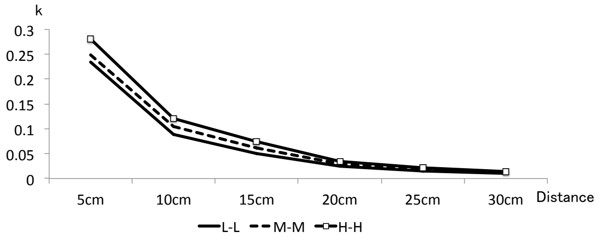

Figure 6 Coupling coefficient k of each set resonators at different transmission distances.

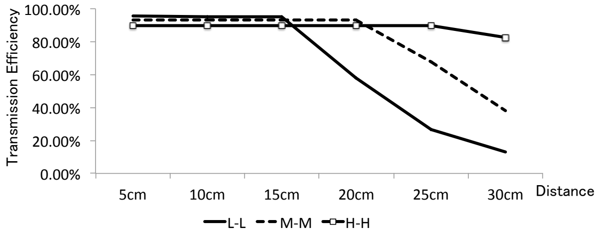

Figure 7 Calculated peak efficiencies in over coupled range.

4. 3 Analysis on critical coupled distance

From Equation 12, by using the parameters shown in Table 1, we could acquire the relationship of each set's $k_{critical}$ as in-equation 19. As shown in Fig.6, at each transmission distance, the relationship of three sets' coupling coefficient k is in-equation 20. By comparing in-equation 19 and 20, we could explain why the critical coupled distance of thicker resonators is farther than that of thinner ones.

$$k_{critical_{LL}} > k_{critical_{MM}} > k_{critical_{HH}} \tag{19}$$

$$k_{LL_{samedistance}} < k_{MM_{samedistance}} < k_{HH_{samedistance}} \tag{20}$$

5. Conclusion

In this paper, we made comparison between three patterns of spiral resonators in their transmission efficiency-distance features. In order to acquire how the thickness of double side spiral resonators affects the transmission efficiency, we fabricated three patterns of resonators, which were Low pattern, Middle pattern and High pattern. We accomplished three sets of measurement and acquired similarity and difference among their performances in both short distance and long distance ranges, which are called as over coupled range and under coupled range in this paper. The measurement results showed that spiral resonators of Low, Middle and High patterns with different thicknesses provided highest transmission efficiency in different distance ranges, which we summarized in three features. In order to explain transmission efficiency-distance features of spiral resonators, we conducted equivalent circuit analysis on all three patterns of resonators, acquiring mathematical analysis and explanation for all three features we figured out in the measurement result section. Our findings and explanations give inspiration to resonator fabrication. With transmission distance and resonator size determined, we should fabricate the resonator with the thickness making its critical coupled distance equal to the determined transmission distance.

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